CHAPTER 13

Project Scheduling: PERT/CPM

CONTENTS

In many situations managers are responsible for planning, scheduling, and controlling projects that consist of numerous separate jobs or tasks performed by a variety of departments and individuals. Often these projects are so large or complex that the manager cannot possibly remember all the information pertaining to the plan, schedule, and progress of the project. In these situations the **program evaluation and review technique (PERT)** and the **critical path method (CPM)** have proven to be extremely valuable.

PERT and CPM can be used to plan, schedule, and control a wide variety of projects:

1. Research and development of new products and processes
2. Construction of plants, buildings, and highways
3. Maintenance of large and complex equipment
4. Design and installation of new systems

Henry L. Gantt developed the Gantt Chart as a graphical aid to scheduling jobs on machines in 1918. This application was the first of what has become known as project scheduling techniques.

In these types of projects, project managers must schedule and coordinate the various jobs or **activities** so that the entire project is completed on time. A complicating factor in carrying out this task is the interdependence of the activities; for example, some activities depend on the completion of other activities before they can be started. Because projects may have as many as several thousand activities, project managers look for procedures that will help them answer questions such as the following:

1. What is the total time to complete the project?
2. What are the scheduled start and finish dates for each specific activity?
3. Which activities are "critical" and must be completed *exactly* as scheduled to keep the project on schedule?
4. How long can "noncritical" activities be delayed before they cause an increase in the total project completion time?

PERT and CPM can help answer these questions.

Although PERT and CPM have the same general purpose and utilize much of the same terminology, the techniques were developed independently. PERT was developed in the late 1950s by the Navy specifically for the Polaris missile project. Many activities associated with this project had never been attempted previously, so PERT was developed to handle uncertain activity times. CPM was developed originally by DuPont and Remington Rand primarily for industrial projects for which activity times were certain and variability was not a concern. CPM offered the option of reducing activity times by adding more workers and/or resources, usually at an increased cost. Thus, a distinguishing feature of CPM was that it identified trade-offs between time and cost for various project activities.

Today's computerized versions of PERT and CPM combine the best features of both approaches. Thus, the distinction between the two techniques is no longer necessary. As a result, we refer to the project scheduling procedures covered in this chapter as PERT/CPM. We begin the discussion of PERT/CPM by considering a project for the expansion of the Western Hills Shopping Center. At the end of the section, we describe how the investment securities firm of Seasongood & Mayer used PERT/CPM to schedule a $31 million hospital revenue bond project.

13.1 Project Scheduling Based on Expected Activity Times

The owner of the Western Hills Shopping Center plans to modernize and expand the current 32-business shopping center complex. The project is expected to provide room for 8 to 10 new businesses. Financing has been arranged through a private investor. All that remains

TABLE 13.1 LIST OF ACTIVITIES FOR THE WESTERN HILLS SHOPPING CENTER PROJECT

Activity	Activity Description	Immediate Predecessor	Expected Activity Time
A	Prepare architectural drawings	—	5
B	Identify potential new tenants	—	6
C	Develop prospectus for tenants	A	4
D	Select contractor	A	3
E	Prepare building permits	A	1
F	Obtain approval for building permits	E	4
G	Perform construction	D, F	14
H	Finalize contracts with tenants	B, C	12
I	Tenants move in	G, H	2
		Total	51

is for the owner of the shopping center to plan, schedule, and complete the expansion project. Let us show how PERT/CPM can help.

The effort that goes into identifying activities, determining interrelationships among activities, and estimating activity times is crucial to the success of PERT/CPM. A significant amount of time may be needed to complete this initial phase of the project scheduling process.

Immediate predecessor information determines whether activities can be completed in parallel (worked on simultaneously) or in series (one completed before another begins). Generally, the more series relationships present in a project, the more time will be required to complete the project.

A project network is extremely helpful in visualizing the interrelationships among the activities. No rules guide the conversion of a list of activities and immediate predecessor information into a project network. The process of constructing a project network generally improves with practice and experience.

The first step in the PERT/CPM scheduling process is to develop a list of the activities that make up the project. Table 13.1 shows the list of activities for the Western Hills Shopping Center expansion project. Nine activities are described and denoted A through I for later reference. Table 13.1 also shows the immediate predecessor(s) and the activity time (in weeks) for each activity. For a given activity, the **immediate predecessor** column identifies the activities that must be completed *immediately prior* to the start of that activity. Activities A and B do not have immediate predecessors and can be started as soon as the project begins; thus, a dash is written in the immediate predecessor column for these activities. The other entries in the immediate predecessor column show that activities C, D, and E cannot be started until activity A has been completed; activity F cannot be started until activity E has been completed; activity G cannot be started until both activities D and F have been completed; activity H cannot be started until both activities B and C have been completed; and, finally, activity I cannot be started until both activities G and H have been completed. The project is finished when activity I is completed.

The last column in Table 13.1 shows the expected number of weeks required to complete each activity. For example, activity A is expected to take 5 weeks, activity B is expected to take 6 weeks, and so on. The sum of expected activity times is 51. As a result, you may think that the total time required to complete the project is 51 weeks. However, as we show, two or more activities often may be scheduled concurrently (assuming sufficient availability of other required resources, such as labor and equipment), thus shortening the completion time for the project. Ultimately, PERT/CPM will provide a detailed activity schedule for completing the project in the shortest time possible.

Using the immediate predecessor information in Table 13.1, we can construct a graphical representation of the project, or the **project network**. Figure 13.1 depicts the project network for Western Hills Shopping Center. The activities correspond to the *nodes* of the network (drawn as rectangles), and the *arcs* (the lines with arrows) show the precedence relationships among the activities. In addition, nodes have been added to the network to denote the start and the finish of the project. A project network will help a manager visualize the activity relationships and provide a basis for carrying out the PERT/CPM computations.

FIGURE 13.1 PROJECT NETWORK FOR THE WESTERN HILLS SHOPPING CENTER

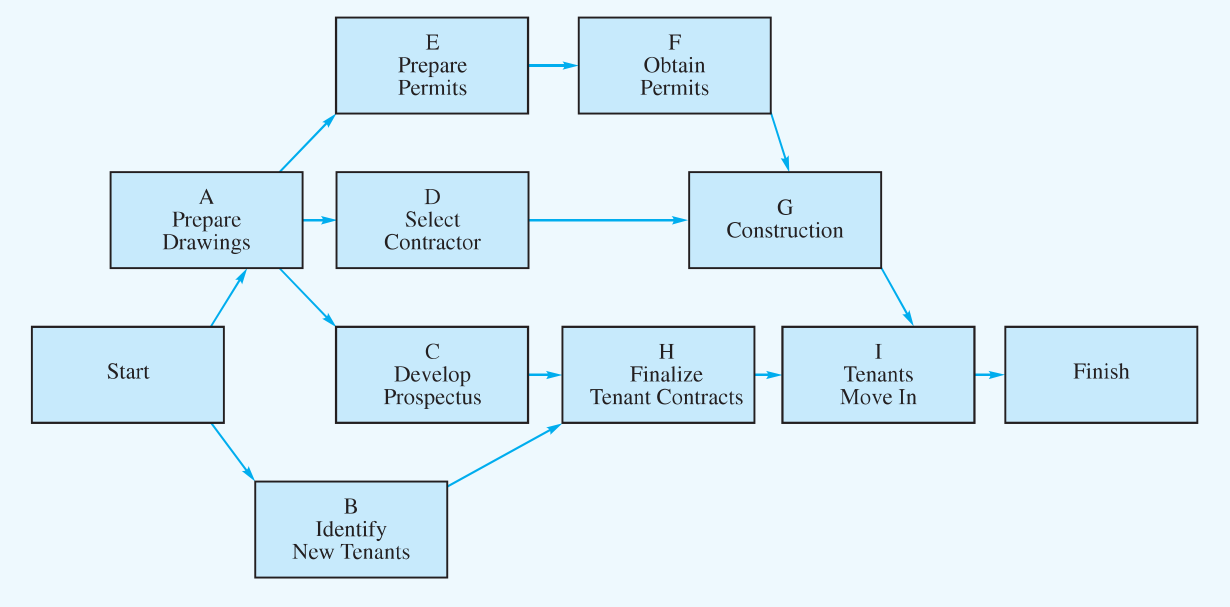
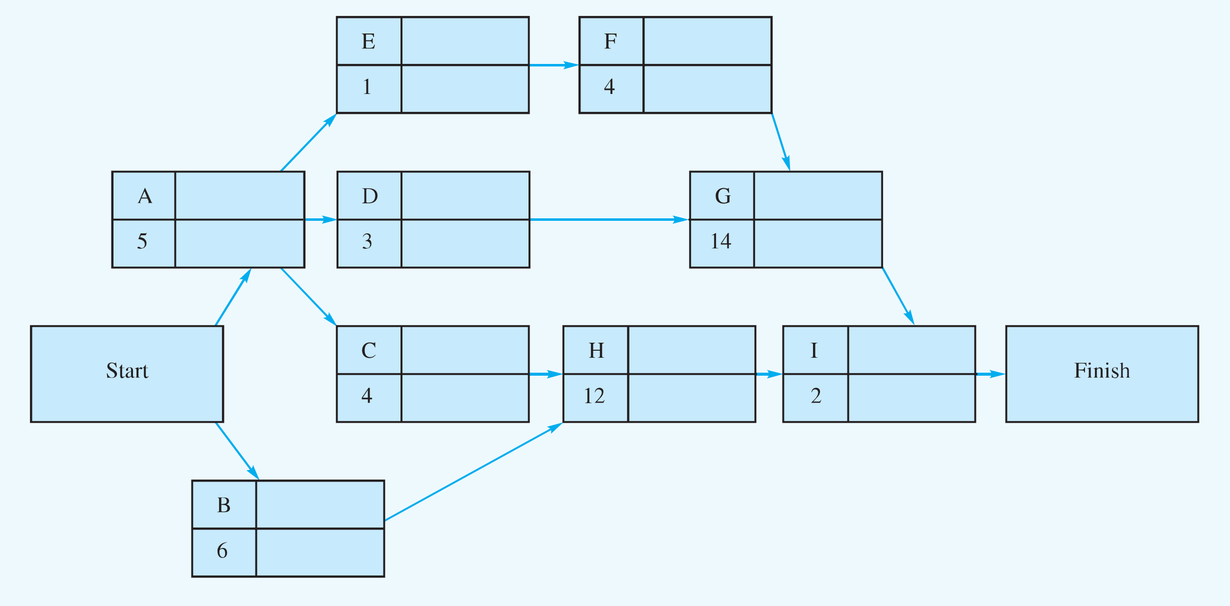


FIGURE 13.2 WESTERN HILLS SHOPPING CENTER PROJECT NETWORK WITH ACTIVITY TIMES



The Concept of a Critical Path

To facilitate the PERT/CPM computations, we modified the project network as shown in Figure 13.2. Note that the upper left-hand corner of each node contains the corresponding activity letter. The activity time appears immediately below the letter.

Problem 3 provides the immediate predecessor information for a project with seven activities and asks you to develop the project network.

For convenience, we use the convention of referencing activities with letters. Generally, we assign the letters in approximate order as we move from left to right through the project network.

To determine the project completion time, we have to analyze the network and identify what is called the **critical path** for the network. However, before doing so, we need to define the concept of a path through the network. A **path** is a sequence of connected nodes that leads from the Start node to the Finish node. For instance, one path for the network in Figure 13.2 is defined by the sequence of nodes A-E-F-G-I. By inspection, we see that other paths are possible, such as A-D-G-I, A-C-H-I, and B-H-I. All paths in the network must be traversed in order to complete the project, so we will look for the path that requires the most time. Because all other paths are shorter in duration, this *longest* path determines the total time required to complete the project. If activities on the longest path are delayed, the entire project will be delayed. Thus, the longest path is the *critical path.* Activities on the critical path are referred to as the **critical activities** for the project. The following discussion presents a step-by-step algorithm for finding the critical path in a project network.

Determining the Critical Path

We begin by finding the **earliest start time** and a **latest start time** for all activities in the network. Let

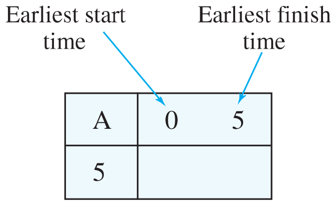
ES = earliest start time for an activity
EF = earliest finish time for an activity
t = expected activity time

The **earliest finish time** for any activity is

$$EF = ES + t \tag{13.1}$$

Activity A can start as soon as the project starts, so we set the earliest start time for activity A equal to 0. With an expected activity time of 5 weeks, the earliest finish time for activity A is $EF = ES + t = 0 + 5 = 5$.

We will write the earliest start and earliest finish times in the node to the right of the activity letter. Using activity A as an example, we have

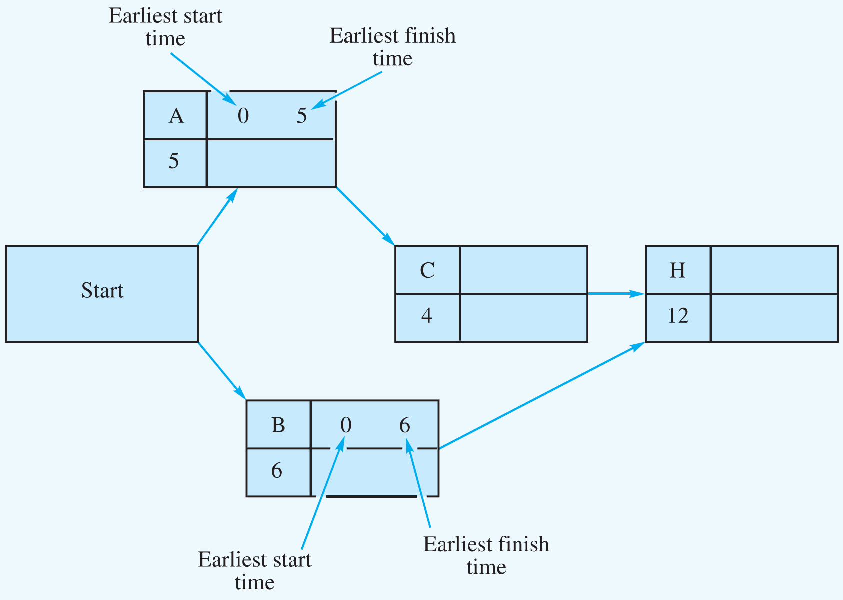


Because an activity cannot be started until *all* immediately preceding activities have been finished, the following rule can be used to determine the earliest start time for each activity:

> The earliest start time for an activity is equal to the *largest* (i.e., *latest*) of the earliest finish times for all its immediate predecessors.

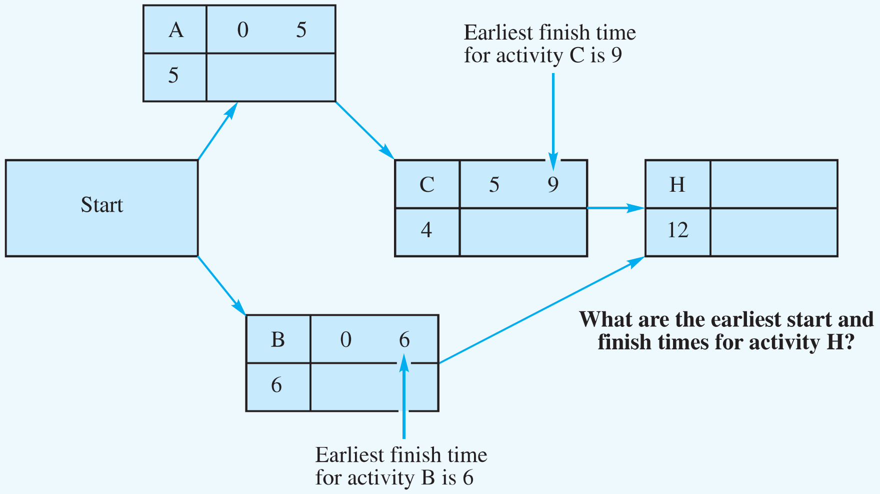
Let us apply the earliest start time rule to the portion of the network involving nodes A, B, C, and H, as shown in Figure 13.3. With an earliest start time of 0 and an activity time of 6 for activity B, we show $ES = 0$ and $EF = ES + t = 0 + 6 = 6$ in the node for

FIGURE 13.3 A PORTION OF THE WESTERN HILLS SHOPPING CENTER PROJECT NETWORK, SHOWING ACTIVITIES A, B, C, AND H



© Cengage Learning 2013

FIGURE 13.4 DETERMINING THE EARLIEST START TIME FOR ACTIVITY H



© Cengage Learning 2013

activity B. Looking at node C, we note that activity A is the only immediate predecessor for activity C. The earliest finish time for activity A is 5, so the earliest start time for activity C must be $ES = 5$. Thus, with an activity time of 4, the earliest finish time for activity C is $EF = ES + t = 5 + 4 = 9$. Both the earliest start time and the earliest finish time can be shown in the node for activity C (see Figure 13.4).

Determining the expected completion time of a project via critical path calculations implicitly assumes there is availability of sufficient resources (labor, equipment, supplies, etc.) to execute activities in parallel. If there is insufficient availability of resources to support the schedule generated by PERT/CPM, then more advanced techniques such as an integer linear programming model (Chapter 11) can be applied.

Continuing with Figure 13.4, we move on to activity H and apply the earliest start time rule for this activity. With both activities B and C as immediate predecessors, the earliest start time for activity H must be equal to the largest of the earliest finish times for activities B and C. Thus, with $EF = 6$ for activity B and $EF = 9$ for activity C, we select the largest value, 9, as the earliest start time for activity H ($ES = 9$). With an activity time of 12 as shown in the node for activity H, the earliest finish time is $EF = ES + t = 9 + 12 = 21$. The $ES = 9$ and $EF = 21$ values can now be entered in the node for activity H in Figure 13.4.

Continuing with this **forward pass** through the network, we can establish the earliest start times and the earliest finish times for each activity in the network. Figure 13.5 shows the Western Hills Shopping Center project network with the *ES* and *EF* values for each activity. Note that the earliest finish time for activity I, the last activity in the project, is 26 weeks. Therefore, we now know that the expected completion time for the entire project is 26 weeks.

We now continue the algorithm for finding the critical path by making a **backward pass** through the network. Because the expected completion time for the entire project is 26 weeks, we begin the backward pass with a **latest finish time** of 26 for activity I. Once the latest finish time for an activity is known, the *latest start time* for an activity can be computed as follows. Let

LS = latest start time for an activity

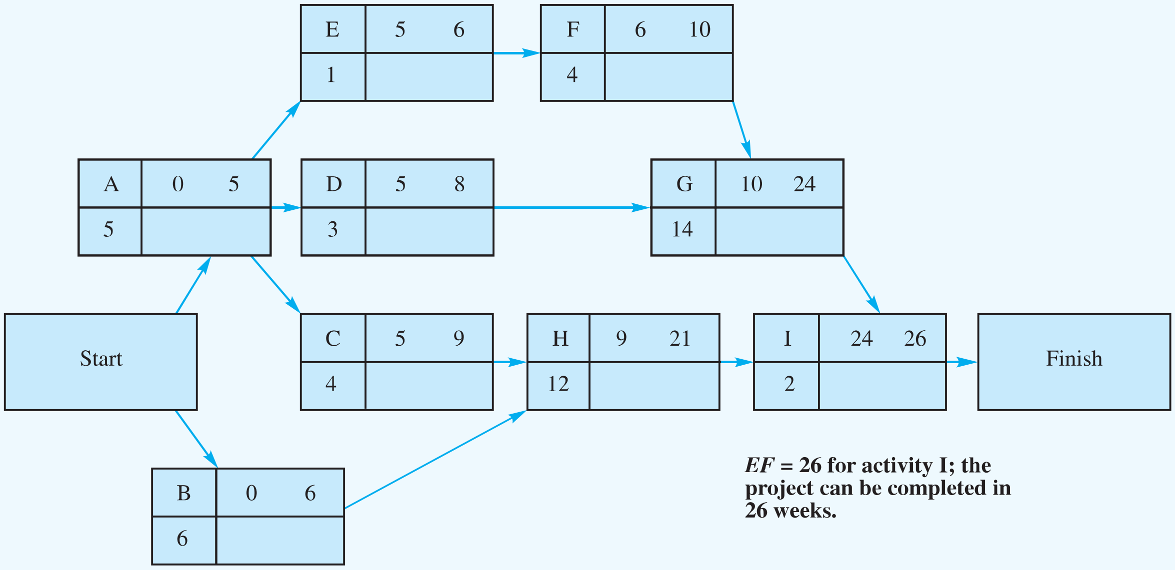
LF = latest finish time for an activity

Then

$$LS = LF - t \tag{13.2}$$

Beginning the backward pass with activity I, we know that the latest finish time is $LF = 26$ and that the activity time is $t = 2$. Thus, the latest start time for activity I is

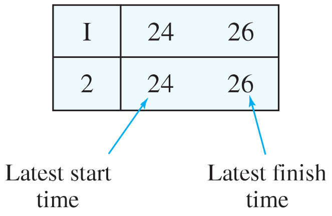
FIGURE 13.5 WESTERN HILLS SHOPPING CENTER PROJECT NETWORK WITH EARLIEST START AND EARLIEST FINISH TIMES SHOWN FOR ALL ACTIVITIES



***EF* = 26 for activity I; the project can be completed in 26 weeks.**

© Cengage Learning 2013

$LS = LF - t = 26 - 2 = 24$. We will write the LS and LF values in the node directly below the earliest start (ES) and earliest finish (EF) times. Thus, for node I, we have

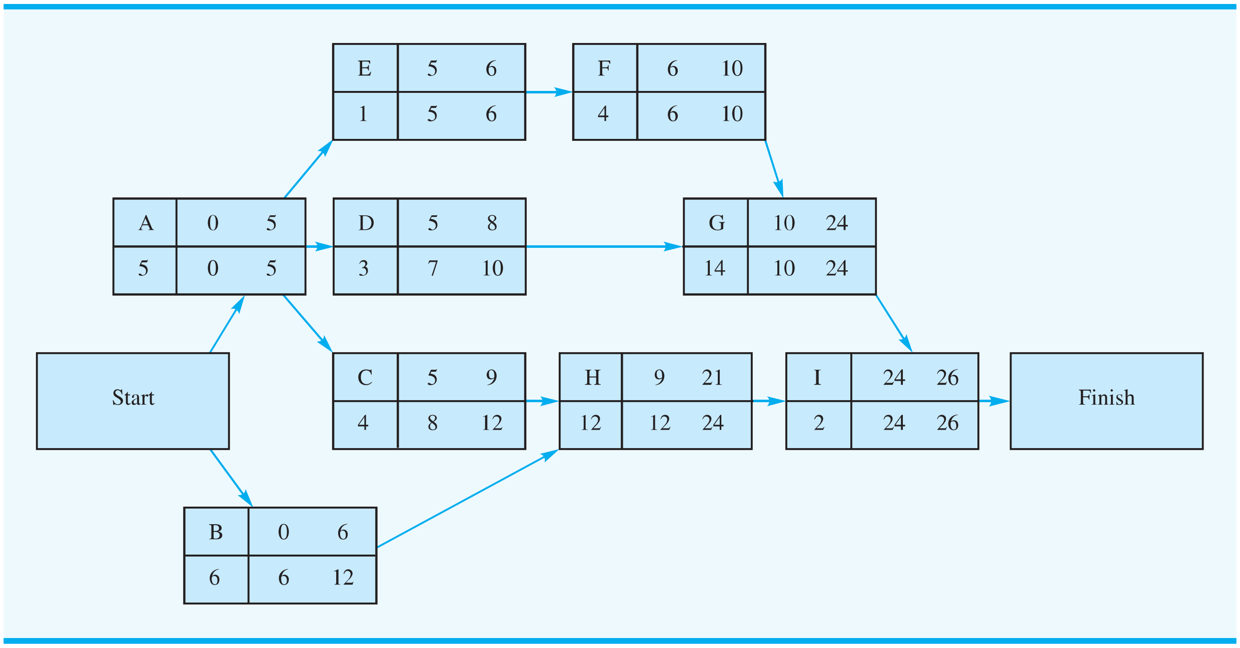


The following rule can be used to determine the latest finish time for each activity in the network:

> The latest finish time for an activity is the smallest of the latest start times for all activities that immediately follow the activity.

Logically, this rule states that the latest time an activity can be finished equals the earliest (smallest) value for the latest start time of following activities. Figure 13.6 shows the complete project network with the LS and LF backward pass results. We can use the latest finish time rule to verify the LS and LF values shown for activity H. The latest finish time for activity H must be the latest start time for activity I. Thus, we set $LF = 24$ for activity H. Using equation (13.2), we find that $LS = LF - t = 24 - 12 = 12$ as the latest start time for activity H. These values are shown in the node for activity H in Figure 13.6.

FIGURE 13.6 WESTERN HILLS SHOPPING CENTER PROJECT NETWORK WITH LATEST START AND LATEST FINISH TIMES SHOWN IN EACH NODE



© Cengage Learning 2013

Activity A requires a more involved application of the latest start time rule. First, note that three activities (C, D, and E) immediately follow activity A. Figure 13.6 shows that the latest start times for activities C, D, and E are $LS = 8$, $LS = 7$, and $LS = 5$, respectively. The latest finish time rule for activity A states that the LF for activity A is the smallest of the latest start times for activities C, D, and E. With the smallest value being 5 for activity E, we set the latest finish time for activity A to $LF = 5$. Verify this result and the other latest start times and latest finish times shown in the nodes in Figure 13.6.

The slack for each activity indicates the length of time the activity can be delayed without increasing the project completion time.

After we complete the forward and backward passes, we can determine the amount of slack associated with each activity. **Slack** is the length of time an activity can be delayed without increasing the project completion time. The amount of slack for an activity is computed as follows:

$$\text{Slack} = LS - ES = LF - EF \tag{13.3}$$

One of the primary contributions of PERT/CPM is the identification of the critical activities. The project manager will want to monitor critical activities closely because a delay in any one of these activities will lengthen the project completion time.

For example, the slack associated with activity C is $LS - ES = 8 - 5 = 3$ weeks. Hence, activity C can be delayed up to 3 weeks, and the entire project can still be completed in 26 weeks. In this sense, activity C is not critical to the completion of the entire project in 26 weeks. Next, we consider activity E. Using the information in Figure 13.6, we find that the slack is $LS - ES = 5 - 5 = 0$. Thus, activity E has zero, or no, slack. Consequently, this activity cannot be delayed without increasing the completion time for the entire project. In other words, completing activity E exactly as scheduled is critical in terms of keeping the project on schedule, and so activity E is a critical activity. In general, the *critical activities* are the activities with zero slack.

The critical path algorithm is essentially a longest path algorithm. From the start node to the finish node, the critical path identifies the path that requires the most time.

The start and finish times shown in Figure 13.6 can be used to develop a detailed start time and finish time schedule for all activities. Putting this information in tabular form provides the activity schedule shown in Table 13.2. Note that the slack column shows that activities A, E, F, G, and I have zero slack. Hence, these activities are the critical activities for the project. The path formed by nodes A-E-F-G-I is the *critical path* in the Western Hills Shopping Center project network. The detailed schedule shown in Table 13.2 indicates the slack or delay that can be tolerated for the noncritical activities before these activities will increase project completion time.

TABLE 13.2 ACTIVITY SCHEDULE FOR THE WESTERN HILLS SHOPPING CENTER PROJECT

Activity	Earliest Start (ES)	Latest Start (LS)	Earliest Finish (EF)	Latest Finish (LF)	Slack ($LS - ES$)	Critical Path?
A	0	0	5	5	0	Yes
B	0	6	6	12	6	
C	5	8	9	12	3	
D	5	7	8	10	2	
E	5	5	6	6	0	Yes
F	6	6	10	10	0	Yes
G	10	10	24	24	0	Yes
H	9	12	21	24	3	
I	24	24	26	26	0	Yes

Contributions of PERT/CPM

If the expected time required to complete the project is too long, judgment about where and how to shorten the time of critical activities must be exercised. If any activity times are altered, the critical path calculations should be repeated to determine the impact on the activity schedule and the impact on the expected project completion time. In Section 13.3 we show how to use linear programming to find the least-cost way to shorten the project completion time.

We previously stated that project managers look for procedures that will help answer important questions regarding the planning, scheduling, and controlling of projects. Let us reconsider these questions in light of the information that the critical path calculations have given us.

1. How long will the project take to complete?
   Answer: The project can be completed in 26 weeks if each activity is completed on schedule.
2. What are the scheduled start and completion times for each activity?
   Answer: The activity schedule (see Table 13.2) shows the earliest start, latest start, earliest finish, and latest finish times for each activity.
3. Which activities are critical and must be completed *exactly* as scheduled to keep the project on schedule?
   Answer: A, E, F, G, and I are the critical activities.
4. How long can noncritical activities be delayed before they cause an increase in the completion time for the project?
   Answer: The activity schedule (see Table 13.2) shows the slack associated with each activity.

Software packages such as Microsoft Project perform the critical path calculations quickly and efficiently. Program inputs include the activities, their immediate predecessors, and expected activity times. The project manager can modify any aspect of the project and quickly determine how the modification affects the activity schedule and the expected time required to complete the project.

Such information is valuable in managing any project. Although the effort required to develop the immediate predecessor relationships and the activity time estimates generally increases with the size of the project, the procedure and contribution of PERT/CPM to larger projects are identical to those shown for the shopping center expansion project. The Q.M. in Action, Hospital Revenue Bond at Seasongood & Mayer, describes a 23-activity project that introduced a \$31 million hospital revenue bond. PERT/CPM identified the critical activities, the expected project completion time of 29 weeks, and the activity start times and finish times necessary to keep the entire project on schedule.

Summary of the PERT/CPM Critical Path Procedure

Before leaving this section, let us summarize the PERT/CPM critical path procedure.

Step 1. Develop a list of the activities that make up the project.
Step 2. Determine the immediate predecessor(s) for each activity in the project.
Step 3. Estimate the expected completion time for each activity.
Step 4. Draw a project network depicting the activities and immediate predecessors listed in steps 1 and 2.
Step 5. Use the project network and the activity time estimates to determine the earliest start and the earliest finish time for each activity by making a forward pass through the network. The earliest finish time for the last activity in the project identifies the expected time required to complete the entire project.
Step 6. Use the expected project completion time identified in step 5 as the latest finish time for the last activity and make a backward pass through the network to identify the latest start and latest finish time for each activity.
Step 7. Use the difference between the latest start time and the earliest start time for each activity to determine the slack for each activity.
Step 8. Find the activities with zero slack; these are the critical activities.
Step 9. Use the information from steps 5 and 6 to develop the activity schedule for the project.

Q.M. *in* ACTION

HOSPITAL REVENUE BOND AT SEASONGOOD & MAYER

Seasongood & Mayer is an investment securities firm located in Cincinnati, Ohio. The firm engages in municipal financing, including the underwriting of new issues of municipal bonds, acting as a market maker for previously issued bonds, and performing other investment banking services.

Seasongood & Mayer provided the underwriting for a $31 million issue of hospital facilities revenue bonds for Providence Hospital in Hamilton County, Ohio. The project of underwriting this municipal bond issue began with activities such as drafting the legal documents, drafting a description of the existing hospital facilities, and completing a feasibility study. A total of 23 activities defined the project that would be completed when the hospital signed the construction contract and then made the bond proceeds available. The immediate predecessor relationships for the activities and the activity times were developed by a project management team.

PERT/CPM analysis of the project network identified the 10 critical path activities. The analysis also provided the expected completion time of 29 weeks, or approximately seven months. The activity schedule showed the start time and finish time for each activity and provided the information necessary to monitor the project and keep it on schedule. PERT/CPM was instrumental in helping Seasongood & Mayer obtain the financing for the project within the time specified in the construction bid.

NOTES AND COMMENTS

1. Suppose that, after analyzing a PERT/CPM network, the project manager finds that the project completion time is unacceptable (i.e., the project is going to take too long). In this case, the manager must take one or both of the following steps. First, review the original PERT/CPM network to see whether any immediate predecessor relationships can be modified so that at least some of the critical path activities can be done simultaneously. Second, consider adding resources to critical path activities in an attempt to shorten the critical path; we discuss this alternative, referred to as *crashing,* in Section 13.3.

13.2 Project Scheduling Considering Uncertain Activity Times

In this section we consider the details of project scheduling for a problem involving new-product research and development. Because many of the activities in such a project have never been attempted, the project manager wants to account for uncertainties in the activity times. Let us show how project scheduling can be conducted with uncertain activity times.

The Daugherty Porta-Vac Project

The H. S. Daugherty Company has manufactured industrial vacuum cleaning systems for many years. Recently, a member of the company's new-product research team submitted a report suggesting that the company consider manufacturing a cordless vacuum cleaner. The new product, referred to as Porta-Vac, could contribute to Daugherty's expansion into the household market. Management hopes that it can be manufactured at a reasonable cost and that its portability and no-cord convenience will make it extremely attractive.

Accurate activity time estimates are important in the development of an activity schedule. When activity times are uncertain, the three time estimates—optimistic, most probable, and pessimistic—allow the project manager to take uncertainty into consideration in determining the critical path and the activity schedule. This approach was developed by the designers of PERT.

Daugherty's management wants to study the feasibility of manufacturing the Porta-Vac product. The feasibility study will provide a recommendation on the action to be taken. To complete this study, information must be obtained from the firm's research and development (R&D), product testing, manufacturing, cost estimating, and market research groups. How long will it take to complete this feasibility study? In the following discussion, we show how to answer this question and provide an activity schedule for the project.

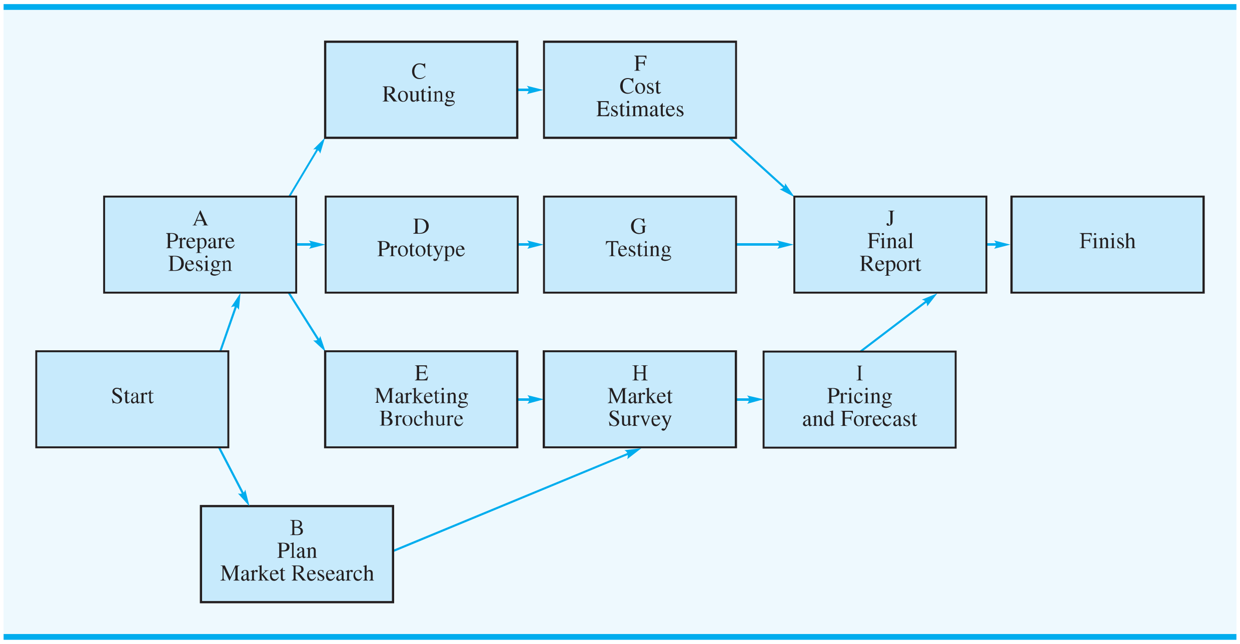
Again, the first step in the project scheduling process is to identify all activities that make up the project and then determine the immediate predecessor(s) for each activity. Table 13.3 shows these data for the Porta-Vac project.

The Porta-Vac project network is shown in Figure 13.7. Verify that the network does in fact maintain the immediate predecessor relationships shown in Table 13.3.

TABLE 13.3 ACTIVITY LIST FOR THE PORTA-VAC PROJECT

Activity	Description	Immediate Predecessor
A	Develop product design	—
B	Plan market research	—
C	Prepare routing (manufacturing engineering)	A
D	Build prototype model	A
E	Prepare marketing brochure	A
F	Prepare cost estimates (industrial engineering)	C
G	Do preliminary product testing	D
H	Complete market survey	B, E
I	Prepare pricing and forecast report	H
J	Prepare final report	F, G, I

FIGURE 13.7 PORTA-VAC CORDLESS VACUUM CLEANER PROJECT NETWORK



Uncertain Activity Times

Once we develop the project network, we will need information on the time required to complete each activity. This information is used in the calculating the total time required to complete the project and in the scheduling of specific activities. For repeat projects, such as construction and maintenance projects, managers may have the experience and historical data necessary to provide accurate activity time estimates. However, for new or unique projects, estimating the time for each activity may be quite difficult. In fact, in many cases activity times are uncertain and are best described by a range of possible values rather than by one specific time estimate. In these instances, the uncertain activity times are treated as random variables with associated probability distributions. As a result, probability statements will be provided about the ability to meet a specific project completion date.

To incorporate uncertain activity times into the analysis, we need to obtain three time estimates for each activity:

Optimistic time a = the minimum activity time if everything progresses ideally
Most probable time m = the most probable activity time under normal conditions
Pessimistic time b = the maximum activity time if significant delays are encountered

To illustrate the PERT/CPM procedure with uncertain activity times, let us consider the optimistic, most probable, and pessimistic time estimates for the Porta-Vac activities as presented in Table 13.4. Using activity A as an example, we see that the most probable time is 5 weeks, with a range from 4 weeks (optimistic) to 12 weeks (pessimistic). If the activity could be repeated a large number of times, what is the average time for the activity? This average or **expected time** (t) is as follows:

$$t = \frac{a + 4m + b}{6} \tag{13.4}$$

For activity A we have an average or expected time of

$$t_{\mathrm{A}} = \frac{4 + 4(5) + 12}{6} = \frac{36}{6} = 6 \text{ weeks}$$

TABLE 13.4 OPTIMISTIC, MOST PROBABLE, AND PESSIMISTIC ACTIVITY TIME ESTIMATES (IN WEEKS) FOR THE PORTA-VAC PROJECT

Activity	Optimistic (a)	Most Probable (m)	Pessimistic (b)
A	4	5	12
B	1	1.5	5
C	2	3	4
D	3	4	11
E	2	3	4
F	1.5	2	2.5
G	1.5	3	4.5
H	2.5	3.5	7.5
I	1.5	2	2.5
J	1	2	3

With uncertain activity times, we can use the *variance* to describe the dispersion or variation in the activity time values. The variance of the activity time is given by the formula[1]

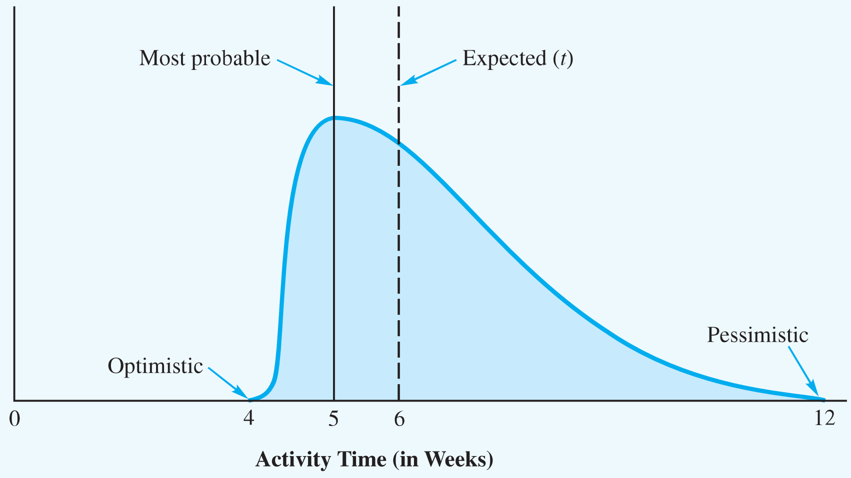
$$\sigma^2 = \left(\frac{b - a}{6}\right)^2 \tag{13.5}$$

The difference between the pessimistic (b) and optimistic (a) time estimates greatly affects the value of the variance. Large differences in these two values reflect a high degree of uncertainty in the activity time. Using equation (13.5), we obtain the measure of uncertainty—that is, the variance—of activity A, denoted σ_A^2:

$$\sigma_A^2 = \left(\frac{12 - 4}{6}\right)^2 = \left(\frac{8}{6}\right)^2 = 1.78$$

Equations (13.4) and (13.5) are based on the assumption that the activity time distribution can be described by a **beta probability distribution**.[2] With this assumption, the probability distribution for the time to complete activity A is as shown in Figure 13.8. Using equations (13.4) and (13.5) and the data in Table 13.4, we calculated the expected times and variances for all Porta-Vac activities; the results are summarized in Table 13.5. The Porta-Vac project network with expected activity times is shown in Figure 13.9.

FIGURE 13.8 ACTIVITY TIME DISTRIBUTION FOR PRODUCT DESIGN (ACTIVITY A) FOR THE PORTA-VAC PROJECT



© Cengage Learning 2013

---

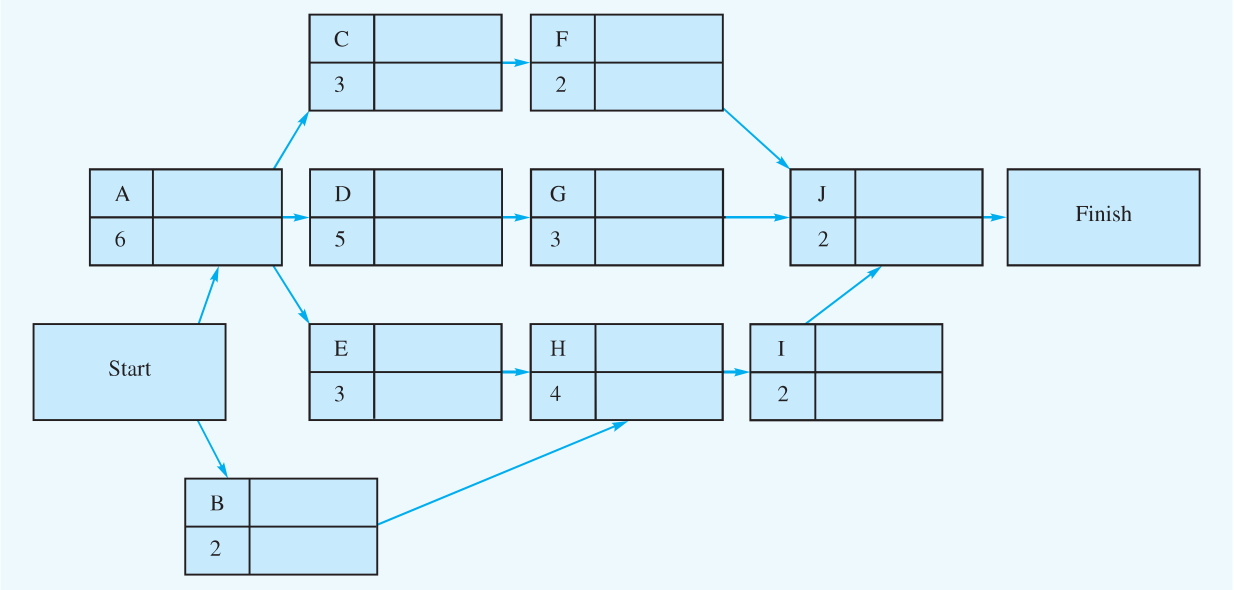
[1] The variance equation is based on the notion that a standard deviation is approximately $\frac{1}{6}$ of the difference between the extreme values of the distribution: $(b - a)/6$. The variance is the square of the standard deviation.

[2] The equations for t and σ^2 require additional assumptions about the parameters of the beta probability distribution. However, even when these additional assumptions are not made, the equations still provide good approximations of t and σ^2.

TABLE 13.5 EXPECTED TIMES AND VARIANCES FOR THE PORTA-VAC PROJECT ACTIVITIES

Activity	Expected Time (weeks)	Variance
A	6	1.78
B	2	0.44
C	3	0.11
D	5	1.78
E	3	0.11
F	2	0.03
G	3	0.25
H	4	0.69
I	2	0.03
J	2	0.11
Total	32	

FIGURE 13.9 PORTA-VAC PROJECT NETWORK WITH EXPECTED ACTIVITY TIMES



When uncertain activity times are considered, the actual time required to complete the project may differ from the expected time to complete the project provided by the critical path calculations. However, for planning purposes, the expected time should be valuable information for the project manager.

The Critical Path

When we have the project network and the expected activity times, we are ready to proceed with the critical path calculations necessary to determine the expected time required to complete the project and determine the activity schedule. In these calculations, we find the critical path for the Porta-Vac project by applying the critical path procedure introduced in Section 13.1 to the expected activity times (Table 13.5). After the critical activities and the expected time to complete the project have been determined, we analyze the effect of the activity time variability.

Proceeding with a forward pass through the network shown in Figure 13.9, we can establish the earliest start (*ES*) and earliest finish (*EF*) times for each activity. Figure 13.10 shows the project network with the *ES* and *EF* values. Note that the earliest finish time for activity J, the last activity, is 17 weeks. Thus, the expected completion time for the project is 17 weeks. Next, we make a backward pass through the network. The backward pass provides the latest start (*LS*) and latest finish (*LF*) times shown in Figure 13.11.

FIGURE 13.10 PORTA-VAC PROJECT NETWORK WITH EARLIEST START AND EARLIEST FINISH TIMES

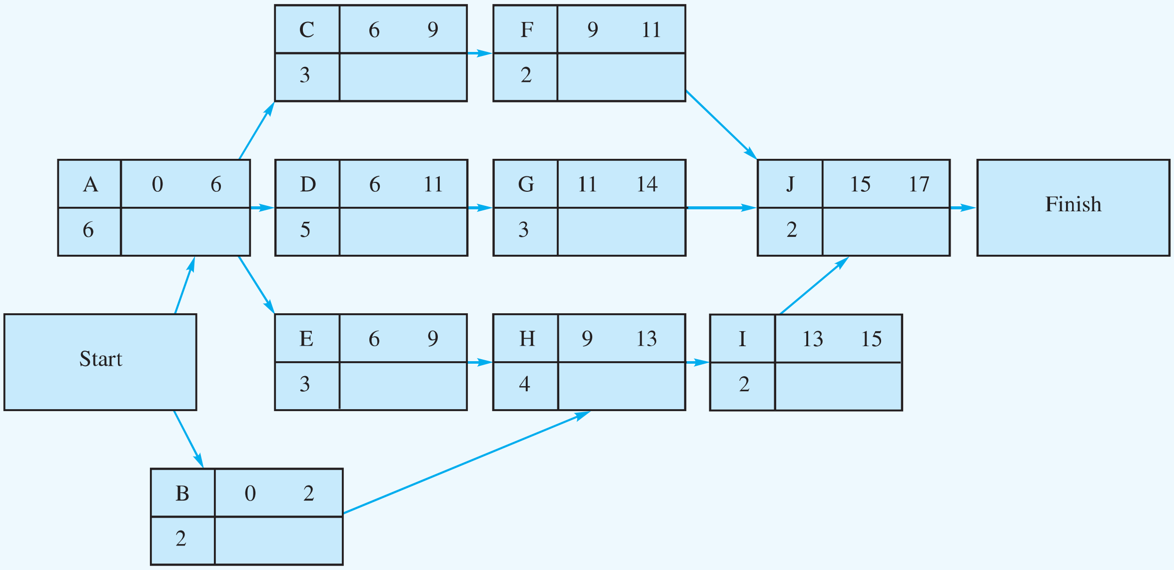


FIGURE 13.11 PORTA-VAC PROJECT NETWORK WITH LATEST START AND LATEST FINISH TIMES

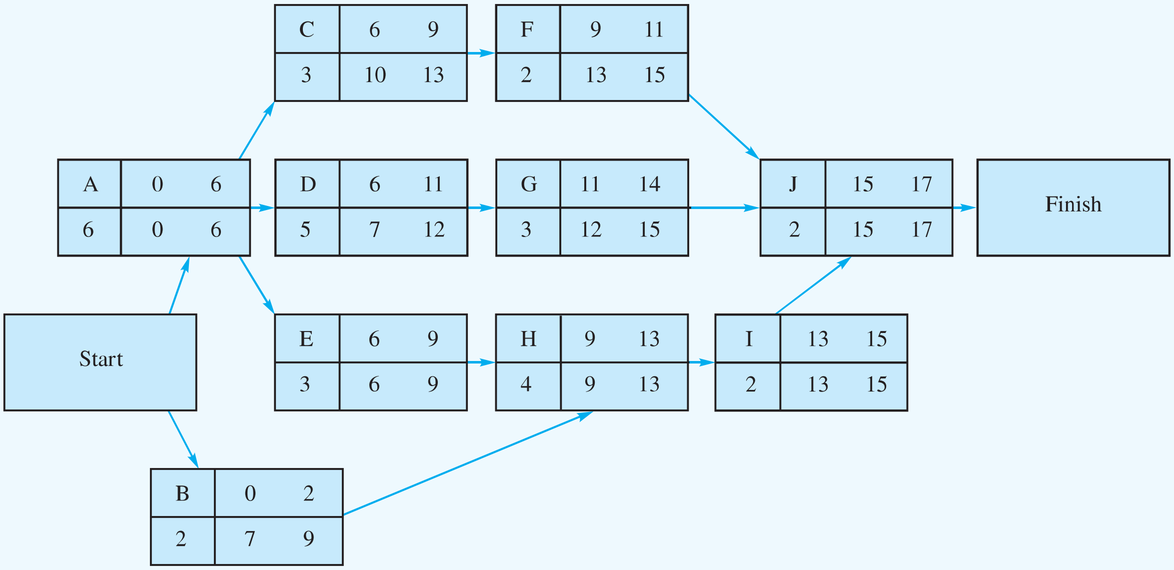


TABLE 13.6 ACTIVITY SCHEDULE FOR THE PORTA-VAC PROJECT

Activity	Earliest Start (*ES*)	Latest Start (*LS*)	Earliest Finish (*EF*)	Latest Finish (*LF*)	Slack (*LS* − *ES*)	Critical Path?
A	0	0	6	6	0	Yes
B	0	7	2	9	7	
C	6	10	9	13	4	
D	6	7	11	12	1	
E	6	6	9	9	0	Yes
F	9	13	11	15	4	
G	11	12	14	15	1	
H	9	9	13	13	0	Yes
I	13	13	15	15	0	Yes
J	15	15	17	17	0	Yes

The activity schedule for the Porta-Vac project is shown in Table 13.6. Note that the slack time (*LS* − *ES*) is also shown for each activity. The activities with zero slack (A, E, H, I, and J) form the critical path for the Porta-Vac project network.

Variability in Project Completion Time

Activities that have larger variances exhibit a greater degree of uncertainty. The project manager should monitor the progress of any activity with a large variance even if the expected time does not identify the activity as a critical activity.

We know that for the Porta-Vac project the critical path of A-E-H-I-J resulted in an expected total project completion time of 17 weeks. However, variation in critical activities can cause variation in the project completion time. Variation in noncritical activities ordinarily has no effect on the project completion time because of the slack time associated with these activities. However, if a noncritical activity is delayed long enough to expend its slack time, it becomes part of a new critical path and may affect the project completion time. Variability leading to a longer-than-expected total time for the critical activities will always extend the project completion time, and, conversely, variability that results in a shorter-than-expected total time for the critical activities will reduce the project completion time, unless other activities become critical. Let us now use the variance in the critical activities to determine the variance in the project completion time.

Let T denote the total time required to complete the project. The expected value of T, which is the sum of the expected times for the critical activities, is

$$\begin{aligned} E(T) &= t_{\mathrm{A}} + t_{\mathrm{E}} + t_{\mathrm{H}} + t_{\mathrm{I}} + t_{\mathrm{J}} \\ &= 6 + 3 + 4 + 2 + 2 = 17 \text{ weeks} \end{aligned}$$

Problem 10 involves a project with uncertain activity times and asks you to compute the expected completion time and the variance for the project.

The variance in the project completion time is the sum of the variances of the critical path activities. Thus, the variance for the Porta-Vac project completion time is

$$\begin{aligned} \sigma^2 &= \sigma^2_{\mathrm{A}} + \sigma^2_{\mathrm{E}} + \sigma^2_{\mathrm{H}} + \sigma^2_{\mathrm{I}} + \sigma^2_{\mathrm{J}} \\ &= 1.78 + 0.11 + 0.69 + 0.03 + 0.11 = 2.72 \end{aligned}$$

where σ^2_{A}, σ^2_{E}, σ^2_{H}, σ^2_{I}, and σ^2_{J} are the variances of the critical activities.

The formula for σ^2 is based on the assumption that the activity times are independent. If two or more activities are dependent, the formula provides only an approximation of the variance of the project completion time. The closer the activities are to being independent, the better the approximation.

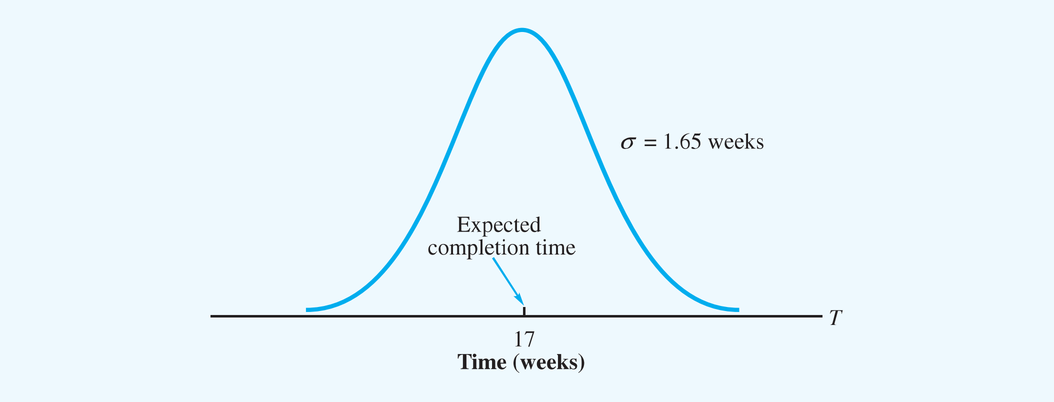
Knowing that the standard deviation is the square root of the variance, we compute the standard deviation σ for the Porta-Vac project completion time as

$$\sigma = \sqrt{\sigma^2} = \sqrt{2.72} = 1.65$$

The normal distribution tends to be a better approximation of the distribution of total time for larger projects, where the critical path has many activities.

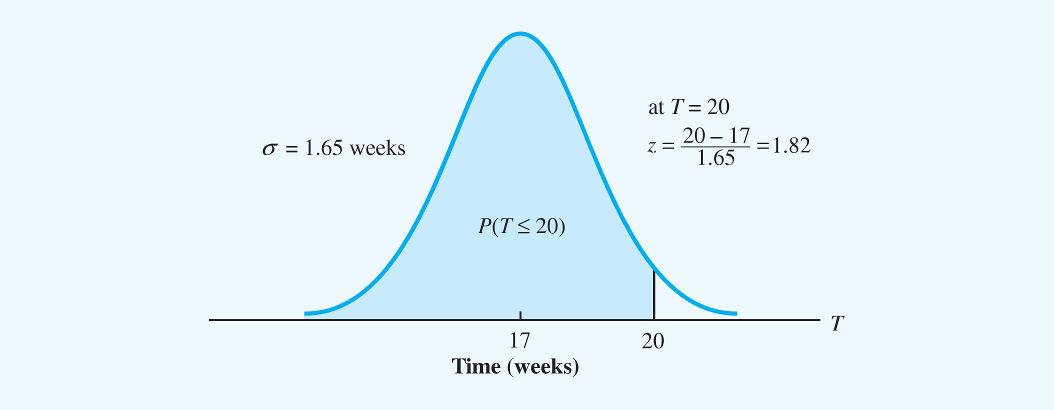
Assuming that the distribution of the project completion time T follows a normal or bell-shaped distribution[3] allows us to draw the distribution shown in Figure 13.12. With this distribution, we can compute the probability of meeting a specified project completion date. For example, suppose that management allotted 20 weeks for the Porta-Vac project. What is the probability that we will meet the 20-week deadline? Using the normal probability distribution shown in Figure 13.13, we are asking for the probability that $T \leq 20$; this

FIGURE 13.12 NORMAL DISTRIBUTION OF THE PROJECT COMPLETION TIME FOR THE PORTA-VAC PROJECT



© Cengage Learning 2013

FIGURE 13.13 PROBABILITY THE PORTA-VAC PROJECT WILL MEET THE 20-WEEK DEADLINE



© Cengage Learning 2013

---

[3] Use of the normal distribution as an approximation is based on the central limit theorem, which indicates that the sum of independent random variables (activity times) follows a normal distribution as the number of random variables becomes large.

probability is shown graphically as the shaded area in the figure. The z value for the normal probability distribution at $T = 20$ is

$$z = \frac{20 - 17}{1.65} = 1.82$$

Using $z = 1.82$ and the table for the normal distribution (see Appendix D), we find that the probability of the project meeting the 20-week deadline is 0.9656. Of course, this result also implies the probability we will not meet the 20-week deadline is $1 - 0.9656 = 0.0344$. Thus, even though activity time variability may cause the completion time to exceed 17 weeks, calculations indicate an excellent chance that the project will be completed before the 20-week deadline. Similar probability calculations can be made for other project deadline alternatives.

NOTES AND COMMENTS

1. For projects involving uncertain activity times, the probability that the project can be completed within a specified amount of time is helpful managerial information. However, remember that this probability estimate is based only on the critical activities. When uncertain activity times exist, longer-than-expected completion times for one or more noncritical activities may cause an original noncritical activity to become critical and hence increase the time required to complete the project. By frequently monitoring the progress of the project to make sure all activities are on schedule, the project manager will be better prepared to take corrective action if a noncritical activity begins to lengthen the duration of the project. The Q.M. in Action, Project Management Helps the U.S. Air Force Reduce Maintenance Time, describes how closely managing the progress of individual activities as well as the assignment of resources led to dramatic improvements in the maintenance of military aircraft.
2. Statistical packages such as Minitab and SAS, as well as Excel, have routines to calculate cumulative probabilities for normally distributed random variables.

Q.M. *in* ACTION

*PROJECT MANAGEMENT HELPS THE U.S. AIR FORCE REDUCE MAINTENANCE TIME**

Warner Robins Air Logistics Center (WR-ALC) provides maintenance and repair services for U.S. Air Force aircraft and ground equipment. To support combat zone efforts, the U.S. Air Force requested that WR-ALC reduce the amount of time it took to complete maintenance service on its C-5 transporter aircraft.

To identify ways to improve the management of its repair and overhaul process, WR-ALC adopted the method of critical chain project management (CCPM) by viewing each aircraft at its facility as a project with a series of tasks, precedence dependencies between these tasks, and resource requirements. Identifying tasks at a level of detail that allowed supervisors to clearly assign mechanics, maintenance tools, and facilities resulted in a project network of approximately 450 activities.

By explicitly accounting for each task's resource requirements (mechanics, aircraft parts, maintenance tools, etc.), CCPM identifies a "critical chain" of activities. Efforts to reduce the critical chain led to the insight that a task should not be started until all resources needed to complete the task are available. While this approach, called "pipelining," often results in an initial delay to the start of a task, it allows for the quicker completion of the task by eliminating delays after the task's launch and by reducing efficiency-robbing multitasking (across tasks) by the mechanics.

---

*M. M. Srinivasan, W. D. Best, and S. Chandrasekaran, "Warner Robins Air Logistics Center Streamlines Aircraft Repair and Overhaul," *Interfaces* 37, no. 1 (2007), pp. 7–21.

13.3 Considering Time–Cost Trade-Offs

Using more resources to reduce activity times was proposed by the developers of CPM. The shortening of activity times is referred to as crashing.

When determining the time estimates for activities in a project, the project manager bases these estimates on the amount of resources (workers, equipment, etc.) that will be assigned to an activity. The original developers of CPM provided the project manager with the option of adding resources to selected activities to reduce project completion time. Added resources (such as more workers, overtime, and so on) generally increase project costs, so the decision to reduce activity times must take into consideration the additional cost involved. In effect, the project manager must make a decision that involves trading reduced activity time for additional project cost.

Table 13.7 defines a two-machine maintenance project consisting of five activities. Management has substantial experience with similar projects and the times for maintenance activities have very little variability; hence, a single time estimate is given for each activity. The project network is shown in Figure 13.14.

The procedure for making critical path calculations for the maintenance project network is the same one used to find the critical path in the networks for both the Western Hills Shopping Center expansion project and the Porta-Vac project. Making the forward pass and backward pass calculations for the network in Figure 13.14, we obtained the activity schedule shown in Table 13.8. The zero slack times, and thus the critical path, are associated with activities A-B-E. The length of the critical path, and thus the total time required to complete the project, is 12 days.

TABLE 13.7 ACTIVITY LIST FOR THE TWO-MACHINE MAINTENANCE PROJECT

Activity	Description	Immediate Predecessor	Expected Time (days)
A	Overhaul machine I	—	7
B	Adjust machine I	A	3
C	Overhaul machine II	—	6
D	Adjust machine II	C	3
E	Test system	B, D	2

FIGURE 13.14 TWO-MACHINE MAINTENANCE PROJECT NETWORK

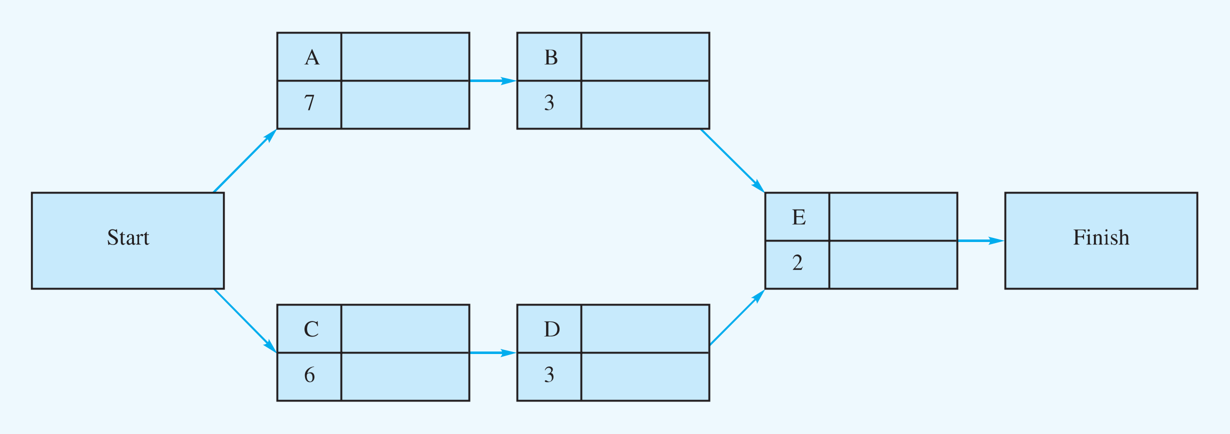


TABLE 13.8 ACTIVITY SCHEDULE FOR THE TWO-MACHINE MAINTENANCE PROJECT

Activity	Earliest Start (*ES*)	Latest Start (*LS*)	Earliest Finish (*EF*)	Latest Finish (*LF*)	Slack (*LS* − *ES*)	Critical Path?
A	0	0	7	7	0	Yes
B	7	7	10	10	0	Yes
C	0	1	6	7	1	
D	6	7	9	10	1	
E	10	10	12	12	0	Yes

Crashing Activity Times

Now suppose that current production levels make completing the maintenance project within 10 days imperative. By looking at the length of the critical path of the network (12 days), we realize that meeting the desired project completion time is impossible unless we can shorten selected activity times. This shortening of activity times, which usually can be achieved by adding resources, is referred to as **crashing**. Because the added resources associated with crashing activity times usually result in added project costs, we will want to identify the activities that cost the least to crash and then crash those activities only the amount necessary to meet the desired project completion time.

To determine just where and how much to crash activity times, we need information on how much each activity can be crashed and how much the crashing process costs. Hence, we must ask for the following information:

1. Activity cost under the normal or expected activity time
2. Time to complete the activity under maximum crashing (i.e., the shortest possible activity time)
3. Activity cost under maximum crashing

Let

τ_i = expected time for activity i
τ_i' = time for activity i under maximum crashing
M_i = maximum possible reduction in time for activity i due to crashing

Given τ_i and τ_i', we can compute M_i:

$$M_i = \tau_i - \tau_i' \tag{13.6}$$

Next, let C_i denote the cost for activity i under the normal or expected activity time and let C_i' denote the cost for activity i under maximum crashing. Thus, per unit of time (e.g., per day), the crashing cost K_i for each activity is given by

$$K_i = \frac{C_i' - C_i}{M_i} \tag{13.7}$$

For example, if the normal or expected time for activity A is 7 days at a cost of $C_A = \$500$ and the time under maximum crashing is 4 days at a cost of $C'_A = \$800$, equations (13.6) and (13.7) show that the maximum possible reduction in time for activity A is

$$M_A = 7 - 4 = 3 \text{ days}$$

with a crashing cost of

$$K_A = \frac{C'_A - C_A}{M_A} = \frac{800 - 500}{3} = \frac{300}{3} = \$100 \text{ per day}$$

We make the assumption that any portion or fraction of the activity crash time can be achieved for a corresponding portion of the activity crashing cost. For example, if we decided to crash activity A by only $1\frac{1}{2}$ days, the added cost would be $1\frac{1}{2}(\$100) = \$150$, which results in a total activity cost of $500 + $150 = $650. Figure 13.15 shows the graph of the time–cost relationship for activity A. The complete normal and crash activity data for the two-machine maintenance project are given in Table 13.9.

FIGURE 13.15 TIME-COST RELATIONSHIP FOR ACTIVITY A

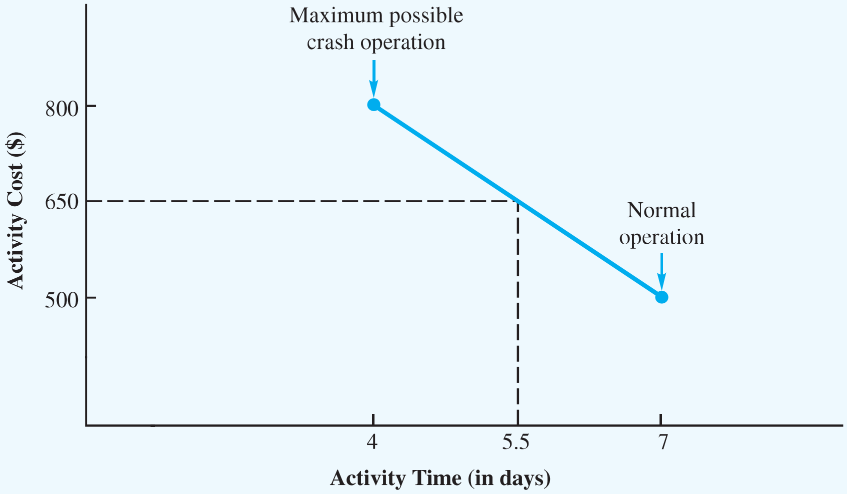


TABLE 13.9 NORMAL AND CRASH ACTIVITY DATA FOR THE TWO-MACHINE MAINTENANCE PROJECT

	Time (days)		Total Cost		Maximum Reduction in Time	Crash Cost per Day
Activity	**Normal**	**Crash**	**Normal (C_i)**	**Crash (C'_i)**	**(M_i)**	$\left(K_i = \frac{C'_i - C_i}{M_i}\right)$
A	7	4	$ 500	$ 800	3	$100
B	3	2	200	350	1	150
C	6	4	500	900	2	200
D	3	1	200	500	2	150
E	2	1	300	550	1	250
			$1700	$3100		

Which activities should be crashed—and by how much—to meet the 10-day project completion deadline at minimum cost? Your first reaction to this question may be to consider crashing the critical activities—A, B, or E. Activity A has the lowest crashing cost per day of the three, and crashing this activity by 2 days will reduce the A-B-E path to the desired 10 days. Keep in mind, however, that as you crash the current critical activities, other paths may become critical. Thus, you will need to check the critical path in the revised network and perhaps either identify additional activities to crash or modify your initial crashing decision. For a small network, this trial-and-error approach can be used to make crashing decisions; in larger networks, however, a mathematical procedure is required to determine the optimal crashing decisions.

Linear Programming Model for Crashing

Let us describe how linear programming can be used to solve the network crashing problem. With PERT/CPM, we know that when an activity starts at its earliest start time, then

$$\text{Finish time} = \text{Earliest start time} + \text{Activity time}$$

However, if slack time is associated with an activity, then the activity need not start at its earliest start time. In this case, we may have

$$\text{Finish time} > \text{Earliest start time} + \text{Activity time}$$

Because we do not know ahead of time whether an activity will start at its earliest start time, we use the following inequality to show the general relationship among finish time, earliest start time, and activity time for each activity:

$$\text{Finish time} \geq \text{Earliest start time} + \text{Activity time}$$

Consider activity A, which has an expected time of 7 days. Let x_A = finish time for activity A, and y_A = amount of time activity A is crashed. If we assume that the project begins at time 0, the earliest start time for activity A is 0. Because the time for activity A is reduced by the amount of time that activity A is crashed, the finish time for activity A must satisfy the relationship

$$x_A \geq 0 + (7 - y_A)$$

Moving y_A to the left side,

$$x_A + y_A \geq 7$$

In general, let

x_i = the finish time for activity i $\quad i$ = A, B, C, D, E
y_i = the amount of time activity i is crashed $\quad i$ = A, B, C, D, E

If we follow the same approach that we used for activity A, the constraint corresponding to the finish time for activity C (expected time = 6 days) is

$$x_C \geq 0 + (6 - y_C) \quad \text{or} \quad x_C + y_C \geq 6$$

Continuing with the forward pass of the PERT/CPM procedure, we see that the earliest start time for activity B is x_A, the finish time for activity A. Thus, the constraint corresponding to the finish time for activity B is

$$x_B \geq x_A + (3 - y_B) \quad \text{or} \quad x_B + y_B - x_A \geq 3$$

Similarly, we obtain the constraint for the finish time for activity D:

$$x_D \geq x_C + (3 - y_D) \quad \text{or} \quad x_D + y_D - x_C \geq 3$$

Finally, we consider activity E. The earliest start time for activity E equals the *largest* of the finish times for activities B and D. Because the finish times for both activities B and D will be determined by the crashing procedure, we must write two constraints for activity E, one based on the finish time for activity B and one based on the finish time for activity D:

$$x_E + y_E - x_B \geq 2 \quad \text{and} \quad x_E + y_E - x_D \geq 2$$

Recall that current production levels made completing the maintenance project within 10 days imperative. Thus, the constraint for the finish time for activity E is

$$x_E \leq 10$$

In addition, we must add the following five constraints corresponding to the maximum allowable crashing time for each activity:

$$y_A \leq 3, \quad y_B \leq 1, \quad y_C \leq 2, \quad y_D \leq 2, \quad \text{and} \quad y_E \leq 1$$

As with all linear programs, we add the usual nonnegativity requirements for the decision variables.

All that remains is to develop an objective function for the model. Because the total project cost for a normal completion time is fixed at $1700 (see Table 13.9), we can minimize the total project cost (normal cost plus crashing cost) by minimizing the total crashing costs. Thus, the linear programming objective function becomes

$$\text{Min } 100y_A + 150y_B + 200y_C + 150y_D + 250y_E$$

Thus, to determine the optimal crashing for each of the activities, we must solve a 10-variable, 12-constraint linear programming model. Optimization software, such as Excel Solver, provides the optimal solution of crashing activity A by 1 day and activity E by 1 day, with a total crashing cost of $100 + $250 = $350. With the minimum cost crashing solution, the activity times are as follows:

Activity	Time in Days	
A	6	(Crash 1 day)
B	3	
C	6	
D	3	
E	1	(Crash 1 day)

The linear programming solution provided the revised activity times, but not the revised earliest start time, latest start time, and slack information. The revised activity times and the usual PERT/CPM procedure must be used to develop the activity schedule for the project.

NOTES AND COMMENTS

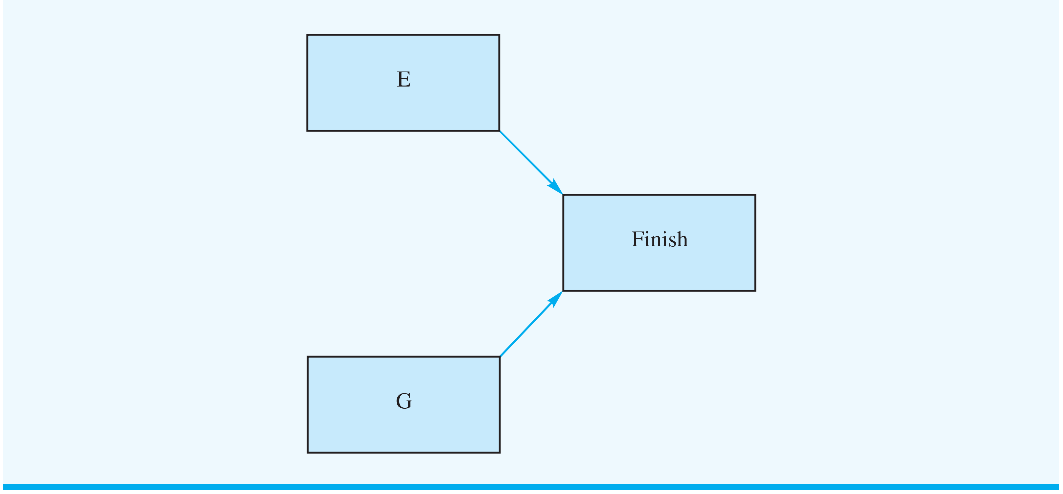
1. Note that the two-machine maintenance project network for the crashing illustration (see Figure 13.14) has only one activity, activity E, leading directly to the Finish node. As a result, the project completion time is equal to the completion time for activity E. Thus, the linear programming constraint requiring the project completion in 10 days or less could be written $x_E \leq 10$.

   If two or more activities lead directly to the Finish node of a project network, a slight modification is required in the linear programming model for crashing. Consider the portion of the project network shown here. In this case, we suggest creating an additional variable, x_{FIN}, which indicates the finish or completion time for the entire project. The fact that the project cannot be finished until both activities E and G are completed can be modeled by the two constraints

$$x_{FIN} \geq x_E \quad \text{or} \quad x_{FIN} - x_E \geq 0$$
$$x_{FIN} \geq x_G \quad \text{or} \quad x_{FIN} - x_G \geq 0$$

The constraint that the project must be finished by time T can be added as $x_{FIN} \leq T$.

Problem 22 gives you practice with this type of project network.



Summary

In this chapter we showed how PERT/CPM can be used to plan, schedule, and control a wide variety of projects. The key to this approach to project scheduling is the development of a PERT/CPM project network that depicts the activities and their precedence relationships. From this project network and activity time estimates, the critical path for the network and the associated critical activities can be identified. In the process, an activity schedule showing the earliest start and earliest finish times, the latest start and latest finish times, and the slack for each activity can be identified.

We showed how we can include capabilities for handling variable or uncertain activity times and how to use this information to provide a probability statement about the chances the project can be completed in a specified period of time. We introduced crashing as a procedure for reducing activity times to meet project completion deadlines, and we showed how a linear programming model can be used to determine the crashing decisions that will minimize the cost of reducing the project completion time.

Glossary

Program evaluation and review technique (PERT) A network-based project scheduling procedure.

Critical path method (CPM) A network-based project scheduling procedure.

Activities Specific jobs or tasks that are components of a project. Activities are represented by nodes in a project network.

Immediate predecessors The activities that must be completed immediately prior to the start of a given activity.

Project network A graphical representation of a project that depicts the activities and shows the predecessor relationships among the activities.

Critical path The longest path in a project network.

Path A sequence of connected nodes that leads from the Start node to the Finish node.

Critical activities The activities on the critical path.

Earliest start time The earliest time an activity may begin.

Latest start time The latest time an activity may begin without increasing the project completion time.

Earliest finish time The earliest time an activity may be completed.

Forward pass Part of the PERT/CPM procedure that involves moving forward through the project network to determine the earliest start and earliest finish times for each activity.

Backward pass Part of the PERT/CPM procedure that involves moving backward through the network to determine the latest start and latest finish times for each activity.

Latest finish time The latest time an activity may be completed without increasing the project completion time.

Slack The length of time an activity can be delayed without affecting the project completion time.

Optimistic time The minimum activity time if everything progresses ideally.

Most probable time The most probable activity time under normal conditions.

Pessimistic time The maximum activity time if significant delays are encountered.

Expected time The average activity time.

Beta probability distribution A probability distribution used to describe activity times.

Crashing The shortening of activity times by adding resources and hence usually increasing cost.

Problems

1. The Mohawk Discount Store is designing a management training program for individuals at its corporate headquarters. The company wants to design the program so that trainees can complete it as quickly as possible. Important precedence relationships must be maintained between assignments or activities in the program. For example, a trainee cannot serve as an assistant to the store manager until the trainee has obtained experience in the credit department and at least one sales department. The following activities are the assignments that must be completed by each trainee in the program. Construct a project network for this problem. Do not perform any further analysis.

Activity	A	B	C	D	E	F	G	H
Immediate Predecessor	—	—	A	A, B	A, B	C	D, F	E, G

2. Bridge City Developers is coordinating the construction of an office complex. As part of the planning process, the company generated the following activity list. Draw a project network that can be used to assist in the scheduling of the project activities.

Activity	A	B	C	D	E	F	G	H	I	J
Immediate Predecessor	—	—	—	A, B	A, B	D	E	C	C	F, G, H, I

SELF test

3. Construct a project network for the following project. The project is completed when activities F and G are both complete.

Activity	A	B	C	D	E	F	G
Immediate Predecessor	—	—	A	A	C, B	C, B	D, E

4. Assume that the project in Problem 3 has the following activity times (in months):

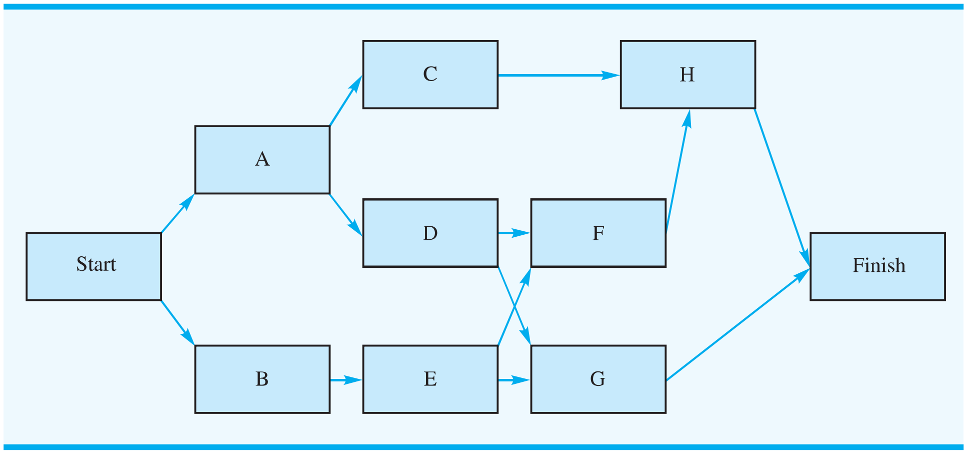
Activity	A	B	C	D	E	F	G
Time	4	6	2	6	3	3	5

a. Find the critical path.
b. The project must be completed in $1^1/_2$ years. Do you anticipate difficulty in meeting the deadline? Explain.

5. Consider the Western Hills Shopping Center Project summarized by Figure 13.6 and Table 13.2. Suppose the project has been underway for seven weeks. Activities A and E have been completed. Activity F has commenced but has three weeks remaining. Activities C and D have not started yet. Activity B has one week remaining (it was not started until week 2). Update the activity schedule for the project. In particular, how has the slack for each activity changed?

SELF test

6. Consider the following project network and activity times (in weeks):



Activity	A	B	C	D	E	F	G	H
Time	5	3	7	6	7	3	10	8

a. Identify the critical path.
b. How much time will be needed to complete this project?
c. Can activity D be delayed without delaying the entire project? If so, by how many weeks?
d. Can activity C be delayed without delaying the entire project? If so, by how many weeks?
e. What is the schedule for activity E?

7. Embassy Club Condominium, located on the west coast of Florida, is undertaking a summer renovation of its main building. The project is scheduled to begin May 1, and a September 1 (17-week) completion date is desired. The condominium manager identified the following renovation activities and their estimated times:

Activity	Immediate Predecessor	Time
A	—	3
B	—	1
C	—	2
D	A, B, C	4
E	C, D	5
F	A	3
G	D, F	6
H	E	4

a. Draw a project network.
b. What are the critical activities?
c. What activity has the most slack time?
d. Will the project be completed by September 1?

8. Colonial State College is considering building a new multipurpose athletic complex on campus. The complex would provide a new gymnasium for intercollegiate basketball games, expanded office space, classrooms, and intramural facilities. The following activities would have to be undertaken before construction can begin:

Activity	Description	Immediate Predecessor	Time (weeks)
A	Survey building site	—	6
B	Develop initial design	—	8
C	Obtain board approval	A, B	12
D	Select architect	C	4
E	Establish budget	C	6
F	Finalize design	D, E	15
G	Obtain financing	E	12
H	Hire contractor	F, G	8

a. Draw a project network.
b. Identify the critical path.
c. Develop the activity schedule for the project.
d. Does it appear reasonable that construction of the athletic complex could begin one year after the decision to begin the project with the site survey and initial design plans? What is the expected completion time for the project?

9. At a local university, the Student Commission on Programming and Entertainment (SCOPE) is preparing to host its first rock concert of the school year. To successfully produce this rock concert, SCOPE has listed the requisite activities and related information in the following table (duration estimates measured in days).

Activity	Immediate Predecessor(s)	Optimistic	Most Probable	Pessimistic
A: Negotiate contract with selected musicians	—	8	10	15
B: Reserve site	—	7	8	9
C: Manage travel logistics for music group	A	5	6	10
D: Screen & hire security personnel	B	3	3	3
E: Arrange advertising & ticketing	B, C	1	5	9
F: Hire parking staff	D	4	7	10
G: Arrange concession sales	E	3	8	10

a. Draw the project network.
b. Compute the expected duration and variance of each activity.
c. Determine the critical path in the project network.
d. What is the expected duration and variance of the critical path?
e. What is the likelihood that the project will be completed within 30 days?
f. If activity B is delayed by six days beyond its early start time, how does this affect the expected project duration?

SELF test

10. The following estimates of activity times (in days) are available for a small project:

Activity	Optimistic	Most Probable	Pessimistic
A	4	5.0	6
B	8	9.0	10
C	7	7.5	11
D	7	9.0	10
E	6	7.0	9
F	5	6.0	7

a. Compute the expected activity completion times and the variance for each activity.
b. An analyst determined that the critical path consists of activities B-D-F. Compute the expected project completion time and the variance of this path.

11. Building a backyard swimming pool consists of nine major activities. The activities and their immediate predecessors are shown. Develop the project network.

Activity	A	B	C	D	E	F	G	H	I
Immediate Predecessor	—	—	A, B	A, B	B	C	D	D, F	E, G, H

12. Assume that the activity time estimates (in days) for the swimming pool construction project in Problem 11 are as follows:

Activity	Optimistic	Most Probable	Pessimistic
A	3	5	6
B	2	4	6
C	5	6	7
D	7	9	10
E	2	4	6
F	1	2	3
G	5	8	10
H	6	8	10
I	3	4	5

a. What are the critical activities?
b. What is the expected time to complete the project?
c. What is the probability that the project can be completed in 25 or fewer days?

SELF test

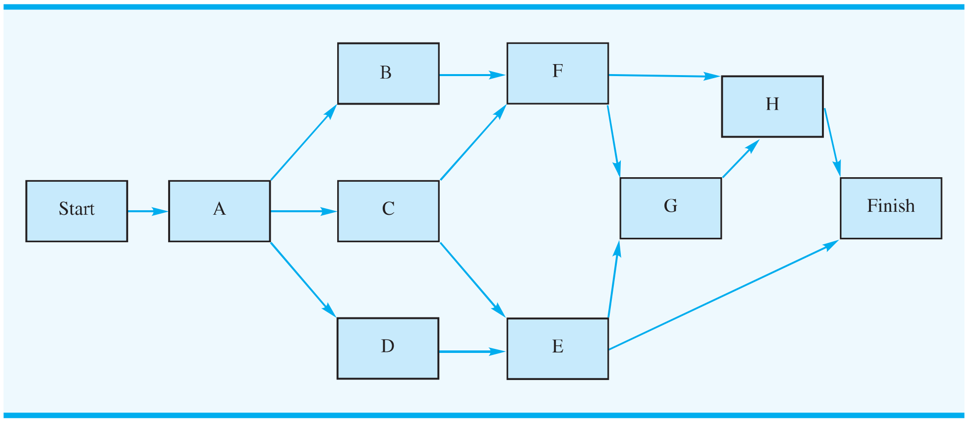
13. Suppose that the following estimates of activity times (in weeks) were provided for the network shown in Problem 6:

Activity	Optimistic	Most Probable	Pessimistic
A	4.0	5.0	6.0
B	2.5	3.0	3.5
C	6.0	7.0	8.0
D	5.0	5.5	9.0
E	5.0	7.0	9.0
F	2.0	3.0	4.0
G	8.0	10.0	12.0
H	6.0	7.0	14.0

What is the probability that the project will be completed

a. Within 21 weeks?
b. Within 22 weeks?
c. Within 25 weeks?

14. Davison Construction Company is building a luxury lakefront home in the Finger Lakes region of New York. Coordination of the architect and subcontractors will require a major effort to meet the 44-week (approximately 10-month) completion date requested by the owner. The Davison project manager prepared the following project network:



Estimates of the optimistic, most probable, and pessimistic times (in weeks) for the activities are as follows:

Activity	Optimistic	Most Probable	Pessimistic
A	4	8	12
B	6	7	8
C	6	12	18
D	3	5	7
E	6	9	18
F	5	8	17
G	10	15	20
H	5	6	13

a. Find the critical path.
b. What is the expected project completion time?
c. What is the probability the project can be completed in the 44 weeks as requested by the owner?
d. What is the probability the building project could run more than 3 months late? Use 57 weeks for this calculation.
e. What should the construction company tell the owner?

15. Doug Casey is in charge of planning and coordinating next spring's sales management training program for his company. Doug listed the following activity information for this project:

Activity	Description	Immediate Predecessor	Time (weeks)		
			Optimistic	Most Probable	Pessimistic
A	Plan topic	—	1.5	2.0	2.5
B	Obtain speakers	A	2.0	2.5	6.0
C	List meeting locations	—	1.0	2.0	3.0
D	Select location	C	1.5	2.0	2.5
E	Finalize speaker travel plans	B, D	0.5	1.0	1.5
F	Make final check with speakers	E	1.0	2.0	3.0
G	Prepare and mail brochure	B, D	3.0	3.5	7.0
H	Take reservations	G	3.0	4.0	5.0
I	Handle last-minute details	F, H	1.5	2.0	2.5

a. Draw a project network.
b. Prepare an activity schedule.
c. What are the critical activities and what is the expected project completion time?
d. If Doug wants a 0.99 probability of completing the project on time, how far ahead of the scheduled meeting date should he begin working on the project?

16. The Daugherty Porta-Vac project discussed in Section 13.2 has an expected project completion time of 17 weeks. The probability that the project could be completed in 20 weeks or less is 0.9656. The noncritical paths in the Porta-Vac project network are

A-D-G-J
A-C-F-J
B-H-I-J

a. Use the information in Table 13.5 to compute the expected time and variance for each path shown.
b. Compute the probability that each path will be completed in the desired 20-week period.
c. Why is the computation of the probability of completing a project on time based on the analysis of the critical path? In what case, if any, would making the probability computation for a noncritical path be desirable?

17. The Porsche Shop, founded in 1985 by Dale Jensen, specializes in the restoration of vintage Porsche automobiles. One of Jensen's regular customers asked him to prepare an estimate for the restoration of a 1964 model 356SC Porsche. To estimate the time and cost to perform such a restoration, Jensen broke the restoration process into four separate activities: disassembly and initial preparation work (A), body restoration (B), engine restoration (C), and final assembly (D). Once activity A has been completed, activities B and C can be performed independently of each other; however, activity D can be started only if both activities B and C have been completed. Based on his inspection of the car, Jensen believes that the following time estimates (in days) are applicable:

Activity	Optimistic	Most Probable	Pessimistic
A	3	4	8
B	5	8	11
C	2	4	6
D	4	5	12

Jensen estimates that the parts needed to restore the body will cost $3000 and that the parts needed to restore the engine will cost $5000. His current labor costs are $400 a day.

a. Develop a project network.
b. What is the expected project completion time?

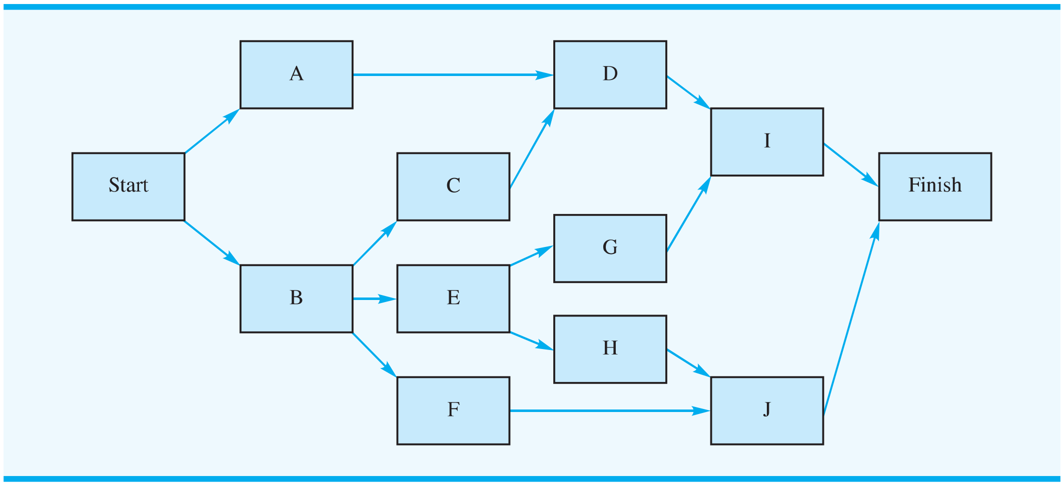
c. Jensen's business philosophy is based on making decisions using a best- and worst-case scenario. Develop cost estimates for completing the restoration based on both a best- and worst-case analysis. Assume that the total restoration cost is the sum of the labor cost plus the material cost.
d. If Jensen obtains the job with a bid that is based on the costs associated with an expected completion time, what is the probability that he will lose money on the job?
e. If Jensen obtains the job based on a bid of $16,800, what is the probability that he will lose money on the job?

18. The manager of the Oak Hills Swimming Club is planning the club's swimming team program. The first team practice is scheduled for May 1. The activities, their immediate predecessors, and the activity time estimates (in weeks) are as follows:

Activity	Description	Immediate Predecessor	Time (weeks) Optimistic	Most Probable	Pessimistic
A	Meet with board	—	1	1	2
B	Hire coaches	A	4	6	8
C	Reserve pool	A	2	4	6
D	Announce program	B, C	1	2	3
E	Meet with coaches	B	2	3	4
F	Order team suits	A	1	2	3
G	Register swimmers	D	1	2	3
H	Collect fees	G	1	2	3
I	Plan first practice	E, H, F	1	1	1

a. Draw a project network.
b. Develop an activity schedule.
c. What are the critical activities, and what is the expected project completion time?
d. If the club manager plans to start the project on February 1, what is the probability the swimming program will be ready by the scheduled May 1 date (13 weeks)? Should the manager begin planning the swimming program before February 1?

19. The product development group at Landon Corporation has been working on a new computer software product that has the potential to capture a large market share. Through outside sources, Landon's management learned that a competitor is working to introduce a similar product. As a result, Landon's top management increased its pressure on the product development group. The group's leader turned to PERT/CPM as an aid to scheduling the activities remaining before the new product can be brought to the market. The project network is as follows:



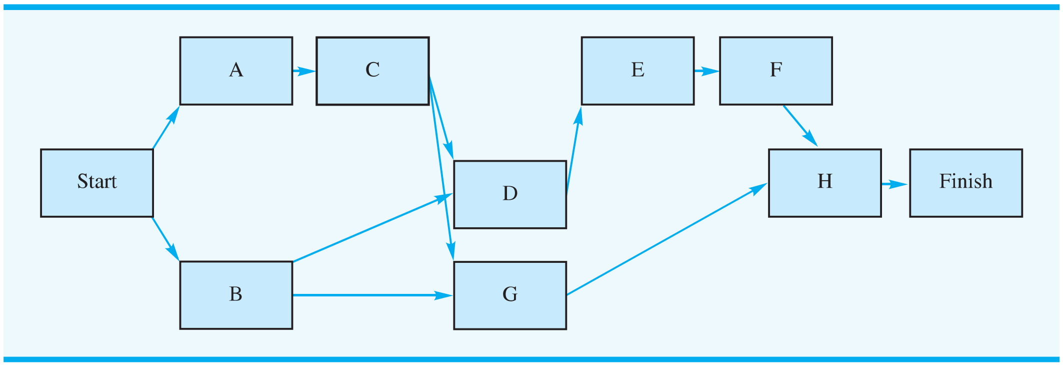
The activity time estimates (in weeks) are as follows:

Activity	Optimistic	Most Probable	Pessimistic
A	3.0	4.0	5.0
B	3.0	3.5	7.0
C	4.0	5.0	6.0
D	2.0	3.0	4.0
E	6.0	10.0	14.0
F	7.5	8.5	12.5
G	4.5	6.0	7.5
H	5.0	6.0	13.0
I	2.0	2.5	6.0
J	4.0	5.0	6.0

a. Develop an activity schedule for this project and identify the critical path activities.
b. What is the probability that the project will be completed so that Landon Corporation may introduce the new product within 25 weeks? Within 30 weeks?

20. Norton Industries is installing a new computer system. The activities, the activity times, and the project network are as follows:

Activity	Time	Activity	Time
A	3	E	4
B	6	F	3
C	2	G	9
D	5	H	3

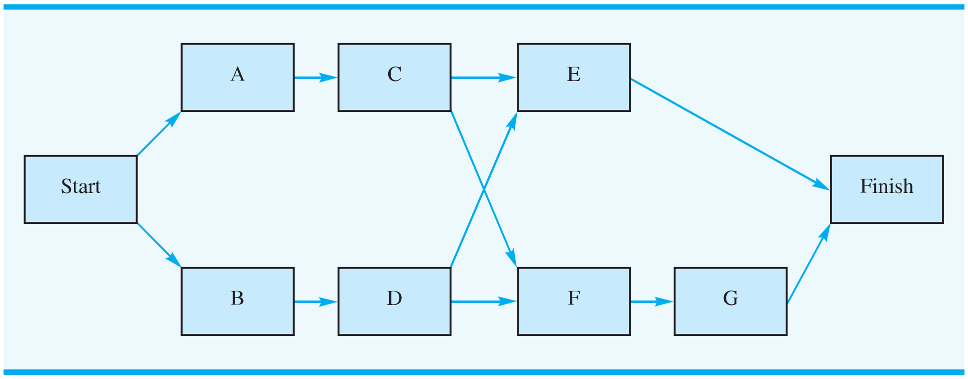


The critical path calculation shows B-D-E-F-H is the critical path, and the expected project completion time is 21 weeks. After viewing this information, management requested overtime be used to complete the project in 16 weeks. Thus, crashing of the project is necessary. The following information is relevant:

	Time (weeks)		Cost ($)	
Activity	Normal	Crash	Normal	Crash
A	3	1	900	1700
B	6	3	2000	4000
C	2	1	500	1000
D	5	3	1800	2400
E	4	3	1500	1850
F	3	1	3000	3900
G	9	4	8000	9800
H	3	2	1000	2000

a. Formulate a linear programming model that can be used to make the crashing decisions for this project.
b. Solve the linear programming model and make the minimum cost crashing decisions. What is the added cost of meeting the 16-week completion time?
c. Develop a complete activity schedule based on the crashed activity times.

21. Consider the following project network and activity times (in days):



Activity	A	B	C	D	E	F	G
Time	3	2	5	5	6	2	2

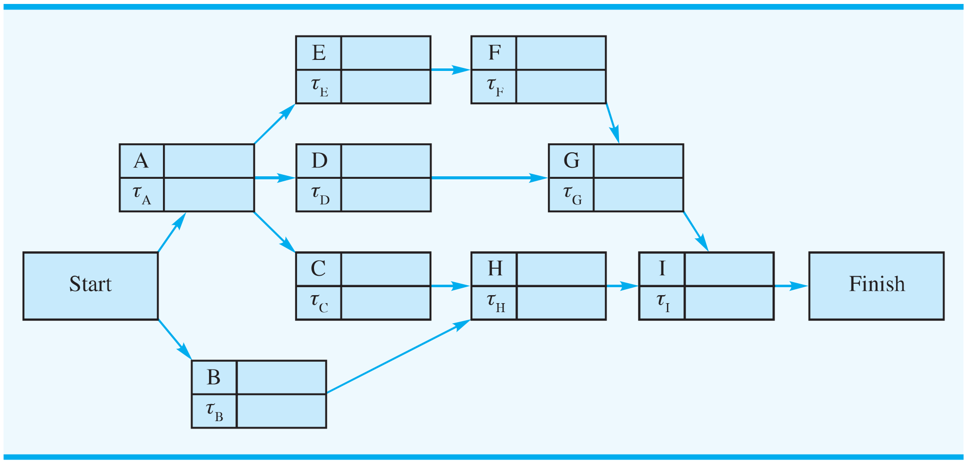
The crashing data for this project are as follows:

	Time (days)		Cost ($)	
Activity	Normal	Crash	Normal	Crash
A	3	2	800	1400
B	2	1	1200	1900
C	5	3	2000	2800
D	5	3	1500	2300
E	6	4	1800	2800
F	2	1	600	1000
G	2	1	500	1000

a. Find the critical path and the expected project completion time.
b. What is the total project cost using the normal times?

22. Refer to Problem 21. Assume that management desires a 12-day project completion time.
a. Formulate a linear programming model that can be used to assist with the crashing decisions.
b. What activities should be crashed?
c. What is the total project cost for the 12-day completion time?

23. Consider the following project network. Note that the normal or expected activity times are denoted τ_i, i = A, B, . . . , I. Let x_i = the earliest finish time for activity i. Formulate a linear programming model that can be used to determine the length of the critical path.



24. Office Automation, Inc., developed a proposal for introducing a new computerized office system that will standardize the electronic archiving of invoices for a particular company. Contained in the proposal is a list of activities that must be accomplished to complete the new office system project. Use the following relevant information about the activities:

Activity	Description	Immediate Predecessor	Time (weeks)		Cost ($1000s)	
			Normal	Crash	Normal	Crash
A	Plan needs	—	10	8	30	70
B	Order equipment	A	8	6	120	150
C	Install equipment	B	10	7	100	160
D	Set up training lab	A	7	6	40	50
E	Conduct training course	D	10	8	50	75
F	Test system	C, E	3	3	60	—

a. Develop a project network.
b. Develop an activity schedule.
c. What are the critical activities, and what is the expected project completion time?
d. Assume that the company wants to complete the project in six months or 26 weeks. What crashing decisions do you recommend to meet the desired completion time at the least possible cost? Work through the network and attempt to make the crashing decisions by inspection.
e. Develop an activity schedule for the crashed project.
f. What added project cost is required to meet the six-month completion time?

25. Because Landon Corporation (see Problem 19) is being pressured to complete the product development project at the earliest possible date, the project leader requested that the possibility of crashing the project be evaluated.
a. Formulate a linear programming model that could be used in making the crashing decisions.
b. What information would have to be provided before the linear programming model could be implemented?

Case Problem R. C. Coleman

R. C. Coleman distributes a variety of food products that are sold through grocery store and supermarket outlets. The company receives orders directly from the individual outlets, with a typical order requesting the delivery of several cases of anywhere from 20 to 50 different products. Under the company's current warehouse operation, warehouse clerks dispatch order-picking personnel to fill each order and have the goods moved to the warehouse shipping area. Because of the high labor costs and relatively low productivity of hand order-picking, management has decided to automate the warehouse operation by installing a computer-controlled order-picking system, along with a conveyor system for moving goods from storage to the warehouse shipping area.

R. C. Coleman's director of material management has been named the project manager in charge of the automated warehouse system. After consulting with members of the engineering staff and warehouse management personnel, the director compiled a list of activities associated with the project. The optimistic, most probable, and pessimistic times (in weeks) have also been provided for each activity.

Activity	Description	Immediate Predecessor
A	Determine equipment needs	—
B	Obtain vendor proposals	—
C	Select vendor	A, B
D	Order system	C
E	Design new warehouse layout	C
F	Design warehouse	E
G	Design computer interface	C
H	Interface computer	D, F, G
I	Install system	D, F
J	Train system operators	H
K	Test system	I, J

	Time (weeks)		
Activity	**Optimistic**	**Most Probable**	**Pessimistic**
A	4	6	8
B	6	8	16
C	2	4	6
D	8	10	24
E	7	10	13
F	4	6	8
G	4	6	20
H	4	6	8
I	4	6	14
J	3	4	5
K	2	4	6

Managerial Report

Develop a report that presents the activity schedule and expected project completion time for the warehouse expansion project. Include a project network in the report. In addition, take into consideration the following issues:

1. R. C. Coleman's top management established a required 40-week completion time for the project. Can this completion time be achieved? Include probability information in your discussion. What recommendations do you have if the 40-week completion time is required?
2. Suppose that management requests that activity times be shortened to provide an 80% chance of meeting the 40-week completion time. If the variance in the project completion time is the same as you found in part (1), how much should the expected project completion time be shortened to achieve the goal of an 80% chance of completion within 40 weeks?
3. Using the expected activity times as the normal times and the following crashing information, determine the activity crashing decisions and revised activity schedule for the warehouse expansion project:

Activity	Crashed Activity Time (weeks)	Cost ($)	
		Normal	Crashed
A	4	1,000	1,900
B	7	1,000	1,800
C	2	1,500	2,700
D	8	2,000	3,200
E	7	5,000	8,000
F	4	3,000	4,100
G	5	8,000	10,250
H	4	5,000	6,400
I	4	10,000	12,400
J	3	4,000	4,400
K	3	5,000	5,500

Appendix 13.1 Finding Cumulative Probabilities for Normally Distributed Random Variables

Excel can be used to find the probability a project with uncertain activity times will be completed in some given completion time (assuming the project completion time is normally distributed). We demonstrate this on the Porta-Vac Project we considered in Section 13.2. Recall that management allotted 20 days to complete the project. We have found the z value that corresponds to $T = 20$:

$$z = \frac{20 - 17}{1.65} = 1.82$$

The Excel function NORM.S.DIST is only recognized by Excel 2010. Earlier versions of Excel use the function name NORMSDIST to compute the same value.

Now we will make use the Excel function

$$=\text{NORM.S.DIST}(z, \text{TRUE})$$

by substituting the value of z we have found into the function (entering “TRUE” for the second argument signifies that we desire the cumulative probability associated with z). Enter the following function into any empty cell in an Excel worksheet:

$$=\text{NORM.S.DIST}(1.82, \text{TRUE})$$

The resulting value is 0.96562, which is the probability that the completion time for the Porta-Vac project will be no more than 20 days.