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Quantitative Methods for Business

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Quantitative Methods for Business,
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CHAPTER 1

Introduction

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This book is concerned with the use of quantitative methods to assist in decision making. It emphasizes not the methods themselves, but rather how they can contribute to better decisions. A variety of names exists for the body of knowledge involving quantitative approaches to decision making. Today, the terms most commonly used—*management science* (MS), *operations research* (OR), *decision science* and *business analytics*—are often used interchangeably.

The scientific management revolution of the early 1900s, initiated by Frederic W. Taylor, provided the foundation for the use of quantitative methods in management. However, modern research in the use of quantitative methods in decision making, for the most part, originated during the World War II period. At that time, teams of people with diverse specialties (e.g., mathematicians, engineers, and behavioral scientists) were formed to deal with strategic and tactical problems faced by the military. After the war, many of these team members continued their research into quantitative approaches to decision making.

Two developments that occurred during the post–World War II period led to the growth and use of quantitative methods in nonmilitary applications. First, continued research resulted in numerous methodological developments. Arguably the most notable of these developments was the discovery by George Dantzig, in 1947, of the simplex method for solving linear programming problems. At the same time these methodological developments were taking place, digital computers prompted a virtual explosion in computing power. Computers enabled practitioners to use the methodological advances to solve a large variety of problems. The computer technology explosion continues, and personal computers can now be used to solve problems larger than those solved on mainframe computers in the 1990s.

Q.M. *in* ACTION

*REVENUE MANAGEMENT AT AT&T PARK**

Imagine the difficult position Russ Stanley, Vice President of Ticket Services for the San Francisco Giants, found himself facing late in the 2010 baseball season. Prior to the season, his organization had adopted a dynamic approach to pricing its tickets similar to the model successfully pioneered by Thomas M. Cook and his operations research group at American Airlines. Stanley desparately wanted the Giants to clinch a playoff birth, but he didn't want the team to do so *too quickly*.

When dynamically pricing a good or service, an organization regularly reviews supply and demand of the product and uses operations research to determine if the price should be changed to reflect these conditions. As the scheduled takeoff date for a flight nears, the cost of a ticket increases if seats for the flight are relatively scarce. On the other hand, the airline discounts tickets for an approaching flight with relatively few ticketed passengers. Through the use of optimization to dynamically set ticket prices, American Airlines generates nearly $1 billion annually in incremental revenue.

The management team of the San Francisco Giants recognized similarities between their primary product (tickets to home games) and the primary product sold by airlines (tickets for flights) and adopted a similar revenue management system. If a particular Giants' game is appealing to fans, tickets sell quickly and demand far exceeds supply as the date of the game approaches; under these conditions fans will be willing to pay more and the Giants charge a premium for the ticket. Similarly, tickets for less attractive games are discounted to reflect relatively low demand by fans. This is why Stanley found himself in a quandary at the end of the 2010 baseball season. The Giants were in the middle of a tight pennant race with the San Diego Padres that effectively increased demand for tickets to Giants' games, and the team was actually scheduled to play the Padres in San Fransisco for the last three

(*continued*)

*Based on Peter Horner, "The Sabre Story," *OR/MS Today* (June 2000); Ken Belson, "Baseball Tickets Too Much? Check Back Tomorrow," *New York Times.com* (May 18, 2009); and Rob Gloster, "Giants Quadruple Price of Cheap Seats as Playoffs Drive Demand," *Bloomberg Businessweek* (September 30, 2010).

games of the season. While Stanley certainly wanted his club to win its division and reach the Major League Baseball playoffs, he also recognized that his team's revenues would be greatly enhanced if it didn't qualify for the playoffs until the last day of the season. "I guess financially it is better to go all the way down to the last game," Stanley said in a late season interview. "Our hearts are in our stomachs; we're pacing watching these games."

Does revenue management and operations research work? Today, virtually every airline uses some sort of revenue-management system, and the cruise, hotel, and car rental industries also now apply revenue-management methods. As for the Giants, Stanley said dynamic pricing provided a 7 to 8% increase in revenue per seat for Giants' home games during the 2010 season. Coincidentally, the Giants did win the National League West division on the last day of the season and ultimately won the World Series. Several professional sports franchises are now looking to the Giants' example and considering implementation of similar dynamic ticket-pricing systems.

To reinforce the applied nature of the text and to provide a better understanding of the variety of applications in which *quantitative methods* (Q.M.) have been used successfully, Q.M. in Action articles are presented throughout the text. Each Q.M. in Action article summarizes an application of quantitative methods in practice. The first Q.M. in Action, Revenue Management at AT&T Park, describes one of the most important applications of quantitative methods in the sports and entertainment industry.

1.1 Problem Solving and Decision Making

Problem solving can be defined as the process of identifying a difference between the actual and the desired state of affairs and then taking action to resolve this difference. For problems important enough to justify the time and effort of careful analysis, the problem-solving process involves the following seven steps:

1. Identify and define the problem.
2. Determine the set of alternative solutions.
3. Determine the criterion or criteria that will be used to evaluate the alternatives.
4. Evaluate the alternatives.
5. Choose an alternative.
6. Implement the selected alternative.
7. Evaluate the results to determine whether a satisfactory solution has been obtained.

Decision making is the term generally associated with the first five steps of the problem-solving process. Thus, the first step of decision making is to identify and define the problem. Decision making ends with the choosing of an alternative, which is the act of making the decision.

Let us consider the following example of the decision-making process. For the moment, assume you are currently unemployed and that you would like a position that will lead to a satisfying career. Suppose your job search results in offers from companies in Rochester, New York; Dallas, Texas; Greensboro, North Carolina; and Pittsburgh, Pennsylvania. Further suppose that it is unrealistic for you to decline all of these offers. Thus, the alternatives for your decision problem can be stated as follows:

1. Accept the position in Rochester.
2. Accept the position in Dallas.
3. Accept the position in Greensboro.
4. Accept the position in Pittsburgh.

The next step of the problem-solving process involves determining the criteria that will be used to evaluate the four alternatives. Obviously, the starting salary is a factor of some importance. If salary were the only criterion important to you, the alternative selected as "best" would be the one with the highest starting salary. Problems in which the objective is to find the best solution with respect to one criterion are referred to as **single-criterion decision problems**.

Suppose that you also conclude that the potential for advancement and the location of the job are two other criteria of major importance. Thus, the three criteria in your decision problem are starting salary, potential for advancement, and location. Problems that involve more than one criterion are referred to as **multicriteria decision problems**.

The next step of the decision-making process is to evaluate each of the alternatives with respect to each criterion. For example, evaluating each alternative relative to the starting salary criterion is done simply by recording the starting salary for each job alternative. However, evaluating each alternative with respect to the potential for advancement and the location of the job is more difficult because these evaluations are based primarily on subjective factors that are often difficult to quantify. Suppose for now that you decide to measure potential for advancement and job location by rating each of these criteria as poor, fair, average, good, or excellent. The data you compile are shown in Table 1.1.

You are now ready to make a choice from the available alternatives. What makes this choice phase so difficult is that the criteria are probably not all equally important, and no one alternative is "best" with regard to all criteria. When faced with a multicriteria decision problem, the third step in the decision-making process often includes an assessment of the relative importance of the criteria. Although we will present a method for dealing with situations like this one later in the text, for now let us suppose that after a careful evaluation of the data in Table 1.1, you decide to select alternative 3. Alternative 3 is thus referred to as the **decision**.

At this point in time, the decision-making process is complete. In summary, we see that this process involves five steps:

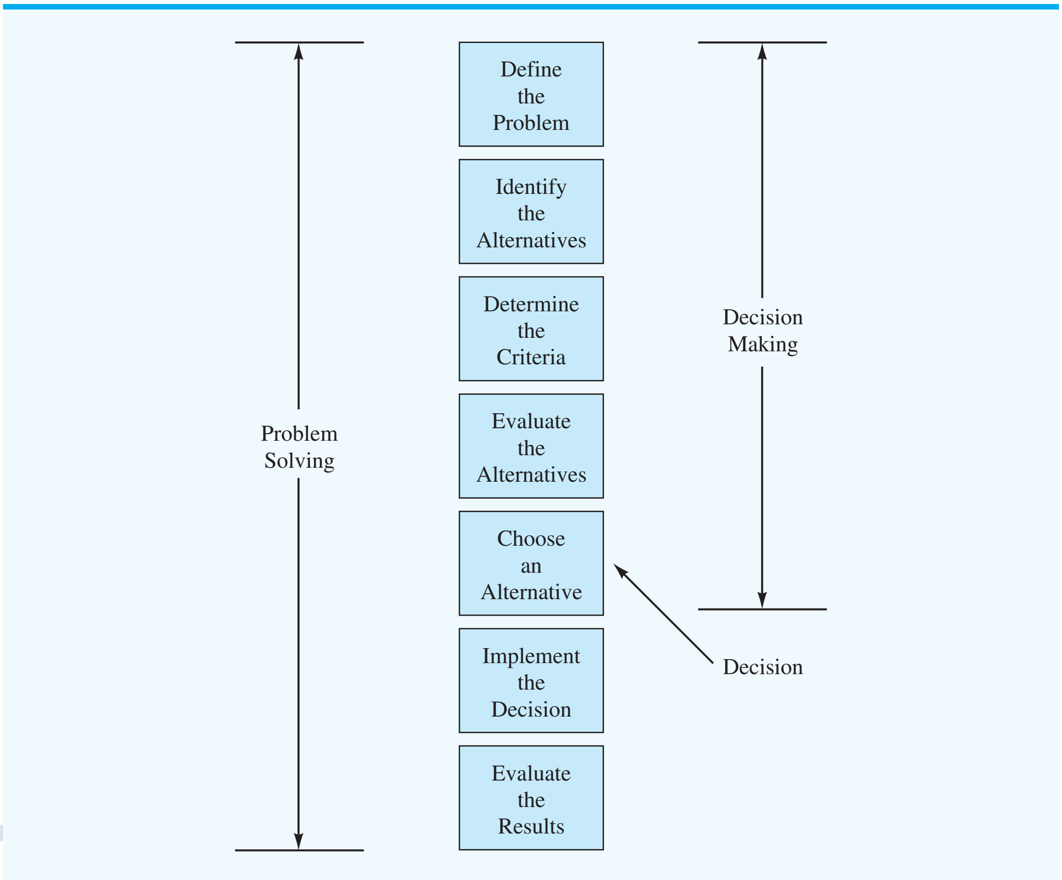
1. Define the problem.
2. Identify the alternatives.
3. Determine the criteria.
4. Evaluate the alternatives.
5. Choose an alternative.

Note that missing from this list are the last two steps in the problem-solving process: implementing the selected alternative and evaluating the results to determine whether a satisfactory solution has been obtained. This omission is not meant to diminish the importance

TABLE 1.1 DATA FOR THE JOB EVALUATION DECISION-MAKING PROBLEM

Alternative	Starting Salary	Potential for Advancement	Job Location
1. Rochester	$48,500	Average	Average
2. Dallas	$46,000	Excellent	Good
3. Greensboro	$46,000	Good	Excellent
4. Pittsburgh	$47,000	Average	Good

FIGURE 1.1 THE RELATIONSHIP BETWEEN PROBLEM SOLVING AND DECISION MAKING



of each of these activities, but to emphasize the more limited scope of the term *decision making* as compared to the term *problem solving*. Figure 1.1 summarizes the relationship between these two concepts.

1.2 Quantitative Analysis and Decision Making

Consider the flowchart presented in Figure 1.2. Note that we combined the first three steps of the decision-making process under the heading of "Structuring the Problem" and the latter two steps under the heading "Analyzing the Problem." Let us now consider in greater detail how to carry out the activities that make up the decision-making process.

Figure 1.3 shows that the analysis phase of the decision-making process may take two basic forms: qualitative and quantitative. Qualitative analysis is based primarily on the manager's judgment and experience; it includes the manager's intuitive "feel" for the problem and is more an art than a science. If the manager has had experience with similar problems, or if the problem is relatively simple, heavy emphasis may be placed upon a qualitative analysis. However, if the manager has had little experience with similar problems, or if the problem

FIGURE 1.2 A SUBCLASSIFICATION OF THE DECISION-MAKING PROCESS

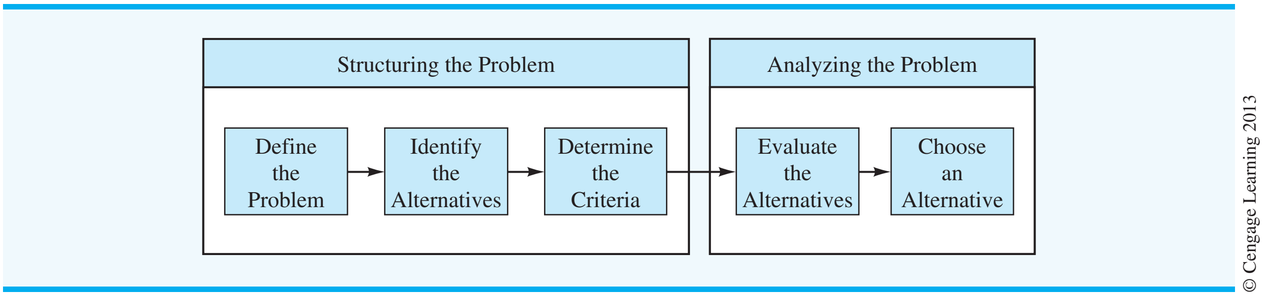
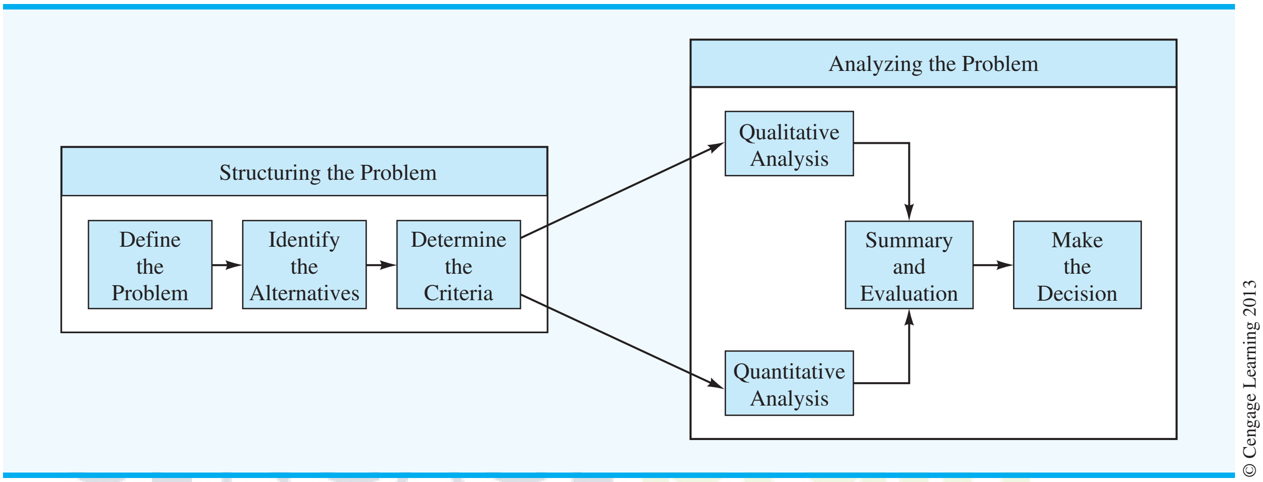


FIGURE 1.3 THE ROLE OF QUALITATIVE AND QUANTITATIVE ANALYSIS



Quantitative methods are especially helpful with large, complex problems. For example, in the coordination of the thousands of tasks associated with landing the Apollo 11 safely on the moon, quantitative techniques helped to ensure that more than 300,000 pieces of work performed by more than 400,000 people were integrated smoothly.

is sufficiently complex, then a quantitative analysis of the problem can be an especially important consideration in the manager's final decision.

When using a quantitative approach, an analyst will concentrate on the quantitative facts or data associated with the problem and develop mathematical expressions that describe the objectives, constraints, and other relationships that exist in the problem. Then, by using one or more mathematical methods, the analyst will make a recommendation based on the quantitative aspects of the problem.

Although skills in the qualitative approach are inherent in the manager and usually increase with experience, the skills of the quantitative approach can be learned only by studying the assumptions and methods of management science. A manager can increase decision-making effectiveness by learning more about quantitative methodology and by better understanding its contribution to the decision-making process. A manager who is knowledgeable in quantitative decision-making procedures is in a much better position to compare and evaluate the qualitative and quantitative sources of recommendations and ultimately to combine the two sources to make the best possible decision.

The box in Figure 1.3 entitled "Quantitative Analysis" encompasses most of the subject matter of this text. We will consider a managerial problem, introduce the appropriate quantitative methodology, and then develop the recommended decision.

Try Problem 4 to test your understanding of why quantitative approaches might be needed in a particular problem.

Some of the reasons why a quantitative approach might be used in the decision-making process include the following:

1. The problem is complex, and the manager cannot develop a good solution without the aid of quantitative analysis.
2. The problem is critical (e.g., a great deal of money is involved), and the manager desires a thorough analysis before making a decision.
3. The problem is new, and the manager has no previous experience from which to draw.
4. The problem is repetitive, and the manager saves time and effort by relying on quantitative procedures to automate routine decision recommendations.

1.3 Quantitative Analysis

From Figure 1.3 we see that quantitative analysis begins once the problem has been structured. It usually takes imagination, teamwork, and considerable effort to transform a rather general problem description into a well-defined problem that can be approached via quantitative analysis. It is important to involve the stakeholders (the decision maker, users of results, etc.) in the process of structuring the problem to improve the likelihood that the ensuing quantitative analysis will make an important contribution to the decision-making process. When those familiar with the problem agree that it has been adequately structured, work can begin on developing a model to represent the problem mathematically. Solution procedures can then be employed to find the best solution for the model. This best solution for the model then becomes a recommendation to the decision maker. The process of developing and solving models is the essence of the quantitative analysis process.

Model Development

Models are representations of real objects or situations and can be presented in various forms. For example, a scale model of an airplane is a representation of a real airplane. Similarly, a child's toy truck is a model of a real truck. The model airplane and toy truck are examples of models that are physical replicas of real objects. In modeling terminology, physical replicas are referred to as **iconic models**.

A second classification includes models that are physical in form but do not have the same physical appearance as the object being modeled. Such models are referred to as **analog models**. The speedometer of an automobile is an analog model; the position of the needle on the dial represents the speed of the automobile. A thermometer is another analog model representing temperature.

A third classification of models—the type we will primarily be studying—includes representations of a problem by a system of symbols and mathematical relationships or expressions. Such models are referred to as **mathematical models** and are a critical part of any quantitative approach to decision making. For example, the total profit from the sale of a product can be determined by multiplying the profit per unit by the quantity sold. Let x represent the number of units produced and sold, and let P represent the total profit. With a profit of \$10 per unit, the following mathematical model defines the total profit earned by producing and selling x units:

$$P = 10x \tag{1.1}$$

The purpose, or value, of any model is that it enables us to make inferences about the real situation by studying and analyzing the model. For example, an airplane designer might test an iconic model of a new airplane in a wind tunnel to learn about the potential flying characteristics of the full-size airplane. Similarly, a mathematical model may be used to make inferences about how much profit will be earned if a specified quantity of a particular product is sold. According to the mathematical model of equation (1.1), we would expect that selling three units of the product ($x = 3$) would provide a profit of $P = 10(3) = \$30$.

In general, experimenting with models requires less time and is less expensive than experimenting with the real object or situation. One can certainly build and study a model airplane in less time and for less money than it would take to build and study the full-size airplane. Similarly, the mathematical model in equation (1.1) allows a quick identification of profit expectations without requiring the manager to actually produce and sell x units. Models also reduce the risks associated with experimenting with the real situation. In particular, bad designs or bad decisions that cause the model airplane to crash or the mathematical model to project a \$10,000 loss can be avoided in the real situation.

Herbert A. Simon, a Nobel Prize winner in economics and an expert in decision making, said that a mathematical model does not have to be exact; it just has to be close enough to provide better results than can be obtained by common sense.

The value of model-based conclusions and decisions depends on how well the model represents the real situation. The more closely the model airplane represents the real airplane, the more accurate will be the conclusions and predictions. Similarly, the more closely the mathematical model represents the company's true profit–volume relationship, the more accurate will be the profit projections.

Because this text deals with quantitative analysis based on mathematical models, let us look more closely at the mathematical modeling process. When initially considering a managerial problem, we usually find that the problem definition phase leads to a specific objective, such as maximization of profit or minimization of cost, and possibly a set of restrictions or **constraints**, which express limitations on resources. The success of the mathematical model and quantitative approach will depend heavily on how accurately the objective and constraints can be expressed in mathematical equations or relationships.

The mathematical expression that defines the quantity to be maximized or minimized is referred to as the **objective function**. For example, suppose x denotes the number of units produced and sold each week, and the firm's objective is to maximize total weekly profit. With a profit of \$10 per unit, the objective function is $10x$. A production capacity constraint would be necessary if, for instance, 5 hours are required to produce each unit and only 40 hours are available per week. The production capacity constraint is given by

$$5x \leq 40 \qquad \textbf{(1.2)}$$

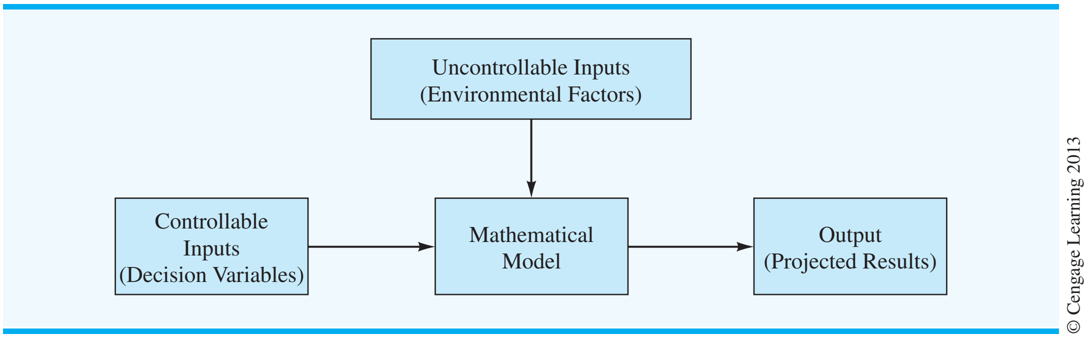
The value of $5x$ is the total time required to produce x units; the symbol $\leq$ indicates that the production time required must be less than or equal to the 40 hours available.

The decision problem or question is the following: How many units of the product should be produced each week to maximize profit? A complete mathematical model for this simple production problem is

$$\begin{aligned} &\text{Maximize} \quad 10x \quad \text{objective function} \\ &\text{subject to (s.t.)} \\ &\left.\begin{aligned} 5x &\leq 40 \\ x &\geq 0 \end{aligned}\right\} \text{constraints} \end{aligned}$$

The $x \geq 0$ constraint requires the production quantity x to be greater than or equal to zero, which simply recognizes the fact that it is not possible to manufacture a negative number

FIGURE 1.4 FLOWCHART OF THE PROCESS OF TRANSFORMING MODEL INPUTS INTO OUTPUT

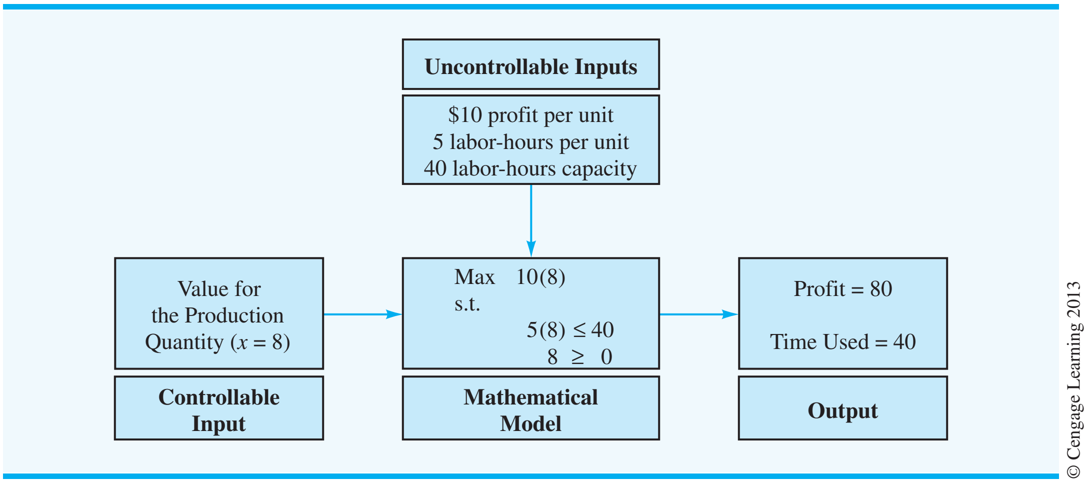


of units. The optimal solution to this simple model can be easily calculated and is given by $x = 8$, with an associated profit of \$80. This model is an example of a linear programming model. In subsequent chapters we will discuss more complicated mathematical models and learn how to solve them in situations for which the answers are not nearly so obvious.

In the preceding mathematical model, the profit per unit (\$10), the production time per unit (5 hours), and the production capacity (40 hours) are factors not under the control of the manager or decision maker. Such factors, which can affect both the objective function and the constraints, are referred to as **uncontrollable inputs** to the model. Inputs that are controlled or determined by the decision maker are referred to as **controllable inputs** to the model. In the example given, the production quantity x is the controllable input to the model. Controllable inputs are the decision alternatives specified by the manager and thus are also referred to as the **decision variables** of the model.

Once all controllable and uncontrollable inputs are specified, the objective function and constraints can be evaluated and the output of the model determined. In this sense, the output of the model is simply the projection of what would happen if those particular factors and decisions occurred in the real situation. A flowchart of how controllable and uncontrollable inputs are transformed by the mathematical model into output is shown in Figure 1.4. A similar flowchart showing the specific details for the production model is shown in Figure 1.5. Note that we have used "Max" as an abbreviation for maximize.

FIGURE 1.5 FLOWCHART FOR THE PRODUCTION MODEL



As stated earlier, the uncontrollable inputs are those the decision maker cannot influence. The specific controllable and uncontrollable inputs of a model depend on the particular problem or decision-making situation. In the production problem, the production time available (40) is an uncontrollable input. However, if it were possible to hire more employees or use overtime, the number of hours of production time would become a controllable input and therefore a decision variable in the model.

Uncontrollable inputs can either be known exactly or be uncertain and subject to variation. If all uncontrollable inputs to a model are known and cannot vary, the model is referred to as a **deterministic model**. Corporate income tax rates are not under the influence of the manager and thus constitute an uncontrollable input in many decision models. Because these rates are known and fixed (at least in the short run), a mathematical model with corporate income tax rates as the only uncontrollable input would be a deterministic model. The distinguishing feature of a deterministic model is that the uncontrollable input values are known in advance.

If any of the uncontrollable inputs are uncertain and subject to variation, the model is referred to as a **stochastic** or **probabilistic model**. An uncontrollable input in many production planning models is demand for the product. Because future demand may be any of a range of values, a mathematical model that treats demand with uncertainty would be considered a stochastic model. In the production model, the number of hours of production time required per unit, the total hours available, and the unit profit were all uncontrollable inputs. Because the uncontrollable inputs were all known to take on fixed values, the model was deterministic. If, however, the number of hours of production time per unit could vary from 3 to 6 hours depending on the quality of the raw material, the model would be stochastic. The distinguishing feature of a stochastic model is that the value of the output cannot be determined even if the value of the controllable input is known because the specific values of the uncontrollable inputs are unknown. In this respect, stochastic models are often more difficult to analyze.

Data Preparation

Another step in the quantitative analysis of a problem is the preparation of the data required by the model. Data in this sense refer to the values of the uncontrollable inputs to the model. All uncontrollable inputs or data must be specified before we can analyze the model and recommend a decision or solution for the problem.

In the production model, the values of the uncontrollable inputs or data were $10 per unit for profit, 5 hours per unit for production time, and 40 hours for production capacity. In the development of the model, these data values were known and incorporated into the model as it was being developed. If the model is relatively small with respect to the number of the uncontrollable input values, the quantitative analyst will probably combine model development and data preparation into one step. In these situations the data values are inserted as the equations of the mathematical model are developed.

However, in many mathematical modeling situations the data or uncontrollable input values are not readily available. In these situations the analyst may know that the model will require profit per unit, production time, and production capacity data, but the values will not be known until the accounting, production, and engineering departments can be consulted. Rather than attempting to collect the required data as the model is being developed, the analyst will usually adopt a general notation for the model development step, and a separate data preparation step will then be performed to obtain the uncontrollable input values required by the model.

Using the general notation

$$c = \text{profit per unit}$$
$$a = \text{production time in hours per unit}$$
$$b = \text{production capacity in hours}$$

the model development step for the production problem would result in the following general model (recall x = the number of units to produce and sell):

$$\begin{aligned} &\text{Max } cx \\ &\text{s.t.} \\ &\qquad ax \leq b \\ &\qquad x \geq 0 \end{aligned}$$

A separate data preparation step to identify the values for c, a, and b would then be necessary to complete the model.

Many inexperienced quantitative analysts assume that once the problem is defined and a general model developed, the problem is essentially solved. These individuals tend to believe that data preparation is a trivial step in the process and can be easily handled by clerical staff. Actually, this is a potentially dangerous assumption that could not be further from the truth, especially with large-scale models that have numerous data input values. For example, a moderate-sized linear programming model with 50 decision variables and 25 constraints could have more than 1300 data elements that must be identified in the data preparation step. The time required to collect and prepare these data and the possibility of data collection errors will make the data preparation step a critical part of the quantitative analysis process. Often, a fairly large database is needed to support a mathematical model, and information systems specialists also become involved in the data preparation step.

Model Solution

Once the model development and data preparation steps are completed, we proceed to the model solution step. In this step, the analyst attempts to identify the values of the decision variables that provide the "best" output for the model. The specific decision-variable value or values providing the "best" output are referred to as the **optimal solution** for the model. For the production problem, the model solution step involves finding the value of the production quantity decision variable x that maximizes profit while not causing a violation of the production capacity constraint.

One procedure that might be used in the model solution step involves a trial-and-error approach in which the model is used to test and evaluate various decision alternatives. In the production model, this procedure would mean testing and evaluating the model using various production quantities or values of x. As noted in Figure 1.5, we could input trial values for x and check the corresponding output for projected profit and satisfaction of the production capacity constraint. If a particular decision alternative does not satisfy one or more of the model constraints, the decision alternative is rejected as being **infeasible**, regardless of the corresponding objective function value. If all constraints are satisfied, the decision alternative is **feasible** and is a candidate for the "best" solution or recommended decision. Through this trial-and-error process of evaluating selected decision alternatives, a decision maker can identify a good—and possibly the best—feasible solution to the problem. This solution would then be the recommended decision for the problem.

TABLE 1.2 TRIAL-AND-ERROR SOLUTION FOR THE PRODUCTION MODEL OF FIGURE 1.5

Decision Alternative (Production Quantity) x	Projected Profit	Total Hours of Production	Feasible Solution? (Hours Used $\leq$ 40)
0	0	0	Yes
2	20	10	Yes
4	40	20	Yes
6	60	30	Yes
8	80	40	Yes
10	100	50	No
12	120	60	No

Table 1.2 shows the results of a trial-and-error approach to solving the production model of Figure 1.5. The recommended decision is a production quantity of 8 because the feasible solution with the highest projected profit occurs at $x = 8$.

Although the trial-and-error solution process is often acceptable and can provide valuable information for the manager, it has the drawbacks of not necessarily providing the best solution and of being inefficient in terms of requiring numerous calculations if many decision alternatives are considered. Thus, quantitative analysts have developed special solution procedures for many models that are much more efficient than the trial-and-error approach. Throughout this text, you will be introduced to solution procedures that are applicable to the specific mathematical models. Some relatively small models or problems can be solved by hand computations, but most practical applications require the use of a computer.

The model development and model solution steps are not completely separable. An analyst will want both to develop an accurate model or representation of the actual problem situation and to be able to find a solution to the model. If we approach the model development step by attempting to find the most accurate and realistic mathematical model, we may find the model so large and complex that it is impossible to obtain a solution. In this case, a simpler and perhaps more easily understood model with a readily available solution procedure is preferred even though the recommended solution may be only a rough approximation of the best decision. As you learn more about quantitative solution procedures, you will form a better understanding of the types of mathematical models that can be developed and solved.

Try Problem 8 to test your understanding of the concept of a mathematical model and what is referred to as the optimal solution to the model.

After obtaining a model solution, the quantitative analyst will be interested in determining the quality of the solution. Even though the analyst has undoubtedly taken many precautions to develop a realistic model, often the usefulness or accuracy of the model cannot be assessed until model solutions are generated. Model testing and validation are frequently conducted with relatively small "test" problems with known or at least expected solutions. If the model generates the expected solutions, and if other output information appears correct or reasonable, the go-ahead may be given to use the model on the full-scale problem. However, if the model test and validation identify potential problems or inaccuracies inherent in the model, corrective action, such as model modification or collection of more accurate input data, may be taken. Whatever the corrective action, the model solution will not be used in practice until the model satisfactorily passes testing and validation.

Report Generation

An important part of the quantitative analysis process is the preparation of managerial reports based on the model's solution. As indicated in Figure 1.3, the solution based on the quantitative analysis of a problem is one of the inputs the manager considers before making a final decision. Thus, the results of the model must appear in a managerial report that can be easily understood by the decision maker. The report includes the recommended decision and other pertinent information about the results that may be useful to the decision maker.

A Note Regarding Implementation

As discussed in Section 1.2, the manager is responsible for integrating the quantitative solution with qualitative considerations to determine the best possible decision. After completing the decision-making process, the manager must oversee the implementation and follow-up evaluation of the decision. During the implementation and follow-up, the manager should continue to monitor the performance of the model. At times, this process may lead to requests for model expansion or refinement that will require the quantitative analyst to return to an earlier step of the process.

Successful implementation of results is critical to any application of quantitative analysis. If the results of the quantitative analysis are not correctly implemented, the entire effort may be of no value. Because implementation often requires people to change the way they do things, it often meets with resistance. People may want to know, "What's wrong with the way we've been doing it?" One of the most effective ways to ensure successful implementation is to include users throughout the modeling process. A user who feels a part of identifying the problem and developing the solution is much more likely to enthusiastically implement the results, and the input the quantitative analyst receives from these users can substantially enhance the models being developed. The success rate for implementing the results of a quantitative analysis project is much greater for those projects characterized by extensive user involvement. The Q.M. in Action, Quantitative Analysis at Merrill Lynch, discusses some of the reasons for the success of quantitative analysis at Merrill Lynch.

Q.M. *in* ACTION

*QUANTITATIVE ANALYSIS AT MERRILL LYNCH**

For over 25 years, the Management Science Group at Merrill Lynch has successfully implemented quantitative models for a wide variety of decision problems. The group has applied quantitative methods for portfolio optimization, asset allocation, financial planning, marketing analysis, credit and liquidity assessment, as well as developing pricing and compensation structures. Although technical expertise and objectivity are clearly important factors in any analytical group, the management science group attributes much of its success to communications skills, teamwork, professional development for its members, and consulting skills.

From the earliest discussion of a potential project, the group focuses on fully understanding the problem and its business impact. Each client is asked, "Whose life will this change?" and "By how much?" The answers to these questions help the group understand who really has responsibility for the project, the processes involved, and how recommendations will be implemented. Analysts assigned to a project are fully engaged from start to finish. They are involved in project scope definition, data collection, analysis, development of recommendations, and

(*continued*)

*Based on R. Nigam, "Structuring and Sustaining Excellence in Management Science at Merrill Lynch," *Interfaces* 38, no. 3 (May/June 2008): 202–209.

marketing those recommendations to the client. The group prides itself on technology transfer; that is, it gives any models it develops to the clients with assistance and training on the use of the models. This leads to longer-term impact through ongoing use of the model. Finally, like any good organization focused on improvement, the Management Science Group seeks feedback from clients after every project it completes.

This approach to problem solving and the implementation of quantitative analysis has been a hallmark of the Management Science Group. The impact and success of the group translates into hard dollars, repeat business, and recognition through a number of prestigious professional awards. The group received the annual Edelman Award given by the Institute for Operations Research and the Management Sciences (INFORMS) for effective use of management science for organizational success as well as the INFORMS Prize, given for long-term and high-impact use of quantitative methods within an organization.

NOTES AND COMMENTS

1. Developments in computer technology have increased the availability of quantitative methods to decision makers. A variety of software packages is now available for personal computers. Versions of Microsoft Excel and LINGO are widely used to apply quantitative methods to business problems. Various chapter appendixes provide step-by-step instructions for using Excel and LINGO to solve problems in the text.

1.4 Models of Cost, Revenue, and Profit

Some of the most basic quantitative models arising in business and economic applications involve the relationships among a volume variable—such as production volume or sales volume—and cost, revenue, and profit. Through the use of these models, a manager can determine the projected cost, revenue, or profit associated with a planned production quantity or a forecasted sales volume. Financial planning, production planning, sales quotas, and other areas of decision making can benefit from such cost, revenue, and profit models.

Cost and Volume Models

The cost of manufacturing or producing a product is a function of the volume produced. This cost can usually be defined as a sum of two costs: fixed cost and variable cost. **Fixed cost** is the portion of the total cost that does not depend on the production volume; this cost remains the same no matter how much is produced. **Variable cost**, on the other hand, is the portion of the total cost that depends on and varies with the production volume. To illustrate how cost and volume models can be developed, we will consider a manufacturing problem faced by Nowlin Plastics.

Nowlin Plastics produces a variety of compact disc (CD) storage cases. Nowlin's best-selling product is the CD-50, a slim plastic CD holder with a specially designed lining that protects the optical surface of each CD. Several products are produced on the same manufacturing line, and a setup cost is incurred each time a changeover is made for a new product. Suppose the setup cost for the CD-50 is $3000; this setup cost is a fixed cost and is incurred regardless of the number of units eventually produced. In addition, suppose that variable labor and material costs are $2 for each unit produced. The cost–volume model for producing x units of the CD-50 can be written as

$$C(x) = 3000 + 2x \tag{1.3}$$

where

x = production volume in units

$C(x)$ = total cost of producing x units

Once a production volume is established, the model in equation (1.3) can be used to compute the total production cost. For example, the decision to produce $x = 1200$ units would result in a total cost of $C(1200) = 3000 + 2(1200) = \$5400$.

Marginal cost is defined as the rate of change of the total cost with respect to production volume; that is, the cost increase associated with a one-unit increase in the production volume. In the cost model of equation (1.3), we see that the total cost $C(x)$ will increase by $2 for each unit increase in the production volume. Thus, the marginal cost is $2. With more complex total cost models, marginal cost may depend on the production volume. In such cases, we could have marginal cost increasing or decreasing with the production volume x.

Revenue and Volume Models

Management of Nowlin Plastics will also want information about projected revenue associated with selling a specified number of units. Thus, a model of the relationship between revenue and volume is also needed. Suppose that each CD-50 storage unit sells for $5. The model for total revenue can be written as

$$R(x) = 5x \tag{1.4}$$

where

x = sales volume in units

$R(x)$ = total revenue associated with selling x units

Marginal revenue is defined as the rate of change of total revenue with respect to sales volume, that is, the increase in total revenue resulting from a one-unit increase in sales volume. In the model of equation (1.4), we see that the marginal revenue is $5. In this case, marginal revenue is constant and does not vary with the sales volume. With more complex models, we may find that marginal revenue increases or decreases as the sales volume x increases.

Profit and Volume Models

One of the most important criteria for management decision making is profit. Managers need to know the profit implications of their decisions. If we assume that we will only produce what can be sold, the production volume and sales volume will be equal. We can then combine equations (1.3) and (1.4) to develop a profit–volume model that determines profit associated with a specified production-sales volume. Total profit is total revenue minus total cost; therefore, the following model provides the profit associated with producing and selling x units:

$$\begin{aligned} P(x) &= R(x) - C(x) \\ &= 5x - (3000 + 2x) = -3000 + 3x \end{aligned} \tag{1.5}$$

Thus, the model for profit $P(x)$ can be derived from the models of the revenue–volume and cost–volume relationships.

Breakeven Analysis

Using equation (1.5), we can now determine the profit associated with any production volume x. For example, suppose that a demand forecast indicates that 500 units of the product can be sold. The decision to produce and sell the 500 units results in a projected profit of

$$P(500) = -3000 + 3(500) = -1500$$

In other words, a loss of $1500 is predicted. If sales are expected to be 500 units, the manager may decide against producing the product. However, a demand forecast of 1800 units would show a projected profit of

$$P(1800) = -3000 + 3(1800) = 2400$$

This profit may be sufficient to justify proceeding with the production and sale of the product.

We see that a volume of 500 units will yield a loss, whereas a volume of 1800 provides a profit. The volume that results in total revenue equaling total cost (providing $0 profit) is called the **breakeven point**. If the breakeven point is known, a manager can quickly infer that a volume above the breakeven point will generate a profit, whereas a volume below the breakeven point will result in a loss. Thus, the breakeven point for a product provides valuable information for a manager who must make a yes/no decision concerning production of the product.

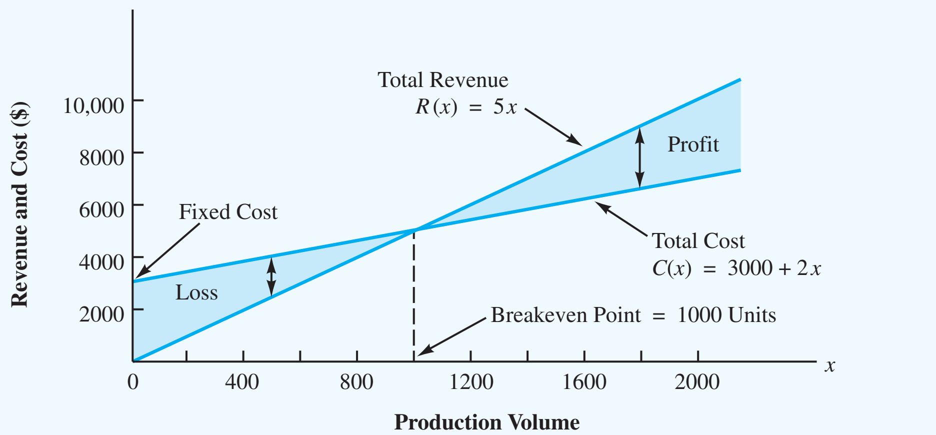
Let us now return to the Nowlin Plastics example and show how the profit model in equation (1.5) can be used to compute the breakeven point. The breakeven point can be found by setting the profit expression equal to zero and solving for the production volume. Using equation (1.5), we have

$$P(x) = -3000 + 3x = 0$$
$$3x = 3000$$
$$x = 1000$$

Try Problem 12 to test your ability to determine the breakeven point for a quantitative model.

With this information, we know that production and sales of the product must exceed 1000 units before a profit can be expected. The graphs of the total cost model, the total revenue model, and the location of the breakeven point are shown in Figure 1.6. In Appendix 1.1 we also show how Excel can be used to perform a breakeven analysis for the Nowlin Plastics production example.

FIGURE 1.6 GRAPH OF THE BREAKEVEN ANALYSIS FOR NOWLIN PLASTICS



1.5 Quantitative Methods in Practice

In this section we present a brief overview of the quantitative methods covered in this text. There are numerous applications for each of the following methods.

Linear Programming Linear programming is a problem-solving approach developed for situations involving maximizing or minimizing a linear function subject to linear constraints that limit the degree to which the objective can be pursued. The production model developed in Section 1.3 (see Figure 1.5) is an example of a simple linear programming model.

Integer Linear Programming Integer linear programming is an approach used for problems that can be set up as linear programs with the additional requirement that some or all of the decision recommendations be integer values.

Project Scheduling: PERT/CPM In many situations managers are responsible for planning, scheduling, and controlling projects that consist of numerous separate jobs or tasks performed by a variety of departments, individuals, and so forth. PERT (Program Evaluation and Review Technique) and CPM (Critical Path Method) help managers carry out their project scheduling and tracking responsibilities.

Inventory Models Inventory models are used by managers faced with the problem of maintaining sufficient inventories to meet demand for goods while incurring the lowest possible inventory holding costs.

Waiting Line or Queueing Models Waiting line or queueing models help managers understand and make better decisions concerning the operation of systems involving waiting lines.

Simulation Simulation is a technique used to model the operation of a complex system. This technique employs a computer program to model the operation and perform simulation computations.

Decision Analysis Decision analysis can be used to determine optimal strategies in situations involving several decision alternatives and an uncertain or risk-filled pattern of future events.

Forecasting Forecasting methods are techniques that can be used to predict future aspects of a business operation.

Markov-Process Models Markov-process models are useful in studying the evolution of certain systems over repeated trials. For example, Markov processes have been used to describe the probability that a machine, functioning in one period, will function or break down in some future period.

Methods Used Most Frequently

We believe barriers to the use of quantitative methods can best be removed by increasing the manager's understanding of how quantitative analysis can be applied. The text will help you develop an understanding of which quantitative methods are most useful, how they are used, and, most importantly, how they can assist managers in making better decisions.

The Q.M. in Action, Impact of Operations Research on Everyday Living, describes some of the many ways quantitative analysis affects our everyday lives.

Q.M. *in* ACTION

*IMPACT OF OPERATIONS RESEARCH ON EVERYDAY LIVING**

Mark Eisner, Communications Associate of the School of Operations Research and Information Engineering at Cornell University, once said that operations research "is probably the most important field nobody has ever heard of." The impact of operations research on everyday living over the past 20 years is substantial.

Suppose you schedule a vacation to Florida and use the Orbitz website to book your flights. An algorithm developed by operations researchers will search among millions of options to find the cheapest fare. Another algorithm will schedule the flight crews and aircraft used by the airline. If you rent a car in Florida, the price you pay for the car is determined by a mathematical model that seeks to maximize revenue for the car rental firm. If you do some shopping on your trip and decide to ship your purchases home using UPS, another algorithm determines the truck on which your packages are loaded, which route the truck should follow, and where your packages should be placed on the truck to minimize loading and unloading time.

If you enjoy watching college basketball, operations research plays a role in what games you see. Michael Trick, a professor at the Tepper School of Business at Carnegie Mellon, designed a system for scheduling each year's Atlantic Coast Conference men's and women's basketball games. Even though it might initially appear that scheduling 16 games among the nine men's teams would be easy, it requires sorting through hundreds of millions of possible combinations of possible schedules. Each of those possibilities entails some desirable and some undesirable characteristics. For example, you do not want to schedule too many consecutive home games for any team, and you want to ensure that each team plays the same number of weekend games.

*Based on Virginia Postrel, "Operations Everything," *The Boston Globe*, June 27, 2004.

NOTES AND COMMENTS

1. In the United States, the Institute for Operations Research and the Management Sciences (INFORMS) and the Decision Sciences Institute (DSI) are two flagship professional societies that publish journals and newsletters dealing with current research and applications of operations research and management science techniques. In Canada, the Canadian Operational Research Society (CORS) provides similar services.
2. Several European countries, including (but not limited to) Great Britain, France, Italy, Germany, Austria, and the Czech Republic, have their own professional operations research and management science societies, and these societies belong to the Association of European Operational Research Societies (EURO). Professional operations research and management science societies from Latin American and Iberian peninsula countries, including (but not limited to) Chile, Brazil, Argentina, Colombia, Spain, Uruguay, Portugal, and Mexico, all belong to the Asociación Latino-Iberoamericana de Investigación Operativa (ALIO). Professional operations research and management science societies from Australia, Japan, China, India, Malaysia, Thailand, New Zealand, and other countries from Asia and the Pacific Rim belong to the Association of Asian Pacific Operational Research Societies (APORS). African operations research societies include the Operations Research Society of South Africa (ORSSA) and the Operations Research Society of Eastern Africa (ORSEA). The International Federation of Operational Research Societies (IFORS) is the global organization to which most of these (and other) professional operations research and management science societies belong.

Summary

This text focuses on the use of quantitative methods to help managers make better decisions. The discussion in this chapter centered on the problem orientation of the decision-making process and an overview of how mathematical models can be used in this type of analysis.

The difference between the model and the situation or managerial problem it represents is an important consideration. Mathematical models are abstractions of real-world situations and, as such, cannot capture all the aspects of the real situation. However, if a model can capture the major relevant aspects of the problem and can then provide a meaningful solution recommendation, it can be a valuable aid to decision making.

One of the characteristics of quantitative analysis that will become increasingly apparent as we proceed through the text is the search for a best solution to the problem. In carrying out the quantitative analysis, we attempt to develop procedures for finding the "best" or optimal solution.

Glossary

Problem solving The process of identifying a difference between the actual and the desired state of affairs and then taking action to resolve the difference.

Decision making The process of defining the problem, identifying the alternatives, determining the criteria, evaluating the alternatives, and choosing an alternative.

Single-criterion decision problem A problem in which the objective is to find the "best" solution with respect to just one criterion.

Multicriteria decision problem A problem that involves more than one criterion; the objective is to find the "best" solution, taking into account all the criteria.

Decision The alternative selected.

Model A representation of a real object or situation.

Iconic model A physical replica, or representation, of a real object.

Analog model Although physical in form, an analog model does not have a physical appearance similar to the real object or situation it represents.

Mathematical model Mathematical symbols and expressions used to represent a real situation.

Constraint A restriction or limitation imposed on a problem.

Objective function The mathematical expression that defines the quantity to be maximized or minimized.

Uncontrollable input The factors that cannot be controlled by the decision maker.

Controllable input The decision alternatives that can be specified by the decision maker.

Decision variable Another term for controllable input.

Deterministic model A model in which all uncontrollable inputs are known and cannot vary.

Stochastic model A model in which at least one uncontrollable input is uncertain and subject to variation; stochastic models are also referred to as probabilistic models.

Optimal solution The specific decision variable value or values that provide the "best" output for the model.

Infeasible solution A decision alternative or solution that violates one or more constraints.

Feasible solution A decision alternative or solution that satisfies all constraints.

Fixed cost The portion of the total cost that does not depend on the volume; this cost remains the same no matter how much is produced.

Variable cost The portion of the total cost that is dependent on and varies with the volume.

Marginal cost The rate of change of the total cost with respect to volume.

Marginal revenue The rate of change of total revenue with respect to volume.
Breakeven point The volume at which total revenue equals total cost.

Problems

1. Define the terms *management science* and *operations research.*
2. List and discuss the steps of the decision-making process.
3. Discuss the different roles played by the qualitative and quantitative approaches to managerial decision making. Why is it important for a manager or decision maker to have a good understanding of both of these approaches to decision making?
4. SELF test — A firm recently built a new plant that will use more than 50 production lines and machines to produce over 500 different products. The production scheduling decisions are critical because sales will be lost if customer demand is not met on time. If no individual in the firm has had experience with this production operation, and if new production schedules must be generated each week, why should the firm consider a quantitative approach to the production scheduling problem?
5. What are the advantages of analyzing and experimenting with a model as opposed to a real object or situation?
6. Suppose a manager must choose between the following two mathematical models of a given situation: (a) a relatively simple model that is a reasonable approximation of the real situation and (b) a thorough and complex model that is the most accurate mathematical representation of the real situation possible. Why might the model described in part (a) be preferred by the manager?
7. Suppose you are going on a weekend trip to a city that is d miles away. Develop a model that determines your round-trip gasoline costs. What assumptions or approximations are necessary to treat this model as a deterministic model? Are these assumptions or approximations acceptable to you?
8. SELF test — Recall the production model from Section 1.3:

$$\begin{aligned} \text{Max} \quad & 10x \\ \text{s.t.} \quad & \\ & 5x \leq 40 \\ & x \geq 0 \end{aligned}$$

   Suppose the firm in this example considers a second product that has a unit profit of \$5 and requires 2 hours for each unit produced. Assume total production capacity remains 40 units. Use y as the number of units of product 2 produced.
   a. Show the mathematical model when both products are considered simultaneously.
   b. Identify the controllable and uncontrollable inputs for this model.
   c. Draw the flowchart of the input–output process for this model (see Figure 1.5).
   d. What are the optimal solution values of x and y?
   e. Is this model a deterministic or a stochastic model? Explain.
9. Suppose we modify the production model from Section 1.3 to obtain the following mathematical model:

$$\begin{aligned} \text{Max} \quad & 10x \\ \text{s.t.} \quad & \\ & ax \leq 40 \\ & x \geq 0 \end{aligned}$$

where a is the number of hours required for each unit produced. With $a = 5$, the optimal solution is $x = 8$. If we have a stochastic model in which the value of a varies between 3 and 6 (i.e., $a = 3, a = 4, a = 5$, or $a = 6$) as the possible values for the number of hours required per unit, what is the optimal value for x? What problems does this stochastic model cause?

10. A retail store in Des Moines, Iowa, receives shipments of a particular product from Kansas City and Minneapolis. Let

$$x = \text{units of product received from Kansas City}$$
$$y = \text{units of product received from Minneapolis}$$

a. Write an expression for the total units of product received by the retail store in Des Moines.
b. Shipments from Kansas City cost \$0.20 per unit, and shipments from Minneapolis cost \$0.25 per unit. Develop an objective function representing the total cost of shipments to Des Moines.
c. Assuming the monthly demand at the retail store is 5000 units, develop a constraint that requires 5000 units to be shipped to Des Moines.
d. No more than 4000 units can be shipped from Kansas City and no more than 3000 units can be shipped from Minneapolis in a month. Develop constraints to model this situation.
e. Of course, negative amounts cannot be shipped. Combine the objective function and constraints developed to state a mathematical model for satisfying the demand at the Des Moines retail store at minimum cost.

11. For most products, higher prices result in a decreased demand, whereas lower prices result in an increased demand (economists refer to such products as *normal goods*). Let

$$d = \text{annual demand for a product in units}$$
$$p = \text{price per unit}$$

Assume that a firm accepts the following price–demand relationship as being a realistic representation of its market:

$$d = 800 - 10p$$

where p must be between \$20 and \$70.

a. How many units can the firm sell at the \$20 per-unit price? At the \$70 per-unit price?
b. What happens to annual units demanded for the product if the firm increases the per-unit price from \$26 to \$27? From \$42 to \$43? From \$68 to \$69? What is the suggested relationship between per-unit price and annual demand for the product in units?
c. Show the mathematical model for the total revenue (TR), which is the annual demand multiplied by the unit price.
d. Based on other considerations, the firm's management will only consider price alternatives of \$30, \$40, and \$50. Use your model from part (b) to determine the price alternative that will maximize the total revenue.
e. What are the expected annual demand and the total revenue according to your recommended price?

12. The O'Neill Shoe Manufacturing Company will produce a special-style shoe if the order size is large enough to provide a reasonable profit. For each special-style order, the company incurs a fixed cost of \$2000 for the production setup. The variable cost is \$60 per pair, and each pair sells for \$80.

a. Let x indicate the number of pairs of shoes produced. Develop a mathematical model for the total cost of producing x pairs of shoes.

b. Let P indicate the total profit. Develop a mathematical model for the total profit realized from an order for x pairs of shoes.
c. What is the breakeven point?

13. Micromedia offers computer training seminars on a variety of topics. In the seminars each student works at a personal computer, practicing the particular activity that the instructor is presenting. Micromedia is currently planning a two-day seminar on the use of Microsoft Excel in statistical analysis. The projected fee for the seminar is $600 per student. The cost for the conference room, instructor compensation, lab assistants, and promotion is $9600. Micromedia rents computers for its seminars at a cost of $60 per computer per day.
    a. Develop a model for the total cost to put on the seminar. Let x represent the number of students who enroll in the seminar.
    b. Develop a model for the total profit if x students enroll in the seminar.
    c. Micromedia has forecasted an enrollment of 30 students for the seminar. How much profit will be earned if its forecast is accurate?
    d. Compute the breakeven point.

14. Eastman Publishing Company is considering publishing a paperback textbook on spreadsheet applications for business. The fixed cost of manuscript preparation, textbook design, and production setup is estimated to be $160,000. Variable production and material costs are estimated to be $6 per book. Demand over the life of the book is estimated to be 4000 copies. The publisher plans to sell the text to college and university bookstores for $46 each.
    a. What is the breakeven point?
    b. What profit or loss can be anticipated with a demand of 3500 copies?
    c. With a demand of 3500 copies, what is the minimum price per copy that the publisher must charge to break even?
    d. If the publisher believes that the price per copy could be increased to $50.95 and not affect the anticipated demand of 4000 copies, what action would you recommend? What profit or loss can be anticipated?

15. Preliminary plans are underway for construction of a new stadium for a major league baseball team. City officials question the number and profitability of the luxury corporate boxes planned for the upper deck of the stadium. Corporations and selected individuals may purchase a box for $300,000. The fixed construction cost for the upper-deck area is estimated to be $4,500,000, with a variable cost of $150,000 for each box constructed.
    a. What is the breakeven point for the number of luxury boxes in the new stadium?
    b. Preliminary drawings for the stadium show that space is available for the construction of up to 50 luxury boxes. Promoters indicate that buyers are available and that all 50 could be sold if constructed. What is your recommendation concerning the construction of luxury boxes? What profit is anticipated?

16. Financial Analysts, Inc., is an investment firm that manages stock portfolios for a number of clients. A new client has requested that the firm handle an $800,000 portfolio. As an initial investment strategy, the client would like to restrict the portfolio to a mix of the following two stocks:

Stock	Price/Share	Estimated Annual Return/Share
Oil Alaska	$50	$6
Southwest Petroleum	$30	$4

Let

x = number of shares of Oil Alaska
y = number of shares of Southwest Petroleum

a. Develop the objective function, assuming that the client desires to maximize the total annual return.
b. Show the mathematical expression for each of the following three constraints:
(1) Total investment funds available are $800,000.
(2) Maximum Oil Alaska investment is $500,000.
(3) Maximum Southwest Petroleum investment is $450,000.

Note: Adding the $x \geq 0$ and $y \geq 0$ constraints provides a linear programming model for the investment problem. A solution procedure for this model will be discussed in Chapter 7.

17. Models of inventory systems frequently consider the relationships among a beginning inventory, a production quantity, a demand or sales, and an ending inventory. For a given production period j, let

s_{j-1} = beginning inventory for period j (ending inventory from period $j - 1$, the previous period)
x_j = production quantity in period j
d_j = demand in period j
s_j = ending inventory for period j

a. Write the mathematical relationship or model that shows ending inventory as a function of beginning inventory, production, and demand.
b. What constraint should be added if production capacity for period j is given by C_j?
c. What constraint should be added if inventory requirements for period j mandate an ending inventory of at least I_j?

Case Problem Scheduling a Golf League

Chris Lane, the head professional at Royal Oak Country Club, must develop a schedule of matches for the couples' golf league that begins its season at 4:00 P.M. tomorrow. Eighteen couples signed up for the league, and each couple must play every other couple over the course of the 17-week season. Chris thought it would be fairly easy to develop a schedule, but after working on it for a couple of hours, he has been unable to come up with a schedule. Because Chris must have a schedule ready by tomorrow afternoon, he has asked you to help him. A possible complication is that one of the couples told Chris that they may have to cancel for the season. They told Chris they would let him know by 1:00 P.M. tomorrow whether they will be able to play this season.

Managerial Report

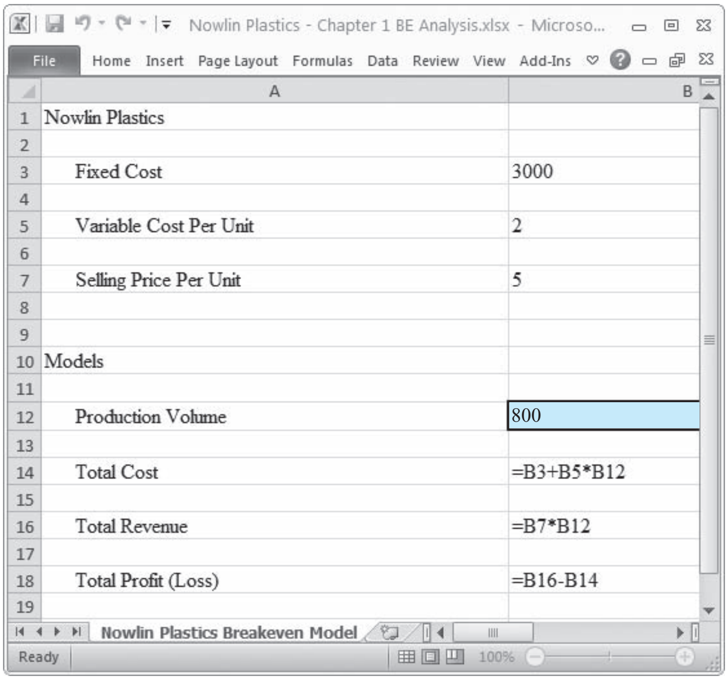
Prepare a report for Chris Lane. Your report should include, at a minimum, the following items:

1. A schedule that will enable each of the 18 couples to play every other couple over the 17-week season.
2. A contingency schedule that can be used if the couple that contacted Chris decides to cancel for the season.

Appendix 1.1 Using Excel for Breakeven Analysis

In Section 1.4 we introduced the Nowlin Plastics production example to illustrate how quantitative models can be used to help a manager determine the projected cost, revenue, and profit associated with an established production quantity or a forecasted sales volume. In this appendix we introduce spreadsheet applications by showing how to use Excel to perform a quantitative analysis of the Nowlin Plastics example.

FIGURE 1.7 FORMULA WORKSHEET FOR THE NOWLIN PLASTICS PRODUCTION EXAMPLE



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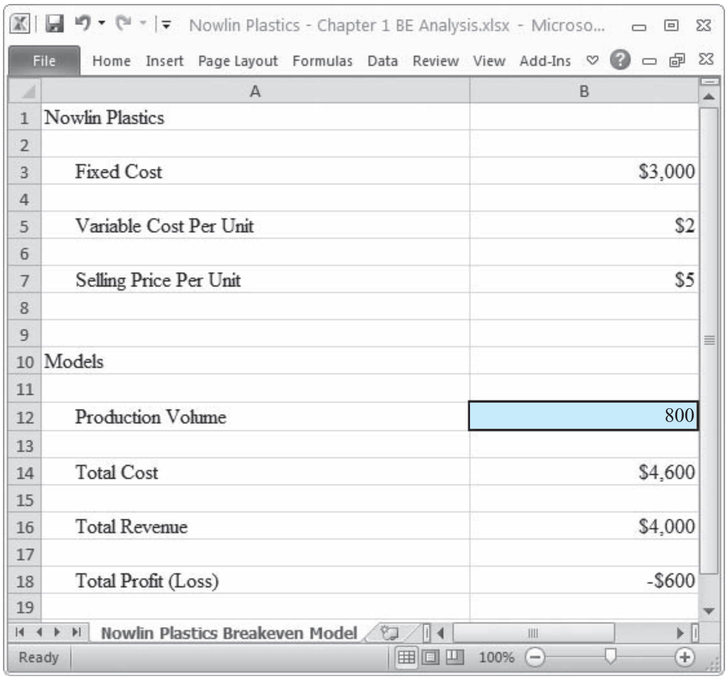
Refer to the worksheet shown in Figure 1.7. We begin by entering the problem data into the top portion of the worksheet. The value of 3000 in cell B3 is the setup cost, the value of 2 in cell B5 is the variable labor and material costs per unit, and the value of 5 in cell B7 is the selling price per unit. In general, whenever we perform a quantitative analysis using Excel, we will enter the problem data in the top portion of the worksheet and reserve the bottom portion for model development. The label "Models" in cell B10 helps to provide a visual reminder of this convention.

Cell B12 in the models portion of the worksheet contains the proposed production volume in units. Because the values for total cost, total revenue, and total profit depend upon the value of this decision variable, we placed a border around cell B12 and screened the cell for emphasis. Based upon the value in cell B12, the cell formulas in cells B14, B16, and B18 are used to compute values for total cost, total revenue, and total profit (loss), respectively. First, recall that the value of total cost is the sum of the fixed cost (cell B3) and the total variable cost. Because the total variable cost is the product of the variable cost per unit (cell B5) and the production volume (cell B12), it is given by B5*B12. Thus, to compute total cost we entered the formula =B3+B5*B12 into cell B14. Next, total revenue is the product of the selling price per unit (cell B7) and the number of units produced (cell B12); therefore in cell B16 we have entered the formula =B7*B12. Finally, the total profit (or loss) is the difference between the total revenue (cell B16) and the total cost (cell B14). Thus, in cell B18 we have entered the formula =B16-B14. The worksheet in Figure 1.7 shows the formulas used to make these computations; we refer to it as a formula worksheet.

To examine the effect of selecting a particular value for the production volume, we have entered a value of 800 in cell B12. The worksheet shown in Figure 1.8 shows the values obtained by the formulas; a production volume of 800 units results in a total cost of $4600, a

FIGURE 1.8 SOLUTION FOR THE NOWLIN PLASTICS PRODUCTION EXAMPLE USING A PRODUCTION VOLUME OF 800 UNITS



	A	B
1	Nowlin Plastics	
2		
3	Fixed Cost	$3,000
4		
5	Variable Cost Per Unit	$2
6		
7	Selling Price Per Unit	$5
8		
9		
10	Models	
11		
12	Production Volume	800
13		
14	Total Cost	$4,600
15		
16	Total Revenue	$4,000
17		
18	Total Profit (Loss)	-$600
19		

WEB file
Nowlin

total revenue of $4000, and a loss of $600. To examine the effect of other production volumes, we only need to enter the other values into cell B12. To examine the effect of different costs and selling prices, we simply enter the appropriate values in the data portion of the worksheet; the results will be displayed in the model section of the worksheet.

In Section 1.4 we illustrated breakeven analysis. Let us now see how a spreadsheet can be used to compute the breakeven point for the Nowlin Plastics production example.

Determining the Breakeven Point Using Excel's Goal Seek Tool

The breakeven point is the production volume that results in total revenue equal to total cost and hence a profit of $0. One way to determine the breakeven point is to use a trial-and-error approach. For example, in Figure 1.8 we saw that a trial production volume of 800 units resulted in a loss of $600. Because this trial solution resulted in a loss, a production volume of 800 units cannot be the breakeven point. We could continue to experiment with other production volumes by simply entering different values into cell B12 and observing the resulting profit or loss in cell B18. A better approach is to use Excel's Goal Seek tool to determine the breakeven point.

Excel's Goal Seek tool allows the user to determine the value for an input cell that will cause the value of a related output cell to equal some specified value (called the *goal*). In the case of breakeven analysis, the "goal" is to set total profit to zero by "seeking" an appropriate value for production volume. Goal Seek will allow us to find the value of production volume

FIGURE 1.9 GOAL SEEK DIALOG BOX FOR THE NOWLIN PLASTICS PRODUCTION EXAMPLE

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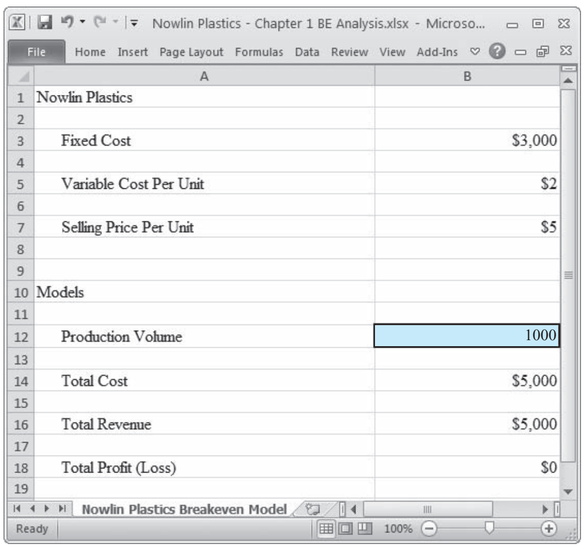
that will set Nowlin Plastics' total profit to zero. The following steps describe how to use Goal Seek to find the breakeven point for Nowlin Plastics:

Step 1. Select the **Data** tab at the top of the Ribbon
Step 2. Select **What-If Analysis** in the **Data Tools** group
Step 3. Select **Goal Seek** in **What-If-Analysis**
Step 4. When the **Goal Seek** dialog box appears (see Figure 1.9):
Enter B18 in the **Set cell** box
Enter 0 in the **To value** box
Enter B12 in the **By changing cell** box
Click **OK**

The completed Goal Seek dialog box is shown in Figure 1.9, and the worksheet obtained is shown in Figure 1.10. The total profit in cell B18 is zero, and the production volume in cell B12 has been set to the breakeven point of 1000.

FIGURE 1.10 BREAKEVEN POINT FOUND USING GOAL SEEK TOOL FOR THE NOWLIN PLASTICS PRODUCTION EXAMPLE



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Appendix F References and Bibliography

Chapter 1 Introduction

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Appendix G Self-Test Solutions and Answers to Even-Numbered Problems

Chapter 1

2. Define the problem; identify the alternatives; determine the criteria; evaluate the alternatives; choose an alternative.

4. A quantitative approach should be considered because the problem is large, complex, important, new, and repetitive.

6. Quicker to formulate, easier to solve, and/or more easily understood

8. a. Max $10x + 5y$

s.t.

$$5x + 2y \le 40$$
$$x \ge 0, y \ge 0$$

b. Controllable inputs: x and y
Uncontrollable inputs: profit (10, 5), labor-hours (5, 2), and labor-hour availability (40)

c. See Figure G1.8c.

d. $x = 0, y = 20$; Profit $= \$100$ (solution by trial and error)

e. Deterministic

10. a. Total units received $= x + y$

b. Total cost $= 0.20x + 0.25y$

c. $x + y = 5000$

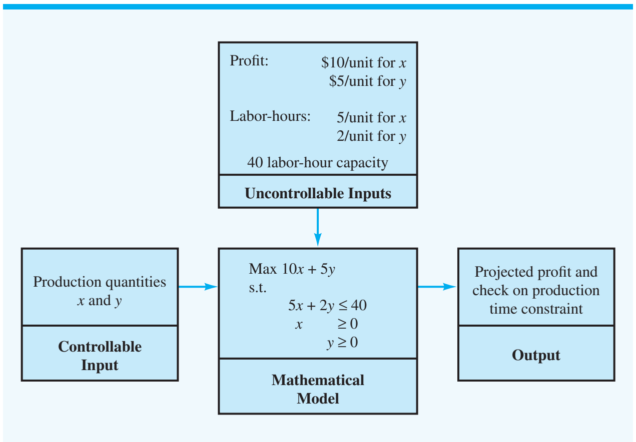
d. $x \le 4000$ Kansas City
$y \le 3000$ Minneapolis

e. Min $0.20x + 0.25y$

s.t.

$$\begin{aligned} x + \quad y &= 5000 \\ x \quad\quad &\le 4000 \\ y &\le 3000 \\ x, y &\ge 0 \end{aligned}$$

FIGURE G1.8c SOLUTION



12. a. If x represents the number of pairs of shoes produced, a mathematical model for the total cost of producing x pairs of shoes is $TC = 2000 + 60x$. The two components of total cost in this model are fixed cost ($2,000) and variable cost ($60x$).

b. If P represents the total profit, the total revenue (TR) is $80x$ and a mathematical model for the total profit realized from an order for x pairs of shoes is $P = TR - TC = 80x - (2000 + 60x) = 20x - 2000$.

c. The breakeven point is the number of shoes produced (x) at the point of no profit ($P = 0$).
Thus the breakeven point is the value of x when $P = 20x - 2000 = 0$. This occurs when $20x = 2000$ or $x = 100$ (i.e., the breakeven point is 100 pairs of shoes).

14. a. If x represents the number of copies of the book that are sold, total revenue (TR) $= 46x$ and total cost (TC) $= 160{,}000 + 6x$, so Profit $= TR - TC = 46x - (160{,}000 + 6x) = 40x - 160{,}000$. The breakeven point is the number of books produced (x) at the point of no profit ($P = 0$). Thus the breakeven point is the value of x when $P = 40x - 160{,}000 = 0$. This occurs when $40x = 160{,}00$ or $x = 4000$ (i.e., the breakeven point is 4000 copies of the book).

b. At a demand of 3800 copies, the publisher can expect a profit of $40(3800) - 160{,}000 = 152{,}000 - 160{,}000 = -8000$ (i.e., a loss of $8,000).

c. Here we know demand ($d = 3800$) and want to determine the price p at which we will breakeven (the point at which profit is 0). The minimum price per copy that the publisher must charge to break even is Profit $= p(3800) - (160{,}000 + 6(3800)) = 3800p - 182{,}800$. This occurs where $3800p = 182{,}800$ or $p = 48.10526316$ or a price of approximately $48.

d. If the publisher believes demand will remain at 4000 copies if the price per copy is increased to $50.95, then the publisher could anticipate a profit of $TR - TC = 50.95(4000) - (160{,}000 + 6(4000)) = 203{,}800 - 184{,}000 = 19{,}800$ or a profit of $19,800. This is a return of $p/TC = 10.8\%$ on the total cost of $184,000, and the publisher should proceed if this return is sufficient.

16. a. The annual return per share of Oil Alaska is $6.00 and the annual return per share of Southwest Petroleum is $4.00, so the objective function that maximizes the total annual return is Max $6x + 4y$.

b. The price per share of Oil Alaska is $50.00 and the price per share of Southwest Petroleum is $30.00, so

(1) the mathematical expression for the constraint that limits total investment funds to $800,000 is $50x + 30y \leq 800000$,
(2) the mathematical expression for the constraint that limits investment in Oil Alaska to $500,000 is $50x \leq 500000$, and
(3) the mathematical expression for the constraint that limits investment in Southwest Petroleum to $450,000 is $30x \leq 450000$.

Quantitative Methods for Business 12e WEBfiles

Chapter 1

Excel Files
Nowlin.xlsx

This page contains webfiles for this chapter only