CHAPTER 14

Inventory Models

CONTENTS

Inventory refers to idle goods or materials held by an organization for use sometime in the future. Items carried in inventory include raw materials, purchased parts, components, subassemblies, work-in-process, finished goods, and supplies. Two primary reasons organizations stock inventory are: (1) to take advantage of economies-of-scale that exist due to the fixed cost of ordering items, and (2) to buffer against uncertainty in customer demand or disruptions in supply. Even though inventory serves an important and essential role, the expense associated with financing and maintaining inventories is a substantial part of the cost of doing business. In large organizations, the cost associated with inventory can run into the millions of dollars.

In applications involving inventory, managers must answer two important questions.

1. *How much* should be ordered when the inventory is replenished?
2. *When* should the inventory be replenished?

The inventory procedure described for the drugstore industry is discussed in detail in Section 14.7.

Virtually every business uses some sort of inventory management model or system to address the preceding questions. Hewlett-Packard works with its retailers to help determine the retailer's inventory replenishment strategies for the printers and other HP products. IBM developed inventory management policies for a range of microelectronic parts that are used in IBM plants as well as sold to a number of outside customers. The Q.M. in Action, Inventory Management at CVS Corporation, describes an inventory system used to determine order quantities in the drugstore industry.

The purpose of this chapter is to show how quantitative models can assist in making the how-much-to-order and when-to-order inventory decisions. We will first consider *deterministic* inventory models, in which we assume that the rate of demand for the item is constant or nearly constant. Later we will consider *probabilistic* inventory models, in which the demand for the item fluctuates and can be described only in probabilistic terms.

Q.M. *in* ACTION

*INVENTORY MANAGEMENT AT CVS CORPORATION**

CVS is one of the largest drugstore chains in the United States. The primary inventory management area in the drugstore involves the numerous basic products that are carried in inventory on an everyday basis. For these items, the most important issue is the replenishment quantity or order size each time an order is placed. In most drugstore chains, basic products are ordered under a periodic review inventory system, with the review period being one week.

The weekly review system uses electronic ordering equipment that scans an order label affixed to the shelf directly below each item. Among other information on the label is the item's replenishment level or order-to-quantity. The store employee placing the order determines the weekly order quantity by counting the number of units of the product on the shelf and subtracting this quantity from the replenishment level. A computer program determines the replenishment quantity for each item in each individual store, based on each store's movement rather than on the company movement. To minimize stock-outs the replenishment quantity is set equal to the store's three-week demand or movement for the product.

*Based on information provided by Bob Carver. (The inventory system described was originally implemented in the CVS stores formerly known as SupeRX.)

14.1 Economic Order Quantity (EOQ) Model

The **economic order quantity (EOQ)** model is applicable when the demand for an item shows a constant, or nearly constant, rate and when the entire quantity ordered arrives in inventory at one point in time. The **constant demand rate** assumption means that the same number of units is taken from inventory each period of time such as 5 units every day, 25 units every week, 100 units every four-week period, and so on.

The cost associated with developing and maintaining inventory is larger than many people think. Models such as the ones presented in this chapter can be used to develop cost-effective inventory management decisions.

To illustrate the EOQ model, let us consider the situation faced by the R&B Beverage Company. R&B Beverage is a distributor of beer, wine, and soft drink products. From a main warehouse located in Columbus, Ohio, R&B supplies nearly 1000 retail stores with beverage products. The beer inventory, which constitutes about 40% of the company's total inventory, averages approximately 50,000 cases. With an average cost per case of approximately $8, R&B estimates the value of its beer inventory to be $400,000.

The warehouse manager decided to conduct a detailed study of the inventory costs associated with Bub Beer, the number-one-selling R&B beer. The purpose of the study is to establish the how-much-to-order and the when-to-order decisions for Bub Beer that will result in the lowest possible total cost. As the first step in the study, the warehouse manager obtained the following demand data for the past 10 weeks:

One of the most criticized assumptions of the EOQ model is the constant demand rate. Obviously, the model would be inappropriate for items with widely fluctuating and variable demand rates. However, as this example shows, the EOQ model can provide a realistic approximation of the optimal order quantity when demand is relatively stable and occurs at a nearly constant rate.

Week	Demand (cases)
1	2000
2	2025
3	1950
4	2000
5	2100
6	2050
7	2000
8	1975
9	1900
10	2000
Total cases	20,000
Average cases per week	2000

Strictly speaking, these weekly demand figures do not show a constant demand rate. However, given the relatively low variability exhibited by the weekly demand, inventory planning with a constant demand rate of 2000 cases per week appears acceptable. In practice, you will find that the actual inventory situation seldom, if ever, satisfies the assumptions of the model exactly. Thus, in any particular application, the manager must determine whether the model assumptions are close enough to reality for the model to be useful. In this situation, because demand varies from a low of 1900 cases to a high of 2100 cases, the assumption of constant demand of 2000 cases per week appears to be a reasonable approximation.

The how-much-to-order decision involves selecting an order quantity that draws a compromise between (1) keeping small inventories and ordering frequently, and (2) keeping large inventories and ordering infrequently. The first alternative can result in undesirably high ordering costs, while the second alternative can result in undesirably high inventory holding costs. To find an optimal compromise between these conflicting alternatives, let us consider a mathematical model that shows the total cost as the sum of the holding cost and the ordering cost.[1]

As with other quantitative models, accurate estimates of cost parameters are critical. In the EOQ model, estimates of both the inventory holding cost and the ordering cost are needed. Also see footnote 1, which refers to relevant costs.

Holding costs are the costs associated with maintaining or carrying a given level of inventory; these costs depend on the size of the inventory. The first holding cost to consider is the cost of financing the inventory investment. When a firm borrows money, it incurs an interest charge. If the firm uses its own money, it experiences an opportunity cost associated with not being able to use the money for other investments. In either case, an interest cost exists for the capital tied up in inventory. This **cost of capital** is usually expressed as a percentage of the amount invested. R&B estimates its cost of capital at an annual rate of 18%.

A number of other holding costs, such as insurance, taxes, breakage, pilferage, and warehouse overhead, also depend on the value of the inventory. R&B estimates these other costs at an annual rate of approximately 7% of the value of its inventory. Thus, the total holding cost for the R&B beer inventory is 18% + 7% = 25% of the value of the inventory. The cost of one case of Bub Beer is $8. With an annual holding cost rate of 25%, the cost of holding one case of Bub Beer in inventory for 1 year is 0.25($8) = $2.00.

The next step in the inventory analysis is to determine the **ordering cost**. This cost, which is considered fixed regardless of the order quantity, covers the preparation of the voucher; and the processing of the order, including payment, postage, telephone, transportation, invoice verification, receiving, and so on. For R&B Beverage, the largest portion of the ordering cost involves the salaries of the purchasers. An analysis of the purchasing process showed that a purchaser spends approximately 45 minutes preparing and processing an order for Bub Beer. With a wage rate and fringe benefit cost for purchasers of $20 per hour, the labor portion of the ordering cost is $15. Making allowances for paper, postage, telephone, transportation, and receiving costs at $17 per order, the manager estimates that the ordering cost is $32 per order. That is, R&B is paying $32 per order regardless of the quantity requested in the order.

The holding cost, ordering cost, and demand information are the three data items that must be known prior to the use of the EOQ model. After developing these data for the R&B problem, we can look at how they are used to develop a total cost model. We begin by defining Q as the order quantity. Thus, the how-much-to-order decision involves finding the value of Q that will minimize the sum of holding and ordering costs.

The inventory for Bub Beer will have a maximum value of Q units when an order of size Q is received from the supplier. R&B will then satisfy customer demand from inventory until the inventory is depleted, at which time another shipment of Q units will be received. Thus, assuming a constant demand, the graph of the inventory for Bub Beer is as shown in Figure 14.1. Note that the graph indicates an average inventory of $\frac{1}{2}Q$ for the period in question. This level should appear reasonable because the maximum inventory is Q, the minimum is zero, and the inventory declines at a constant rate over the period.

Figure 14.1 shows the inventory pattern during one order cycle of length T. As time goes on, this pattern will repeat. The complete inventory pattern is shown in Figure 14.2. If the average inventory during each cycle is $\frac{1}{2}Q$, the average inventory over any number of cycles is also $\frac{1}{2}Q$.

Most inventory cost models use an annual cost. Thus, demand should be expressed in units per year, and inventory holding cost should be based on an annual rate.

The holding cost can be calculated using the average inventory. That is, we can calculate the holding cost by multiplying the average inventory by the cost of carrying one unit in inventory for the stated period. The period selected for the model is up to you; it could be one week, one month, one year, or more. However, because the holding cost for many

[1]Even though analysts typically refer to "total cost" models for inventory systems, often these models describe only the total variable or total relevant costs for the decision being considered. Costs that are not affected by the how-much-to-order decision are considered fixed or constant and are not included in the model.

FIGURE 14.1 INVENTORY FOR BUB BEER

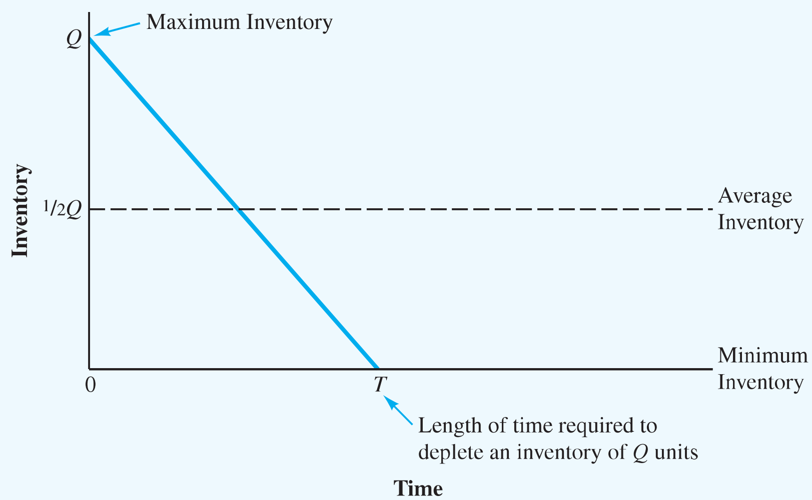
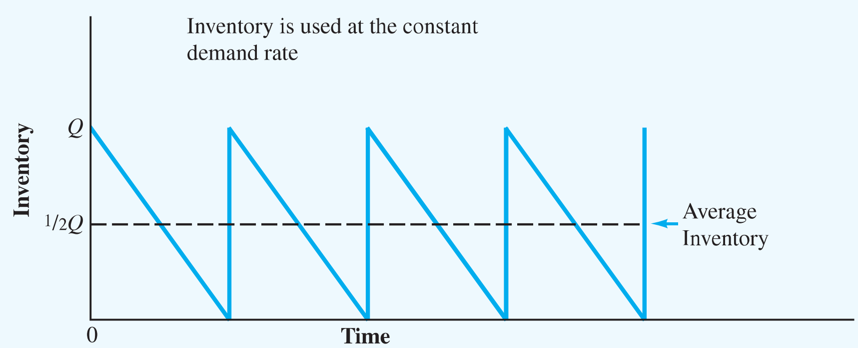


FIGURE 14.2 INVENTORY PATTERN FOR THE EOQ INVENTORY MODEL



industries and businesses is expressed as an *annual* percentage, most inventory models are developed on an *annual* cost basis.

Let

I = annual holding cost rate

C = unit cost of the inventory item

C_h = annual cost of holding one unit in inventory

The annual cost of holding one unit in inventory is

$$C_h = IC \quad (14.1)$$

C_h is the cost of holding one unit in inventory for one year. Because smaller order quantities Q will result in lower inventory, total annual holding cost can be reduced by using smaller order quantities.

The general equation for the annual holding cost for the average inventory of $\frac{1}{2}Q$ units is as follows:

$$\begin{aligned}\text{Annual holding cost} &= \begin{pmatrix}\text{Average}\\\text{inventory}\end{pmatrix}\begin{pmatrix}\text{Annual holding}\\\text{cost}\\\text{per unit}\end{pmatrix}\\ &= \frac{1}{2}QC_h \qquad (14.2)\end{aligned}$$

To complete the total cost model, we must now include the annual ordering cost. The goal is to express the annual ordering cost in terms of the order quantity Q. The first question is, How many orders will be placed during the year? Let D denote the annual demand for the product. For R&B Beverage, D = (52 weeks)(2000 cases per week) = 104,000 cases per year. We know that by ordering Q units every time we order, we will have to place D/Q orders per year. If C_o is the cost of placing one order, the general equation for the annual ordering cost is as follows:

C_o, the fixed cost per order, is independent of the amount ordered. For a given annual demand of D units, the total annual ordering cost can be reduced by using larger order quantities.

$$\begin{aligned}\text{Annual ordering cost} &= \begin{pmatrix}\text{Number of}\\\text{orders}\\\text{per year}\end{pmatrix}\begin{pmatrix}\text{Cost}\\\text{per}\\\text{order}\end{pmatrix}\\ &= \left(\frac{D}{Q}\right)C_o \qquad (14.3)\end{aligned}$$

Thus, the total annual cost, denoted TC, can be expressed as follows:

$$\begin{aligned}\text{Total annual cost} &= \text{Annual holding cost} + \text{Annual ordering cost}\\ TC &= \frac{1}{2}QC_h + \frac{D}{Q}C_o \qquad (14.4)\end{aligned}$$

Using the Bub Beer data [$C_h = IC = (0.25)(\$8) = \$2$, $C_o = \$32$, and $D = 104{,}000$], the total annual cost model is

$$TC = \frac{1}{2}Q(\$2) + \frac{104{,}000}{Q}(\$32) = Q + \frac{3{,}328{,}000}{Q}$$

The development of the total cost model goes a long way toward solving the inventory problem. We now are able to express the total annual cost as a function of *how much* should be ordered. The development of a realistic total cost model is perhaps the most important part of the application of quantitative methods to inventory decision making. Equation (14.4) is the general total cost equation for inventory situations in which the assumptions of the economic order quantity model are valid.

The How-Much-to-Order Decision

The next step is to find the order quantity Q that will minimize the total annual cost for Bub Beer. Using a trial-and-error approach, we can compute the total annual cost for several possible order quantities. As a starting point, let us consider $Q = 8000$. The total annual cost for Bub Beer is

$$TC = Q + \frac{3,328,000}{Q}$$
$$= 8000 + \frac{3,328,000}{8000} = \$8416$$

A trial order quantity of 5000 gives

$$TC = 5000 + \frac{3,328,000}{5000} = \$5666$$

The results of several other trial order quantities are shown in Table 14.1. It shows the lowest cost solution to be about 2000 cases. Graphs of the annual holding and ordering costs and total annual costs are shown in Figure 14.3.

The advantage of the trial-and-error approach is that it is rather easy to do and provides the total annual cost for a number of possible order quantity decisions. In this case, the minimum cost order quantity appears to be approximately 2000 cases. The disadvantage of this approach, however, is that it does not provide the exact minimum cost order quantity.

The EOQ formula determines the optimal order quantity by balancing the annual holding cost and the annual ordering cost.

In 1915 F. W. Harris derived the mathematical formula for the economic order quantity. It was the first application of quantitative methods to the area of inventory management.

Refer to Figure 14.3. The minimum total cost order quantity is denoted by an order size of Q^*. By using differential calculus, it can be shown (see Appendix 14.1) that the value of Q^* that minimizes the total annual cost is given by the formula

$$Q^* = \sqrt{\frac{2DC_o}{C_h}} \tag{14.5}$$

This formula is referred to as the *economic order quantity (EOQ) formula*.

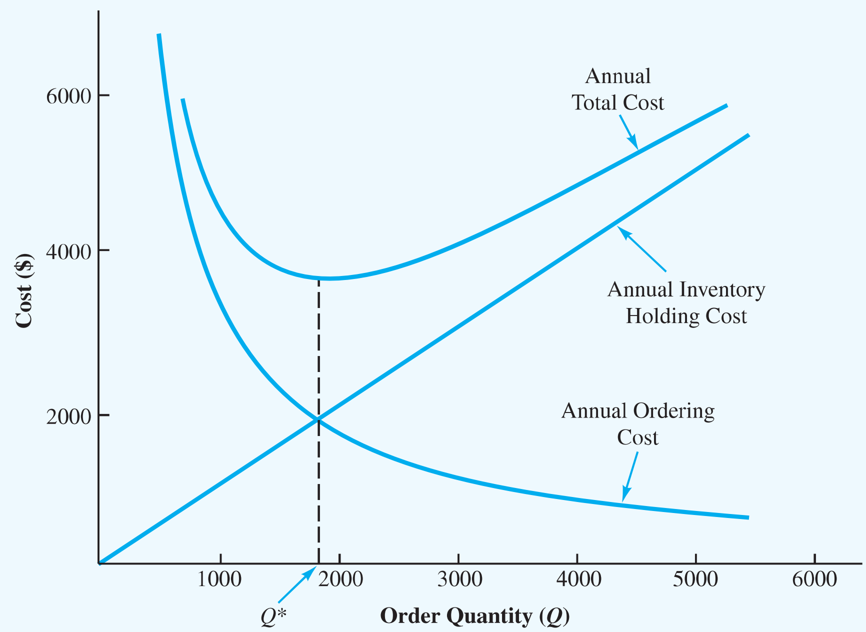
Using equation (14.5), the minimum total annual cost order quantity for Bub Beer is

$$Q^* = \sqrt{\frac{2(104,000)32}{2}} = 1824 \text{ cases}$$

TABLE 14.1 ANNUAL HOLDING, ORDERING, AND TOTAL COSTS FOR VARIOUS ORDER QUANTITIES OF BUB BEER

	Annual Cost		
Order Quantity	**Holding**	**Ordering**	**Total**
5000	$5000	$ 666	$5666
4000	$4000	$ 832	$4832
3000	$3000	$1109	$4109
2000	$2000	$1664	$3664
1000	$1000	$3328	$4328

FIGURE 14.3 ANNUAL HOLDING, ORDERING, AND TOTAL COSTS FOR BUB BEER



Problem 2 at the end of the chapter asks you to show that equal holding and ordering costs is a property of the EOQ model.

The use of an order quantity of 1824 in equation (14.4) shows that the minimum cost inventory policy for Bub Beer has a total annual cost of $3649. Note that $Q^* = 1824$ balances the holding and ordering costs. Check for yourself to see that these costs are equal.[2]

The When-to-Order Decision

The reorder point is expressed in terms of inventory position, the amount of inventory on hand plus the amount on order. Some people think that the reorder point is expressed in terms of inventory on hand. With short lead times, inventory position is usually the same as the inventory on hand. However, with long lead times, inventory position may be larger than inventory on hand.

Now that we know how much to order, we want to address the question of *when* to order. To answer this question, we need to introduce the concept of inventory position. The **inventory position** is defined as the amount of inventory on hand plus the amount of inventory on order. The when-to-order decision is expressed in terms of a **reorder point**—the inventory position at which a new order should be placed.

The manufacturer of Bub Beer guarantees a two-day delivery on any order placed by R&B Beverage. Hence, assuming R&B Beverage operates 250 days per year, the annual demand of 104,000 cases implies a daily demand of $104{,}000/250 = 416$ cases. Thus, we expect (2 days)(416 cases per day) $= 832$ cases of Bub to be sold during the two days it takes a new order to reach the R&B warehouse. In inventory terminology, the two-day delivery period is referred to as the **lead time** for a new order, and the 832-case demand anticipated during this period is referred to as the **lead-time demand**. Thus, R&B should order a new shipment of Bub Beer from the manufacturer when the inventory reaches 832 cases. For inventory systems using the constant demand rate assumption and a fixed

[2]Actually, Q^* from equation (14.5) is 1824.28, but because we cannot order fractional cases of beer, a Q^* of 1824 is shown. This value of Q^* may cause a few cents deviation between the two costs. If Q^* is used at its exact value, the holding and ordering costs will be exactly the same.

lead time, the reorder point is the same as the lead-time demand. For these systems, the general expression for the reorder point is as follows:

$$r = dm \tag{14.6}$$

where

r = reorder point
d = demand per day
m = lead time for a new order in days

The question of how frequently the order will be placed can now be answered. The period between orders is referred to as the **cycle time**. Previously in equation (14.3), we defined D/Q as the number of orders that will be placed in a year. Thus, $D/Q^* = 104{,}000/1824 = 57$ is the number of orders R&B Beverage will place for Bub Beer each year. If R&B places 57 orders over 250 working days, it will order approximately every $250/57 = 4.39$ working days. Thus, the cycle time is 4.39 working days. The general expression for a cycle time[3] of T days is given by

$$T = \frac{250}{D/Q^*} = \frac{250Q^*}{D} \tag{14.7}$$

Sensitivity Analysis for the EOQ Model

Even though substantial time may have been spent in arriving at the cost per order ($32) and the holding cost rate (25%), we should realize that these figures are at best good estimates. Thus, we may want to consider how much the recommended order quantity would change with different estimated ordering and holding costs. To determine the effects of various cost scenarios, we can calculate the recommended order quantity under several different cost conditions. Table 14.2 shows the minimum total cost order quantity for several cost possibilities. As you can see from the table, the value of Q^* appears relatively stable, even with some variations in the cost estimates. Based on these results, the best order quantity for Bub Beer is in the range of 1700–2000 cases. If operated properly, the total cost for the Bub Beer inventory system should be close to $3400–$3800 per year. We also note that little risk is associated with implementing the calculated order quantity of 1824. For example, if R&B implements an order quantity of 1824 cases (using cost estimates based on $32 per order and 25% annual holding rate), but the actual cost per order turns out to be $34 and the actual annual holding rate turns out to be 24%, then R&B experiences only a $5 increase ($3690–$3685) in the total annual cost.

From the preceding analysis, we would say that this EOQ model is insensitive to small variations or errors in the cost estimates. This insensitivity is a property of EOQ models in general, which indicates that if we have at least reasonable estimates of ordering cost and holding cost, we can expect to obtain a good approximation of the true minimum cost order quantity.

[3]This general expression for cycle time is based on 250 working days per year. If the firm operated 300 working days per year and wanted to express cycle time in terms of working days, the cycle time would be given by $T = 300Q^*/D$.

TABLE 14.2 OPTIMAL ORDER QUANTITIES FOR SEVERAL COST POSSIBILITIES

Possible Inventory Holding Cost (%)	Possible Cost per Order	Optimal Order Quantity (Q^*)	Projected Total Annual Cost Using Q^*	Projected Total Annual Cost Using $Q = 1824$
24	$30	1803	$3461	$3462
24	34	1919	3685	3690
26	30	1732	3603	3607
26	34	1844	3835	3836

Excel Solution of the EOQ Model

Inventory models such as the EOQ model are easily implemented with the aid of spreadsheets. The Excel EOQ formula worksheet for Bub Beer is shown in Figure 14.4. Data on annual demand, ordering cost, annual inventory holding cost rate, cost per unit, working days per year, and lead time in days are input in cells B3 to B8. The appropriate EOQ model formulas, which determine the optimal inventory policy, are placed in cells B13 to B21. For example, cell B13 computes the optimal economic order quantity 1824.28, and cell B16

FIGURE 14.4 FORMULA WORKSHEET FOR THE BUB BEER EOQ INVENTORY MODEL

WEB file

EOQ

	A	B
1	**Economic Order Quantity**	
2		
3	Annual Demand	104000
4	Ordering Cost	32
5	Annual Inventory Holding Rate %	25
6	Cost per Unit	8
7	Working Days per Year	250
8	Lead Time (Days)	2
9		
10		
11	**Optimal Inventory Policy**	
12		
13	Economic Order Quanity	=SQRT(2*B3*B4/(B5/100*B6))
14	Annual Inventory Holding Cost	=(1/2)*B13*(B5/100*B6)
15	Annual Ordering Cost	=(B3/B13)*B4
16	Total Annual Cost	=B14+B15
17	Maximum Inventory Level	=B13
18	Average Inventory Level	=B17/2
19	Reorder Point	=(B3/B7)*B8
20	Number of Orders per Year	=B3/B13
21	Cycle Time (Days)	=B7/B20
22		

TABLE 14.3 THE EOQ MODEL ASSUMPTIONS

1. Demand D is deterministic and occurs at a constant rate.
2. The order quantity Q is the same for each order. The inventory level increases by Q units each time an order is received.
3. The cost per order, C_o, is constant and does not depend on the quantity ordered.
4. The purchase cost per unit, C, is constant and does not depend on the quantity ordered.
5. The inventory holding cost per unit per time period, C_h, is constant. The total inventory holding cost depends on both C_h and the size of the inventory.
6. Shortages such as stock-outs or backorders are not permitted.
7. The lead time for an order is constant.
8. The inventory position is reviewed continuously. As a result, an order is placed as soon as the inventory position reaches the reorder point.

computes the total annual cost $3648.56. If sensitivity analysis is desired, one or more of the input data values can be modified. The impact of any change or changes on the optimal inventory policy will then appear in the worksheet.

The Excel worksheet in Figure 14.4 is a template that can be used for the EOQ model. This worksheet and similar Excel worksheets for the other inventory models presented in this chapter are available at the WEBfiles link on the website that accompanies this text.

Summary of the EOQ Model Assumptions

You should carefully review the assumptions of the inventory model before applying it in an actual situation. Several inventory models discussed later in this chapter alter one or more of the assumptions of the EOQ model.

To use the optimal order quantity and reorder point model described in this section, an analyst must make assumptions about how the inventory system operates. The EOQ model with its economic order quantity formula is based on some specific assumptions about the R&B inventory system. A summary of the assumptions for this model is provided in Table 14.3. Before using the EOQ formula, carefully review these assumptions to ensure that they are applicable to the inventory system being analyzed. If the assumptions are not reasonable, seek a different inventory model.

Various types of inventory systems are used in practice, and the inventory models presented in the following sections alter one or more of the EOQ model assumptions shown in Table 14.3. When the assumptions change, a different inventory model with different optimal operating policies becomes necessary.

NOTES AND COMMENTS

1. With relatively long lead times, the lead-time demand and the resulting reorder point r, determined by equation (14.6), may exceed Q^*. If this condition occurs, at least one order will be outstanding when a new order is placed. For example, assume that Bub Beer has a lead time of $m = 6$ days. With a daily demand of $d = 432$ cases, equation (14.6) shows that the reorder point would be $r = dm = 6 \times 432 = 2592$ cases. Thus, a new order for Bub Beer should be placed whenever the inventory position (the amount of inventory on hand plus the amount of inventory on order) reaches 2592. With an order quantity of $Q = 2000$ cases, the inventory position of 2592 cases occurs when one order of 2000 cases is outstanding and $2592 - 2000 = 592$ cases are on hand.

14.2 Economic Production Lot Size Model

The inventory model in this section alters assumption 2 of the EOQ model (see Table 14.3). The assumption concerning the arrival of Q units each time an order is received is changed to a constant production supply rate.

The inventory model presented in this section is similar to the EOQ model in that we are attempting to determine *how much* we should order and *when* the order should be placed. We again assume a constant demand rate. However, instead of assuming that the order arrives in a shipment of size Q^*, as in the EOQ model, we assume that units are supplied to inventory at a constant rate over several days or several weeks. The **constant supply rate** assumption implies that the same number of units is supplied to inventory each period of time (e.g., 10 units every day or 50 units every week). This model is designed for production situations in which, once an order is placed, production begins and a constant number of units is added to inventory each day until the production run has been completed.

If we have a production system that produces 50 units per day and we decide to schedule 10 days of production, we have a $50(10) = 500$-unit production lot size. The **lot size** is the number of units in an order. In general, if we let Q indicate the production lot size, the approach to the inventory decisions is similar to the EOQ model; that is, we build a holding and ordering cost model that expresses the total cost as a function of the production lot size. Then we attempt to find the production lot size that minimizes the total cost.

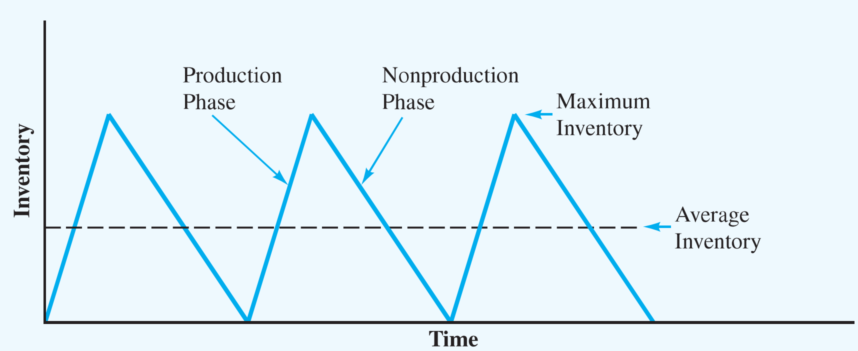
One other condition that should be mentioned at this time is that the model only applies to situations where the production rate is greater than the demand rate; the production system must be able to satisfy demand. For instance, if the constant demand rate is 400 units per day, the production rate must be at least 400 units per day to satisfy demand.

During the production run, demand reduces the inventory while production adds to inventory. Because we assume that the production rate exceeds the demand rate, each day during a production run we produce more units than are demanded. Thus, the excess production causes a gradual inventory buildup during the production period. When the production run is completed, the continuing demand causes the inventory to gradually decline until a new production run is started. The inventory pattern for this system is shown in Figure 14.5.

This model differs from the EOQ model in that a setup cost replaces the ordering cost, and the saw-tooth inventory pattern shown in Figure 14.5 differs from the inventory pattern shown in Figure 14.2.

As in the EOQ model, we are now dealing with two costs, the holding cost and the ordering cost. Here the holding cost is identical to the definition in the EOQ model, but the interpretation of the ordering cost is slightly different. In fact, in a production situation the ordering cost is more correctly referred to as the production **setup cost**. This cost, which includes labor, material, and lost production costs incurred while preparing the production

FIGURE 14.5 INVENTORY PATTERN FOR THE PRODUCTION LOT SIZE INVENTORY MODEL



system for operation, is a fixed cost that occurs for every production run regardless of the production lot size.

Total Cost Model

Let us begin building the production lot size model by writing the holding cost in terms of the production lot size Q. Again, the approach is to develop an expression for average inventory and then establish the holding costs associated with the average inventory. We use a one-year time period and an annual cost for the model.

In the EOQ model the average inventory is one-half the maximum inventory, or $^1/_2 Q$. Figure 14.5 shows that for a production lot size model, a constant inventory buildup rate occurs during the production run, and a constant inventory depletion rate occurs during the nonproduction period; thus, the average inventory will be one-half the maximum inventory. However, in this inventory system the production lot size Q does not go into inventory at one point in time, and thus the inventory never reaches a level of Q units.

To show how we can compute the maximum inventory, let

$$d = \text{daily demand rate}$$
$$p = \text{daily production rate}$$
$$t = \text{number of days for a production run}$$

At this point, the logic of the production lot size model is easier to follow using a daily demand rate d and a daily production rate p. However, when the total annual cost model is eventually developed, we recommend that inputs to the model be expressed in terms of the annual demand rate D and the annual production rate P.

Because we are assuming that p will be larger than d, the daily inventory buildup rate during the production phase is $p - d$. If we run production for t days and place $p - d$ units in inventory each day, the inventory at the end of the production run will be $(p - d)t$. From Figure 14.5 we can see that the inventory at the end of the production run is also the maximum inventory. Thus,

$$\text{Maximum inventory} = (p - d)t \tag{14.8}$$

If we know we are producing a production lot size of Q units at a daily production rate of p units, then $Q = pt$, and the length of the production run t must be

$$t = \frac{Q}{p} \text{ days} \tag{14.9}$$

Thus,

$$\begin{aligned} \text{Maximum inventory} &= (p - d)t = (p - d)\left(\frac{Q}{p}\right) \\ &= \left(1 - \frac{d}{p}\right)Q \end{aligned} \tag{14.10}$$

The average inventory, which is one-half the maximum inventory, is given by

$$\text{Average inventory} = \frac{1}{2}\left(1 - \frac{d}{p}\right)Q \tag{14.11}$$

With an annual per-unit holding cost of C_h, the general equation for annual holding cost is as follows:

$$\begin{aligned}\text{Annual holding cost} &= \begin{pmatrix}\text{Average}\\ \text{inventory}\end{pmatrix}\begin{pmatrix}\text{Annual}\\ \text{cost}\\ \text{per unit}\end{pmatrix}\\ &= \frac{1}{2}\left(1 - \frac{d}{p}\right)QC_h\end{aligned} \tag{14.12}$$

If D is the annual demand for the product and C_o is the setup cost for a production run, then the annual setup cost, which takes the place of the annual ordering cost in the EOQ model, is as follows:

$$\begin{aligned}\text{Annual setup cost} &= \begin{pmatrix}\text{Number of production}\\ \text{runs per year}\end{pmatrix}\begin{pmatrix}\text{Setup cost}\\ \text{per run}\end{pmatrix}\\ &= \frac{D}{Q}C_o\end{aligned} \tag{14.13}$$

Thus, the total annual cost (TC) model is

$$TC = \frac{1}{2}\left(1 - \frac{d}{p}\right)QC_h + \frac{D}{Q}C_o \tag{14.14}$$

Suppose that a production facility operates 250 days per year. Then we can write daily demand d in terms of annual demand D as follows:

$$d = \frac{D}{250}$$

Now let P denote the annual production for the product if the product were produced every day. Then

$$P = 250p \qquad \text{and} \qquad p = \frac{P}{250}$$

Thus,[4]

$$\frac{d}{p} = \frac{D/250}{P/250} = \frac{D}{P}$$

Therefore, we can write the total annual cost model as follows:

$$TC = \frac{1}{2}\left(1 - \frac{D}{P}\right)QC_h + \frac{D}{Q}C_o \tag{14.15}$$

[4]The ratio $d/p = D/P$ holds regardless of the number of days of operation; 250 days is used here merely as an illustration.

Equations (14.14) and (14.15) are equivalent. However, equation (14.15) may be used more frequently because an *annual* cost model tends to make the analyst think in terms of collecting *annual* demand data (D) and *annual* production data (P) rather than daily data.

Economic Production Lot Size

As the production rate P approaches infinity, D/P approaches zero. In this case, equation (14.16) is equivalent to the EOQ model in equation (14.5).

Given estimates of the holding cost (C_h), setup cost (C_o), annual demand rate (D), and annual production rate (P), we could use a trial-and-error approach to compute the total annual cost for various production lot sizes (Q). However, trial and error is not necessary; we can use the minimum cost formula for Q^* that has been developed using differential calculus (see Appendix 14.2). The equation is as follows:

$$Q^* = \sqrt{\frac{2DC_o}{(1 - D/P)C_h}} \tag{14.16}$$

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Lot Size

An Example Beauty Bar Soap is produced on a production line that has an annual capacity of 60,000 cases. The annual demand is estimated at 26,000 cases, with the demand rate essentially constant throughout the year. The cleaning, preparation, and setup of the production line cost approximately \$135. The manufacturing cost per case is \$4.50, and the annual holding cost is figured at a 24% rate. Thus, $C_h = IC = 0.24(\$4.50) = \$1.08$. What is the recommended production lot size?

Using equation (14.16), we have

$$Q^* = \sqrt{\frac{2(26{,}000)(135)}{(1 - 26{,}000/60{,}000)(1.08)}} = 3387$$

Work Problem 13 as an example of an economic production lot size model.

The total annual cost using equation (14.15) and $Q^* = 3387$ is \$2073.

Other relevant data include a five-day lead time to schedule and set up a production run and 250 working days per year. Thus, the lead-time demand of (26,000/250)(5) = 520 cases is the reorder point. The cycle time is the time between production runs. Using equation (14.7), the cycle time is $T = 250Q^*/D = [(250)(3387)]/26{,}000$, or 33 working days. Thus, we should plan a production run of 3387 units every 33 working days.

14.3 Inventory Model with Planned Shortages

A **shortage** or **stock-out** occurs when demand exceeds the amount of inventory on hand. In many situations, shortages are undesirable and should be avoided if at all possible. However, in other cases it may be desirable—from an economic point of view—to plan for and allow shortages. In practice, these types of situations are most commonly found where the value of the inventory per unit is high and hence the holding cost is high. An example of this type of situation is a new car dealer's inventory. Often a specific car that a customer wants is not in stock. However, if the customer is willing to wait a few weeks, the dealer is usually able to order the car.

The assumptions of the EOQ model in Table 14.3 apply to this inventory model with the exception that shortages, referred to as backorders, are now permitted.

The model developed in this section takes into account a type of shortage known as a **backorder**. In a backorder situation, we assume that when a customer places an order and discovers that the supplier is out of stock, the customer waits until the new shipment arrives, and then the order is filled. Frequently, the waiting period in backorder situations is relatively short. Thus, by promising the customer top priority and immediate delivery when the goods become available, companies may be able to convince the customer to wait until the order arrives. In these cases, the backorder assumption is valid.

The backorder model that we develop is an extension of the EOQ model presented in Section 14.1. We use the EOQ model in which all goods arrive in inventory at one time and are subject to a constant demand rate. If we let S indicate the number of backorders that have accumulated by the time a new shipment of size Q is received, then the inventory system for the backorder case has the following characteristics:

- If S backorders exist when a new shipment of size Q arrives, then S backorders are shipped to the appropriate customers, and the remaining $Q - S$ units are placed in inventory. Therefore, $Q - S$ is the maximum inventory.
- The inventory cycle of T days is divided into two distinct phases: t_1 days when inventory is on hand and orders are filled as they occur, and t_2 days when stock-outs occur and all new orders are placed on backorder.

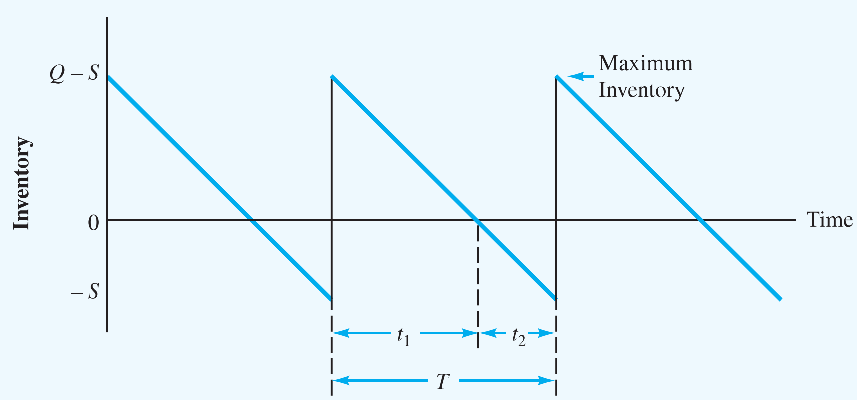
The inventory pattern for the inventory model with backorders, where negative inventory represents the number of backorders, is shown in Figure 14.6.

With the inventory pattern now defined, we can proceed with the basic step of all inventory models—namely, the development of a total cost model. For the inventory model with backorders, we encounter the usual holding costs and ordering costs. We also incur a backorder cost in terms of the labor and special delivery costs directly associated with the handling of the backorders. Another portion of the backorder cost accounts for the loss of goodwill because some customers will have to wait for their orders. Because the **goodwill cost** depends on how long a customer has to wait, it is customary to adopt the convention of expressing backorder cost in terms of the cost of having a unit on backorder for a stated period of time. This method of costing backorders on a time basis is similar to the method used to compute the inventory holding cost, and we can use it to compute a total annual cost of backorders once the average backorder level and the backorder cost per unit per period are known.

Let us begin the development of a total cost model by calculating the average inventory for a hypothetical problem. If we have an average inventory of two units for three days and no inventory on the fourth day, what is the average inventory over the four-day period? It is

$$\frac{2 \text{ units (3 days)} + 0 \text{ units (1 day)}}{4 \text{ days}} = \frac{6}{4} = 1.5 \text{ units}$$

FIGURE 14.6 INVENTORY PATTERN FOR AN INVENTORY MODEL WITH BACKORDERS



Refer to Figure 14.6. You can see that this situation is what happens in the backorder model. With a maximum inventory of $Q - S$ units, the t_1 days we have inventory on hand will have an average inventory of $(Q - S)/2$. No inventory is carried for the t_2 days in which we experience backorders. Thus, over the total cycle time of $T = t_1 + t_2$ days, we can compute the average inventory as follows:

$$\text{Average inventory} = \frac{\frac{1}{2}(Q - S)t_1 + 0t_2}{t_1 + t_2} = \frac{\frac{1}{2}(Q - S)t_1}{T} \quad \textbf{(14.17)}$$

Can we find other ways of expressing t_1 and T? Because we know that the maximum inventory is $Q - S$ and that d represents the constant daily demand, we have

$$t_1 = \frac{Q - S}{d} \text{ days} \quad \textbf{(14.18)}$$

That is, the maximum inventory of $Q - S$ units will be used up in $(Q - S)/d$ days. Because Q units are ordered each cycle, we know the length of a cycle must be

$$T = \frac{Q}{d} \text{ days} \quad \textbf{(14.19)}$$

Combining equations (14.18) and (14.19) with equation (14.17), we can compute the average inventory as follows:

$$\text{Average inventory} = \frac{\frac{1}{2}(Q - S)[(Q - S)/d]}{Q/d} = \frac{(Q - S)^2}{2Q} \quad \textbf{(14.20)}$$

Thus, the average inventory is expressed in terms of two inventory decisions: how much we will order (Q) and the maximum number of backorders (S).

The formula for the annual number of orders placed using this model is identical to that for the EOQ model. With D representing the annual demand, we have

$$\text{Annual number of orders} = \frac{D}{Q} \quad \textbf{(14.21)}$$

The next step is to develop an expression for the average backorder level. Because we know the maximum for backorders is S, we can use the same logic we used to establish average inventory in finding the average number of backorders. We have an average number of backorders during the period t_2 of $\frac{1}{2}$ the maximum number of backorders or $\frac{1}{2}S$. We do not have any backorders during the t_1 days we have inventory; therefore we can calculate the average backorders in a manner similar to equation (14.17). Using this approach, we have

$$\text{Average backorders} = \frac{0t_1 + (S/2)t_2}{T} = \frac{(S/2)t_2}{T} \quad \textbf{(14.22)}$$

When we let the maximum number of backorders reach an amount S at a daily rate of d, the length of the backorder portion of the inventory cycle is

$$t_2 = \frac{S}{d} \quad \textbf{(14.23)}$$

Using equations (14.23) and (14.19) in equation (14.22), we have

$$\text{Average backorders} = \frac{(S/2)(S/d)}{Q/d} = \frac{S^2}{2Q} \quad \textbf{(14.24)}$$

Let

C_h = cost to hold one unit in inventory for one year
C_o = cost per order
C_b = cost to maintain one unit on backorder for one year

The total annual cost (TC) for the inventory model with backorders becomes

$$TC = \frac{(Q - S)^2}{2Q} C_h + \frac{D}{Q} C_o + \frac{S^2}{2Q} C_b \quad \textbf{(14.25)}$$

Given C_h, C_o, and C_b and the annual demand D, differential calculus can be used to show that the minimum cost values for the order quantity Q^* and the planned backorders S^* are as follows:

$$Q^* = \sqrt{\frac{2DC_o}{C_h}\left(\frac{C_h + C_b}{C_b}\right)} \quad \textbf{(14.26)}$$

$$S^* = Q^*\left(\frac{C_h}{C_h + C_b}\right) \quad \textbf{(14.27)}$$

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Shortage

An Example Suppose that the Higley Radio Components Company has a product for which the assumptions of the inventory model with backorders are valid. Information obtained by the company is as follows:

D = 2000 units per year
I = 20%
C = \$50 per unit
$C_h = IC = (0.20)(\$50) = \$10$ per unit per year
C_o = \$25 = per order

An inventory situation that incorporates backorder costs is considered in Problem 15.

The company is considering the possibility of allowing some backorders to occur for the product. The annual backorder cost is estimated to be \$30 per unit per year. Using equations (14.26) and (14.27), we have

$$Q^* = \sqrt{\frac{2(2000)(25)}{10}\left(\frac{10 + 30}{30}\right)} = 115$$

and

$$S^* = 115\left(\frac{10}{10 + 30}\right) = 29$$

The backorder cost C_b is one of the most difficult costs to estimate in inventory models. The reason is that it attempts to measure the cost associated with the loss of goodwill when a customer must wait for an order. Expressing this cost on an annual basis adds to the difficulty.

If this solution is implemented, the system will operate with the following properties:

$$\text{Maximum inventory} = Q - S = 115 - 29 = 86$$

$$\text{Cycle time} = T = \frac{Q}{D}(250) = \frac{115}{2000}(250) = 14 \text{ working days}$$

The total annual cost is

$$\text{Holding cost} = \frac{(86)^2}{2(115)}(10) = \$322$$

$$\text{Ordering cost} = \frac{2000}{115}(25) = \$435$$

$$\text{Backorder cost} = \frac{(29)^2}{2(115)}(30) = \$110$$

$$\text{Total cost} = \$867$$

If backorders can be tolerated, the total cost including the backorder cost will be less than the total cost of the EOQ model. Some people think the model with backorders will have a greater cost because it includes a backorder cost in addition to the usual inventory holding and ordering costs. You can point out the fallacy in this thinking by noting that the backorder model leads to lower inventory and hence lower inventory holding costs.

If the company chooses to prohibit backorders and adopts the regular EOQ model, the recommended inventory decision would be

$$Q^* = \sqrt{\frac{2(2000)(25)}{10}} = \sqrt{10{,}000} = 100$$

This order quantity would result in a holding cost and an ordering cost of \$500 each or a total annual cost of \$1000. Thus, in this problem, allowing backorders is projecting a \$1000 − \$867 = \$133, or 13.3%, savings in cost from the no-stock-out EOQ model. The preceding comparison and conclusion are based on the assumption that the backorder model with an annual cost per backordered unit of \$30 is a valid model for the actual inventory situation. If the company is concerned that stock-outs might lead to lost sales, then the savings might not be enough to warrant switching to an inventory policy that allows for planned shortages.

NOTES AND COMMENTS

1. Equation (14.27) shows that the optimal number of planned backorders S^* is proportional to the ratio $C_h/(C_h + C_b)$, where C_h is the annual holding cost per unit and C_b is the annual backorder cost per unit. Whenever C_h increases, this ratio becomes larger, and the number of planned backorders increases. This relationship explains why items that have a high per-unit cost and a correspondingly high annual holding cost are more economically handled on a backorder basis. On the other hand, whenever the backorder cost C_b increases, the ratio becomes smaller, and the number of planned backorders decreases. Thus, the model provides the intuitive result that items with high backorder costs will be handled with few backorders. In fact, with high backorder costs, the backorder model and the EOQ model with no backordering allowed provide similar inventory policies.

14.4 Quantity Discounts for the EOQ Model

In the quantity discount model, assumption 4 of the EOQ model in Table 14.3 is altered. The cost per unit varies depending on the quantity ordered.

Quantity discounts occur in numerous situations in which suppliers provide an incentive for large order quantities by offering a lower purchase cost when items are ordered in larger quantities. In this section we show how the EOQ model can be used when quantity discounts are available.

Assume that we have a product in which the basic EOQ model (see Table 14.3) is applicable. Instead of a fixed unit cost, the supplier quotes the following discount schedule:

Discount Category	Order Size	Discount (%)	Unit Cost
1	0 to 999	0	$5.00
2	1000 to 2499	3	4.85
3	2500 and over	5	4.75

The 5% discount for the 2500-unit minimum order quantity looks tempting. However, realizing that higher order quantities result in higher inventory holding costs, we should prepare a thorough cost analysis before making a final ordering and inventory policy recommendation.

Suppose that the data and cost analyses show an annual holding cost rate of 20%, an ordering cost of $49 per order, and an annual demand of 5000 units; what order quantity should we select? The following three-step procedure shows the calculations necessary to make this decision. In the preliminary calculations, we use Q_1 to indicate the order quantity for discount category 1, Q_2 for discount category 2, and Q_3 for discount category 3.

Step 1. For each discount category, compute a Q^* using the EOQ formula based on the unit cost associated with the discount category.

Recall that the EOQ model provides $Q^* = \sqrt{2DC_o/C_h}$, where $C_h = IC = (0.20)C$. With three discount categories providing three different unit costs C, we obtain

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Discount

$$Q_1^* = \sqrt{\frac{2(5000)49}{(0.20)(5.00)}} = 700$$

$$Q_2^* = \sqrt{\frac{2(5000)49}{(0.20)(4.85)}} = 711$$

$$Q_3^* = \sqrt{\frac{2(5000)49}{(0.20)(4.75)}} = 718$$

Because the only differences in the EOQ formulas come from slight differences in the holding cost, the economic order quantities resulting from this step will be approximately the same. However, these order quantities will usually not all be of the size necessary to qualify for the discount price assumed. In the preceding case, both Q_2^* and Q_3^* are insufficient order quantities to obtain their assumed discounted costs of $4.85 and $4.75, respectively.

For those order quantities for which the assumed price cannot be obtained, the following procedure must be used:

Step 2. For the Q^* that is too small to qualify for the assumed discount price, adjust the order quantity upward to the nearest order quantity that will allow the product to be purchased at the assumed price.

In our example, this adjustment causes us to set

$$Q_2^* = 1000$$

and

$$Q_3^* = 2500$$

If a calculated Q^* for a given discount price is large enough to qualify for a bigger discount, that value of Q^* cannot lead to an optimal solution. Although the reason may not be obvious, it does turn out to be a property of the EOQ quantity discount model.

In the EOQ model with quantity discounts, the annual purchase cost must be included because purchase cost depends on the order quantity. Thus, it is a relevant cost.

In the previous inventory models considered, the annual purchase cost of the item was not included because it was constant and never affected by the inventory order policy decision. However, in the quantity discount model, the annual purchase cost depends on the order quantity and the associated unit cost. Thus, annual purchase cost (annual demand $D \times$ unit cost C) is included in the equation for total cost as shown here.

$$TC = \frac{Q}{2} C_h + \frac{D}{Q} C_o + DC \qquad (14.28)$$

Using this total cost equation, we can determine the optimal order quantity for the EOQ discount model in step 3.

Problem 21 will give you practice in applying the EOQ model to situations with quantity discounts.

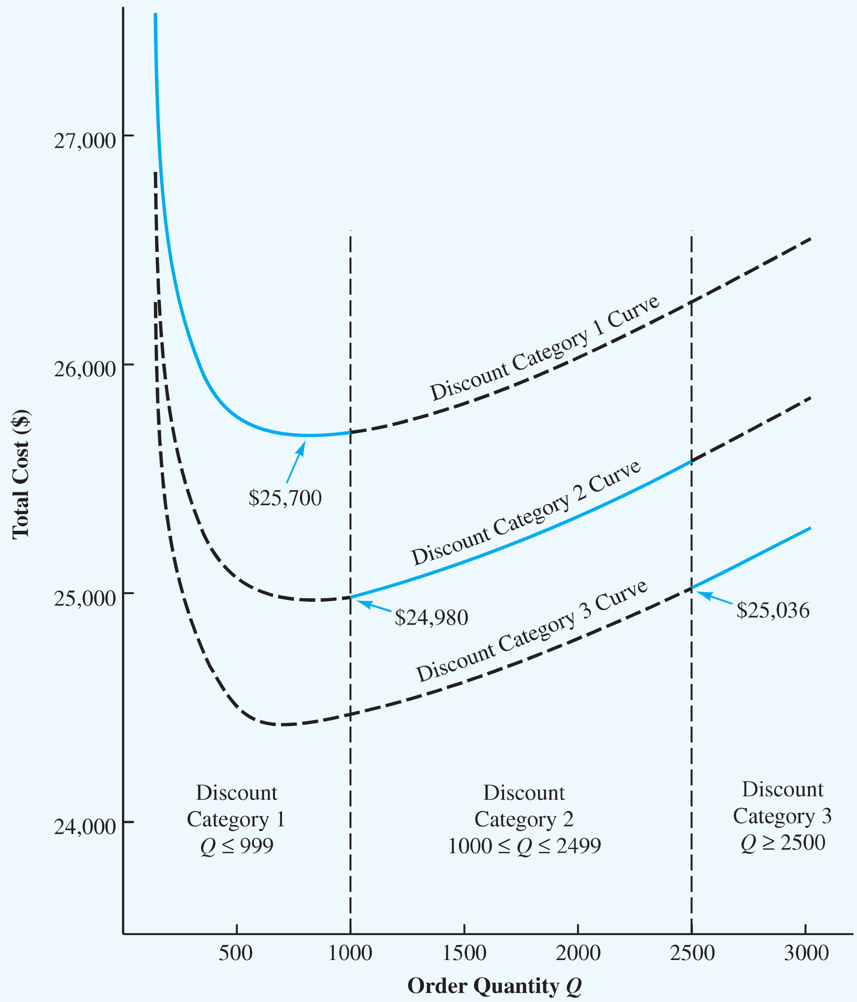
Step 3. For each order quantity resulting from steps 1 and 2, compute the total annual cost using the unit price from the appropriate discount category and equation (14.28). The order quantity yielding the minimum total annual cost is the optimal order quantity.

The step 3 calculations for the example problem are summarized in Table 14.4. As you can see, a decision to order 1000 units at the 3% discount rate yields the minimum cost solution. Even though the 2500-unit order quantity would result in a 5% discount, its excessive holding cost makes it the second-best solution. Figure 14.7 shows the total cost curve for each of the three discount categories. Note that $Q^* = 1000$ provides the minimum cost order quantity.

TABLE 14.4 TOTAL ANNUAL COST CALCULATIONS FOR THE EOQ MODEL WITH QUANTITY DISCOUNTS

Discount Category	Unit Cost	Order Quantity	Annual Cost			
			Holding	Ordering	Purchase	Total
1	$5.00	700	$ 350	$350	$25,000	$25,700
2	4.85	1000	$ 485	$245	$24,250	$24,980
3	4.75	2500	$1188	$ 98	$23,750	$25,036

FIGURE 14.7 TOTAL COST CURVES FOR THE THREE DISCOUNT CATEGORIES



The overall minimum cost of \$24,980 occurs at $Q^* = 1000$.

14.5 Single-Period Inventory Model with Probabilistic Demand

The inventory models discussed thus far were based on the assumption that the demand rate is constant and **deterministic** throughout the year. We developed minimum cost order quantity and reorder point policies based on this assumption. In situations in which the demand rate is not deterministic, other models treat demand as **probabilistic** and best described by a probability distribution. In this section we consider a single-period inventory model with probabilistic demand.

The single-period inventory model refers to inventory situations in which *one* order is placed for the product; at the end of the period, the product has either sold out, or a surplus of

This inventory model is the first in the chapter that explicitly treats probabilistic demand. Unlike the EOQ model, it is for a single period, and unused inventory is not carried over to future periods.

unsold items will be sold for a salvage value. The single-period inventory model is applicable in situations involving seasonal or perishable items that cannot be carried in inventory and sold in future periods. Seasonal clothing (such as bathing suits and winter coats) are typically handled in a single-period manner. In these situations, a buyer places one preseason order for each item and then experiences a stock-out or holds a clearance sale on the surplus stock at the end of the season. No items are carried in inventory and sold the following year. Newspapers are another example of a product that is ordered one time and is either sold or not sold during the single period. Although newspapers are ordered daily, they cannot be carried in inventory and sold in later periods. Thus, newspaper orders may be treated as a sequence of single-period models; that is, each day or period is separate, and a single-period inventory decision must be made each period (day). Because we order only once for the period, the only inventory decision we must make is *how much* of the product to order at the start of the period.

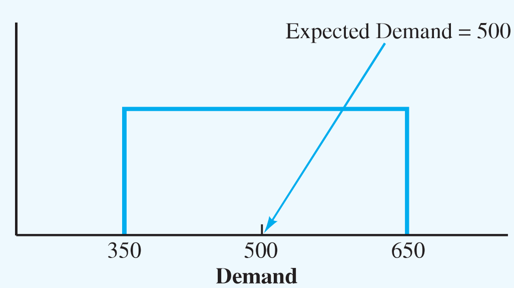
Obviously, if the demand were known for a single-period inventory situation, the solution would be easy; we would simply order the amount we knew would be demanded. However, in most single-period models, the exact demand is not known. In fact, forecasts may show that demand can have a wide variety of values. If we are going to analyze this type of inventory problem in a quantitative manner, we need information about the probabilities associated with the various demand values. Thus, the single-period model presented in this section is based on probabilistic demand.

Neiman Marcus

Let us consider a single-period inventory model that could be used to make a how-much-to-order decision for Neiman Marcus, a high-end fashion store. The buyer for Neiman Marcus decided to order Manolo Blahnik heels shown at a buyers' meeting in New York City. The shoe will be part of the company's spring–summer promotion and will be sold through nine retail stores in the Chicago area. Because the shoe is designed for spring and summer months, it cannot be expected to sell in the fall. Neiman Marcus plans to hold a special August clearance sale in an attempt to sell all shoes not sold by July 31. The shoes cost $700 a pair and retail for $900 a pair. At the sale price of $600 a pair, all surplus shoes can be expected to sell during the August sale. If you were the buyer for Neiman Marcus, how many pairs of the shoes would you order?

To answer the question of how much to order, we need information on the demand for the shoe. Specifically, we would need to construct a probability distribution for the possible values of demand. Let us suppose that the uniform probability distribution shown in Figure 14.8 can be used to describe the demand for the Manolo Blahnik heels. In particular,

FIGURE 14.8 UNIFORM PROBABILITY DISTRIBUTION OF DEMAND FOR NEIMAN MARCUS PROBLEM



note that the range of demand is from 350 to 650 pairs of shoes, with an average, or expected, demand of 500 pairs of shoes.

Incremental analysis is a method that can be used to determine the optimal order quantity for a single-period inventory model. Incremental analysis addresses the how-much-to-order question by comparing the cost or loss of *ordering one additional unit* with the cost or loss of *not ordering one additional unit.* The costs involved are defined as follows:

c_o = cost per unit of *overestimating* demand. This cost represents the loss of ordering one additional unit and finding that it cannot be sold.

c_u = cost per unit of *underestimating* demand. This cost represents the opportunity loss of not ordering one additional unit and finding that it could have been sold.

In the Neiman Marcus problem, the company will incur the cost of overestimating demand whenever it orders too many pairs and has to sell the extra shoes during the August clearance sale. Thus, the cost per unit of overestimating demand is equal to the purchase cost per unit minus the August sales price per unit; that is, $c_o = \$700 - \$600 = \$100$. Therefore, Neiman Marcus will lose $100 for each pair of shoes that it orders over the quantity demanded. The cost of underestimating demand is the lost profit (often referred to as an opportunity cost) because a pair of shoes that could have been sold was not available in inventory. Thus, the per-unit cost of underestimating demand is the difference between the regular selling price per unit and the purchase cost per unit; that is, $c_u = \$900 - \$700 = \$200$.

The cost of underestimating demand is usually harder to determine than the cost of overestimating demand. The reason is that the cost of underestimating demand includes a lost profit and may include a customer goodwill cost because the customer is unable to purchase the item when desired.

The key to incremental analysis is to focus on the costs that are different when comparing an order quantity $Q + 1$ to an order quantity Q.

Because the exact level of demand for the Manolo Blahnik heels is unknown, we have to consider the probability of demand and thus the probability of obtaining the associated costs or losses. For example, let us assume that Neiman Marcus management wishes to consider an order quantity equal to the average or expected demand for 500 pairs of shoes. In incremental analysis, we consider the possible losses associated with an order quantity of 501 (ordering one additional unit) and an order quantity of 500 (not ordering one additional unit). The order quantity alternatives and the possible losses are summarized here.

Order Quantity Alternatives	Loss Occurs If	Possible Loss	Probability Loss Occurs
$Q = 501$	Demand overestimated; the additional unit *cannot* be sold	$c_o = \$100$	$P(\text{demand} \leq 500)$
$Q = 500$	Demand underestimated; an additional unit *could have* been sold	$c_u = \$200$	$P(\text{demand} > 500)$

Using the demand probability distribution in Figure 14.8, we see that $P(\text{demand} \leq 500) = 0.50$ and that $P(\text{demand} > 500) = 0.50$. By multiplying the possible losses, $c_o = \$100$ and $c_u = \$200$, by the probability of obtaining the loss, we can compute the expected value of the loss, or simply the *expected loss* (EL), associated with the order quantity alternatives. Thus,

$$\text{EL}(Q = 501) = c_o P(\text{demand} \leq 500) = \$100(0.50) = \$50$$
$$\text{EL}(Q = 500) = c_u P(\text{demand} > 500) = \$200(0.50) = \$100$$

Based on these expected losses, do you prefer an order quantity of 501 or 500 pairs of shoes? Because the expected loss is greater for $Q = 500$, and because we want to avoid this higher cost or loss, we should make $Q = 501$ the preferred decision. We could now consider incrementing the order quantity one additional unit to $Q = 502$ and repeating the expected loss calculations.

Although we could continue this unit-by-unit analysis, it would be time-consuming and cumbersome. We would have to evaluate $Q = 502$, $Q = 503$, $Q = 504$, and so on until we found the value of Q where the expected loss of ordering one incremental unit is equal to the expected loss of not ordering one incremental unit; that is, the optimal order quantity Q^* occurs when the incremental analysis shows that

$$\text{EL}(Q^* + 1) = \text{EL}(Q^*) \qquad \textbf{(14.29)}$$

When this relationship holds, increasing the order quantity by one additional unit has no economic advantage. Using the logic with which we computed the expected losses for the order quantities of 501 and 500, the general expressions for $\text{EL}(Q^* + 1)$ and $\text{EL}(Q^*)$ can be written

$$\text{EL}(Q^* + 1) = c_\text{o} P(\text{demand} \leq Q^*) \qquad \textbf{(14.30)}$$

$$\text{EL}(Q^*) = c_\text{u} P(\text{demand} > Q^*) \qquad \textbf{(14.31)}$$

Because demand $\leq Q^*$ and demand $> Q^*$ are complimentary events, we know from basic probability that

$$P(\text{demand} \leq Q^*) + P(\text{demand} > Q^*) = 1 \qquad \textbf{(14.32)}$$

and we can write

$$P(\text{demand} > Q^*) = 1 - P(\text{demand} \leq Q^*) \qquad \textbf{(14.33)}$$

Using this expression, equation (14.31) can be rewritten as

$$\text{EL}(Q^*) = c_\text{u}[1 - P(\text{demand} \leq Q^*)] \qquad \textbf{(14.34)}$$

Equations (14.30) and (14.34) can be used to show that $\text{EL}(Q^* + 1) = \text{EL}(Q^*)$ whenever

$$c_\text{o} P(\text{demand} \leq Q^*) = c_\text{u}[1 - P(\text{demand} \leq Q^*)] \qquad \textbf{(14.35)}$$

Solving for $P(\text{demand} \leq Q^*)$, we have

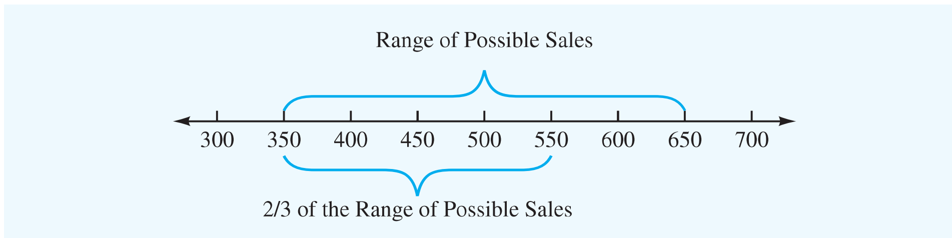
$$P(\text{demand} \leq Q^*) = \frac{c_\text{u}}{c_\text{u} + c_\text{o}} \qquad \textbf{(14.36)}$$

This expression provides the general condition for the optimal order quantity Q^* in the single-period inventory model.

In the Neiman Marcus problem, $c_\text{o} = \$100$ and $c_\text{u} = \$200$. Thus, equation (14.36) shows that the optimal order size for the Manolo Blahnik heels must satisfy the following condition:

$$P(\text{demand} \leq Q^*) = \frac{c_\text{u}}{c_\text{u} + c_\text{o}} = \frac{200}{200 + 100} = \frac{200}{300} = \frac{2}{3}$$

We can find the optimal order quantity Q^* by referring to the probability distribution shown in Figure 14.8 and finding the value of Q that will provide $P(\text{demand} \le Q^*) = {}^2/_3$. To find this solution, we note that in the uniform distribution the probability is evenly distributed over the entire range of 350–650 pairs of shoes. Thus, we can satisfy the expression for Q^* by moving two-thirds of the way from 350 to 650. Because this range is 650 – 350 = 300, we move 200 units from 350 toward 650.



Doing so provides the optimal order quantity of 550 pairs of shoes.

In summary, the key to establishing an optimal order quantity for single-period inventory models is to identify the probability distribution that describes the demand for the item and to calculate the per-unit costs of overestimation and underestimation. Then, using the information for the per-unit costs of overestimation and underestimation, equation (14.36) can be used to find the location of Q^* in the probability distribution.

Nationwide Car Rental

As another example of a single-period inventory model with probabilistic demand, consider the situation faced by Nationwide Car Rental. Nationwide must decide how many automobiles to have available at each car rental location at specific points in time throughout the year. Using the Myrtle Beach, South Carolina, location as an example, management would like to know the number of full-sized automobiles to have available for the Labor Day weekend. Based on previous experience, customer demand for full-sized automobiles for the Labor Day weekend has a normal distribution with a mean of 150 automobiles and a standard deviation of 14 automobiles.

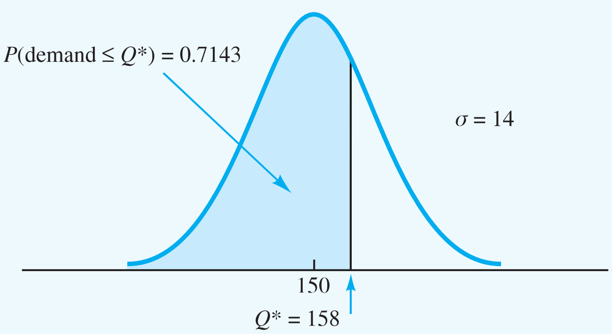
The Nationwide Car Rental situation can benefit from use of a single-period inventory model. The company must establish the number of full-sized automobiles to have available prior to the weekend. Customer demand over the weekend will then result in either a stock-out or a surplus. Let us denote the number of full-sized automobiles available by Q. If Q is greater than customer demand, Nationwide will have a surplus of cars. The cost of a surplus is the cost of overestimating demand. This cost is set at \$80 per car, which reflects, in part, the opportunity cost of not having the car available for rent elsewhere.

If Q is less than customer demand, Nationwide will rent all available cars and experience a stock-out or shortage. A shortage results in an underestimation cost of \$200 per car. This figure reflects the cost due to lost profit and the lost goodwill of not having a car available for a customer. Given this information, how many full-sized automobiles should Nationwide make available for the Labor Day weekend?

Using the cost of underestimation, $c_u = \$200$, and the cost of overestimation, $c_o = \$80$, equation (14.36) indicates that the optimal order quantity must satisfy the following condition:

$$P(\text{demand} \le Q^*) = \frac{c_u}{(c_u + c_o)} = \frac{200}{200 + 80} = 0.7143$$

FIGURE 14.9 PROBABILITY DISTRIBUTION OF DEMAND FOR THE NATIONWIDE CAR RENTAL PROBLEM SHOWING THE LOCATION OF Q^*



We can use the normal probability distribution for demand as shown in Figure 14.9 to find the order quantity that satisfies the condition that $P(\text{demand} \le Q^*) = 0.7143$. From Appendix D, we see that 0.7143 of the area in the left tail of the normal probability distribution occurs at $z = 0.57$ standard deviations *above* the mean. With a mean demand of $\mu = 150$ automobiles and a standard deviation of $\sigma = 14$ automobiles, we have

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Single-Period

An example of a single-period inventory model with probabilistic demand described by a normal probability distribution is considered in Problem 25.

$$Q^* = \mu + 0.57\sigma$$
$$= 150 + 0.57(14) = 158$$

Thus, Nationwide Car Rental should plan to have 158 full-sized automobiles available in Myrtle Beach for the Labor Day weekend. Note that in this case the cost of overestimation is less than the cost of underestimation. Thus, Nationwide is willing to risk a higher probability of overestimating demand and hence a higher probability of a surplus. In fact, Nationwide's optimal order quantity has a 0.7143 probability of a surplus and a $1 - 0.7143 = 0.2857$ probability of a stock-out. As a result, the probability is 0.2857 that all 158 full-sized automobiles will be rented during the Labor Day weekend.

NOTES AND COMMENTS

1. In any probabilistic inventory model, the assumption about the probability distribution for demand is critical and can affect the recommended inventory decision. In the problems presented in this section, we used the uniform and the normal probability distributions to describe demand. In some situations, other probability distributions may be more appropriate. In using probabilistic inventory models, we must exercise care in selecting the probability distribution that most realistically describes demand.
2. In the single-period inventory model, the value of $c_u/(c_u + c_o)$ plays a critical role in selecting the order quantity [see equation (14.36)]. Whenever $c_u = c_o$, $c_u/(c_u + c_o)$ equals 0.50; in this case, we should select an order quantity corresponding to the median demand. With this choice, a stock-out is just as likely as a surplus because the two costs are equal. However, whenever $c_u < c_o$, a smaller order quantity will be recommended. In this case, the smaller order quantity will provide a higher probability of a stock-out; however, the more expensive cost of overestimating demand and having a surplus will tend to be avoided. Finally, whenever $c_u > c_o$, a larger order quantity will be recommended. In this case, the larger order quantity provides a lower probability of a stock-out in an attempt to avoid the more expensive cost of underestimating demand and experiencing a stock-out.

14.6 Order-Quantity, Reorder Point Model with Probabilistic Demand

In the previous section we considered a single-period inventory model with probabilistic demand. In this section we extend our discussion to a multiperiod order-quantity, reorder point inventory model with probabilistic demand. In the multiperiod model, the inventory system operates continuously with many repeating periods or cycles; inventory can be carried from one period to the next. Whenever the inventory position reaches the reorder point, an order for Q units is placed. Because demand is probabilistic, the time the reorder point will be reached, the time between orders, and the time the order of Q units will arrive in inventory cannot be determined in advance.

The inventory model in this section is based on the assumptions of the EOQ model shown in Table 14.3, with the exception that demand is probabilistic rather than deterministic. With probabilistic demand, occasional shortages may occur.

The inventory pattern for the order-quantity, reorder point model with probabilistic demand will have the general appearance shown in Figure 14.10. Note that the increases, or jumps, in the inventory occur whenever an order of Q units arrives. The inventory decreases at a nonconstant rate based on the probabilistic demand. A new order is placed whenever the reorder point is reached. At times, the order quantity of Q units will arrive before inventory reaches zero. However, at other times, higher demand will cause a stock-out before a new order is received. As with other order-quantity, reorder point models, the manager must determine the order quantity Q and the reorder point r for the inventory system.

The exact mathematical formulation of an order-quantity, reorder point inventory model with probabilistic demand is beyond the scope of this text. However, we present a procedure that can be used to obtain good, workable order-quantity and reorder point inventory policies. The solution procedure can be expected to provide only an approximation of the optimal solution, but it can yield good solutions in many practical situations.

Let us consider the inventory problem of Dabco Industrial Lighting Distributors. Dabco purchases a special high-intensity lightbulb for industrial lighting systems from a well-known

FIGURE 14.10 INVENTORY PATTERN FOR AN ORDER-QUANTITY, REORDER POINT MODEL WITH PROBABILISTIC DEMAND

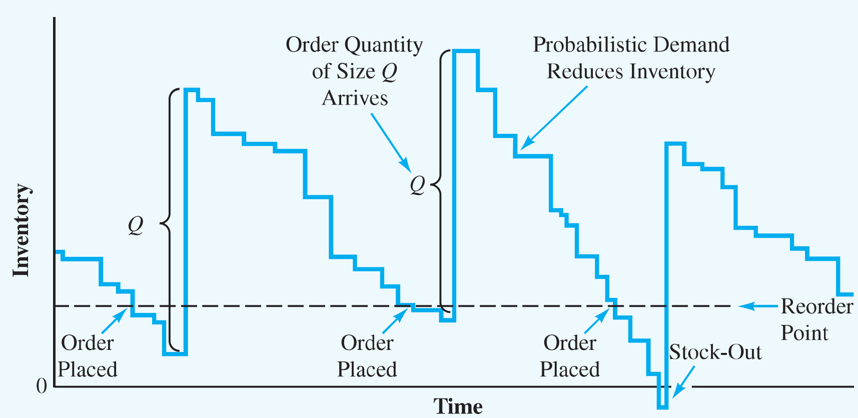
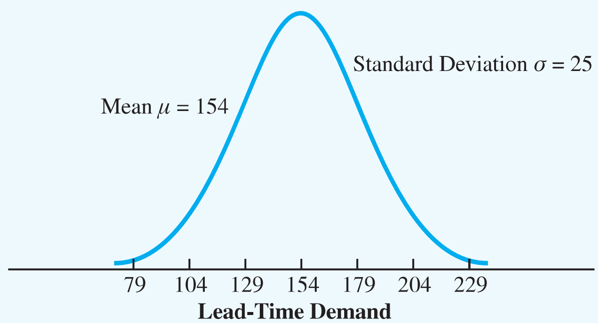


FIGURE 14.11 LEAD-TIME DEMAND PROBABILITY DISTRIBUTION FOR DABCO LIGHTBULBS



lightbulb manufacturer. Dabco would like a recommendation on how much to order and when to order so that a low-cost inventory policy can be maintained. Pertinent facts are that the ordering cost is \$12 per order, one bulb costs \$6, and Dabco uses a 20% annual holding cost rate for its inventory ($C_h = IC = 0.20 \times \$6 = \$1.20$). Dabco, which has more than 1000 customers, experiences a probabilistic demand; in fact, the number of units demanded varies considerably from day to day and from week to week. The lead time for a new order of lightbulbs is one week. Historical sales data indicate that demand during a one-week lead time can be described by a normal probability distribution with a mean of 154 lightbulbs and a standard deviation of 25 lightbulbs. The normal distribution of demand during the lead time is shown in Figure 14.11. Because the mean demand during one week is 154 units, Dabco can anticipate a mean or expected annual demand of 154 units per week × 52 weeks per year = 8008 units per year.

The How-Much-to-Order Decision

Although we are in a probabilistic demand situation, we have an estimate of the expected annual demand of 8008 units. We can apply the EOQ model from Section 14.1 as an approximation of the best order quantity, with the expected annual demand used for *D*. In Dabco's case

WEB file
Q Prob

$$Q^* = \sqrt{\frac{2DC_o}{C_h}} = \sqrt{\frac{2(8008)(12)}{(1.20)}} = 400 \text{ units}$$

When we studied the sensitivity of the EOQ model, we learned that the total cost of operating an inventory system was relatively insensitive to order quantities that were in the neighborhood of Q^*. Using this knowledge, we expect 400 units per order to be a good approximation of the optimal order quantity. Even if annual demand were as low as 7000 units or as high as 9000 units, an order quantity of 400 units should be a relatively good low-cost order size. Thus, given our best estimate of annual demand at 8008 units, we will use $Q^* = 400$.

We have established the 400-unit order quantity by ignoring the fact that demand is probabilistic. Using $Q^* = 400$, Dabco can anticipate placing approximately $D/Q^* = 8008/400 = 20$ orders per year with an average of approximately $250/20 = 12.5$ working days between orders.

The When-to-Order Decision

The probability of a stock-out during any one inventory cycle is easiest to estimate by first determining the number of orders that are expected during the year. The inventory manager can usually state a willingness to allow perhaps one, two, or three stock-outs during the year. The allowable stock-outs per year divided by the number of orders per year will provide the desired probability of a stock-out.

We now want to establish a when-to-order decision rule or reorder point that will trigger the ordering process. With a mean lead-time demand of 154 units, you might first suggest a 154-unit reorder point. However, considering the probability of demand now becomes extremely important. If 154 is the mean lead-time demand, and if demand is symmetrically distributed about 154, then the lead-time demand will be more than 154 units roughly 50% of the time. When the demand during the one-week lead time exceeds 154 units, Dabco will experience a shortage or stock-out. Thus, using a reorder point of 154 units, approximately 50% of the time (10 of the 20 orders a year) Dabco will be short of bulbs before the new supply arrives. This shortage rate would most likely be viewed as unacceptable.

Refer to the **lead-time demand distribution** shown in Figure 14.11. Given this distribution, we can now determine how the reorder point r affects the probability of a stock-out. Because stock-outs occur whenever the demand during the lead time exceeds the reorder point, we can find the probability of a stock-out by using the lead-time demand distribution to compute the probability that demand will exceed r.

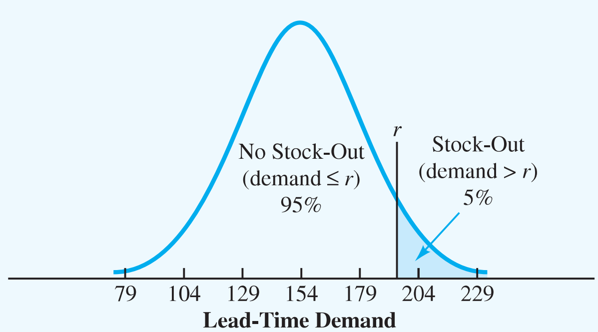
We could now approach the when-to-order problem by defining a cost per stock-out and then attempting to include this cost in a total cost equation. Alternatively, we can ask management to specify the average number of stock-outs that can be tolerated per year. If demand for a product is probabilistic, a manager who will never tolerate a stock-out is being somewhat unrealistic because attempting to avoid stock-outs completely will require high reorder points, high inventory, and an associated high holding cost.

Suppose in this case that Dabco management is willing to tolerate an average of one stock-out per year. Because Dabco places 20 orders per year, this decision implies that management is willing to allow demand during lead time to exceed the reorder point one time in 20, or 5% of the time. The reorder point r can be found by using the lead-time demand distribution to find the value of r with a 5% chance of having a lead-time demand that will exceed it. This situation is shown graphically in Figure 14.12.

From the standard normal probability distribution table in Appendix D, we see that $1 - 0.05 = 0.95$ of the area in the left tail of the normal probability distribution occurs at $z = 1.645$ standard deviations above the mean. Therefore, for the assumed normal distribution for lead-time demand with $\mu = 154$ and $\sigma = 25$, the reorder point r is

$$r = 154 + 1.645(25) = 195$$

FIGURE 14.12 REORDER POINT r THAT ALLOWS A 5% CHANCE OF A STOCK-OUT FOR DABCO LIGHTBULBS



If a normal distribution is used for lead-time demand, the general equation for r is

$$r = \mu + z\sigma \quad (14.37)$$

where z is the number of standard deviations necessary to obtain the acceptable stock-out probability.

Try Problem 29 as an example of an order-quantity, reorder point model with probabilistic demand.

Thus, the recommended inventory decision is to order 400 units whenever the inventory reaches the reorder point of 195. Because the mean or expected demand during the lead time is 154 units, the $195 - 154 = 41$ units serve as a **safety stock**, which absorbs higher-than-usual demand during the lead time. Roughly 95% of the time, the 195 units will be able to satisfy demand during the lead time. The anticipated annual cost for this system is as follows:

Holding cost, normal inventory	$(Q/2)C_h$	$= (400/2)(1.20)$	$= \$240$
Holding cost, safety stock	$(41)C_h$	$= 41(1.20)$	$= \$\ 49$
Ordering cost	$(D/Q)C_o$	$= (8008/400)12$	$= \$240$
		Total	$529

If Dabco could assume that a known, constant demand rate of 8008 units per year existed for the lightbulbs, then $Q^* = 400$, $r = 154$, and a total annual cost of $\$240 + \$240 = \$480$ would be optimal. When demand is uncertain and can only be expressed in probabilistic terms, a larger total cost can be expected. The larger cost occurs in the form of larger holding costs because more inventory must be maintained to limit the number of stock-outs. For Dabco, this additional inventory or safety stock was 41 units, with an additional annual holding cost of $49. The Q.M. in Action, Inventory Models at Microsoft, describes how Microsoft has employed inventory models to increase customer service levels as well as reduce inventory costs.

Q.M. in ACTION

*INVENTORY MODELS AT MICROSOFT**

While known more for its operating system software, Microsoft has steadily increased its presence in consumer electronics. Microsoft produces Xbox video game consoles, Zune music players, and a variety of personal-computer accessories such as mice and keyboards. In 2008 the consumer-electronics division of Microsoft generated over $8 billion in revenue compared to $52 billion in revenue from software. While products such as the Xbox are sold year-round, approximately 40% of annual sales occur in October, November, and December. Therefore, it is critical that Microsoft have sufficient inventory available to meet demand during this holiday season.

In conjunction with the supply-chain-services company Optiant, Microsoft began an ambitious effort in 2005 to improve its inventory management systems. Microsoft developed new forecasting techniques to better estimate future demand for its products. It then set service-level requirements for each product based on profit margins and demand forecasts. These service levels were used in safety-stock model calculations to determine target inventory levels that drove production plans. The new safety-stock models were used for more than 10,000 different consumer-electronics products sold by Microsoft.

Microsoft has experienced substantial inventory level reductions since implementing its new models and policies. Corporate-wide, Microsoft has reduced its inventories by $1.5 billion (60%). The consumer-electronics division of Microsoft posted its first ever profitable year in 2008. Microsoft largely credits these cost savings and profitability to superior forecasting and inventory models.

*Based on J.J. Neale and S.P. Willems, "Managing Inventory in Supply Chains with Nonstationary Demand," Interfaces 39, no. 5 (September 2009): 388–399.

NOTES AND COMMENTS

1. The safety stock required at Microsoft in the Q.M. in Action, Inventory Models at Microsoft, was based on a service level defined by the probability of being able to satisfy all customer demand during an order cycle. If Microsoft wanted to guarantee that it would be able to meet all demand in 95% of all order cycles, then we would say that Microsoft has a 95% service level. This is sometimes referred to as a *Type-I* service level or a *cycle service level*. However, other definitions of *service level* may include the percentage of all customer demand that can be satisfied from inventory. Thus, when an inventory manager expresses a desired service level, it is a good idea to clarify exactly what the manager means by the term *service level*.

14.7 Periodic Review Model with Probabilistic Demand

Up to this point, we have assumed that the inventory position is reviewed continuously so that an order can be placed as soon as the inventory position reaches the reorder point. The inventory model in this section assumes probabilistic demand and a periodic review of the inventory position.

The order-quantity, reorder point inventory models previously discussed require a **continuous review inventory system**. In a continuous review inventory system, the inventory position is monitored continuously so that an order can be placed whenever the reorder point is reached. Computerized inventory systems can easily provide the continuous review required by the order-quantity, reorder point models.

An alternative to the continuous review system is the **periodic review inventory system**. With a periodic review system, the inventory is checked and reordering is done only at specified points in time. For example, inventory may be checked and orders placed on a weekly, biweekly, monthly, or some other periodic basis. When a firm or business handles multiple products, the periodic review system offers the advantage of requiring that orders for several items be placed at the same preset periodic review time. With this type of inventory system, the shipping and receiving of orders for multiple products are easily coordinated. Under the previously discussed order-quantity, reorder point systems, the reorder points for various products can be encountered at substantially different points in time, making the coordination of orders for multiple products more difficult.

To illustrate this system, let us consider Dollar Discounts, a firm with several retail stores that carry a wide variety of products for household use. The company operates its inventory system with a two-week periodic review. Under this system, a retail store manager may order any number of units of any product from the Dollar Discounts central warehouse every two weeks. Orders for all products going to a particular store are combined into one shipment. When making the order quantity decision for each product at a given review period, the store manager knows that a reorder for the product cannot be made until the next review period.

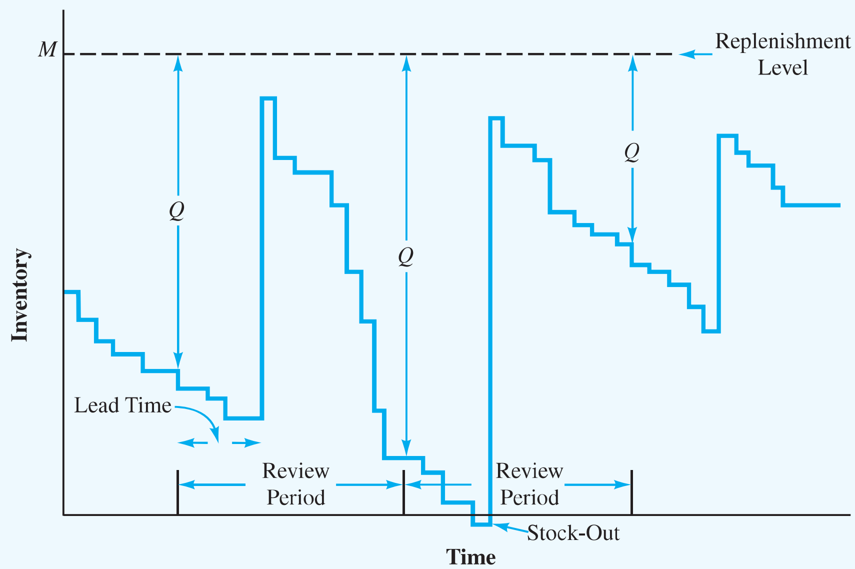
Assuming that the lead time is less than the length of the review period, an order placed at a review period will be received prior to the next review period. In this case, the how-much-to-order decision at any review period is determined using the following:

$$Q = M - H \tag{14.38}$$

where

Q = the order quantity
M = the replenishment level
H = the inventory on hand at the review period

FIGURE 14.13 INVENTORY PATTERN FOR PERIODIC REVIEW MODEL WITH PROBABILISTIC DEMAND



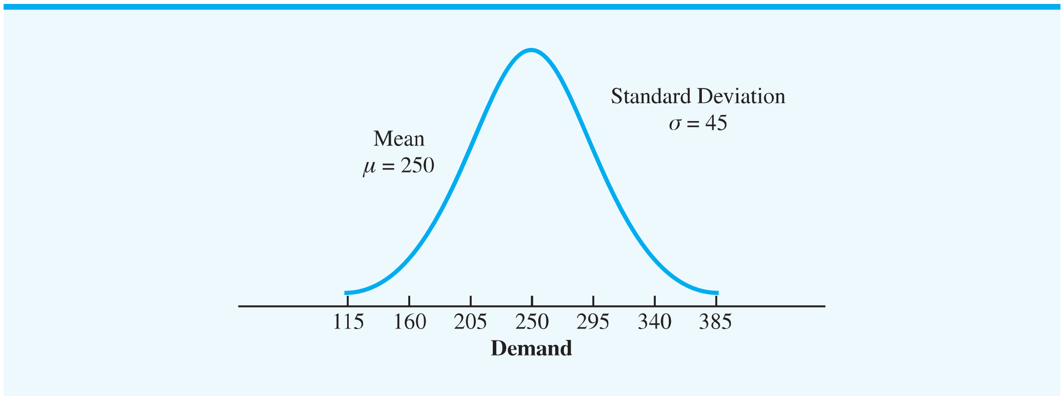
Because the demand is probabilistic, the inventory on hand at the review period, H, will vary. Thus, the order quantity that must be sufficient to bring the inventory position back to its maximum or replenishment level M can be expected to vary each period. For example, if the replenishment level for a particular product is 50 units and the inventory on hand at the review period is $H = 12$ units, an order of $Q = M - H = 50 - 12 = 38$ units should be made. Thus, under the periodic review model, enough units are ordered each review period to bring the inventory position back up to the replenishment level.

A typical inventory pattern for a periodic review system with probabilistic demand is shown in Figure 14.13. Note that the time between periodic reviews is predetermined and fixed. The order quantity Q at each review period can vary and is shown to be the difference between the replenishment level and the inventory on hand. Finally, as with other probabilistic models, an unusually high demand can result in an occasional stock-out.

The decision variable in the periodic review model is the replenishment level M. To determine M, we could begin by developing a total cost model, including holding, ordering, and stock-out costs. Instead, we describe an approach that is often used in practice. In this approach, the objective is to determine a replenishment level that will meet a desired performance level, such as a reasonably low probability of stock-out or a reasonably low number of stock-outs per year.

In the Dollar Discounts problem, we assume that management's objective is to determine the replenishment level with only a 1% chance of a stock-out. In the periodic review model, the order quantity at each review period must be sufficient to cover *demand for the review period plus the demand for the following lead time.* That is, the order quantity that brings the inventory position up to the replenishment level M must last until the order made at the next review period is received in inventory. The length of this time is equal to the review period plus the lead time. Figure 14.14 shows the normal probability

FIGURE 14.14 PROBABILITY DISTRIBUTION OF DEMAND DURING THE REVIEW PERIOD AND LEAD TIME FOR THE DOLLAR DISCOUNTS PROBLEM



distribution of demand during the review period plus the lead-time period for one of the Dollar Discounts products. The mean demand is 250 units, and the standard deviation of demand is 45 units. Given this situation, the logic used to establish M is similar to the logic used to establish the reorder point in Section 14.6. Figure 14.15 shows the replenishment level M with a 1% chance that demand will exceed that replenishment level. In other words, Figure 14.15 shows the replenishment level that allows a 1% chance of a stock-out associated with the replenishment decision. Using the normal probability distribution table in Appendix D, we see that $1 - 0.01 = 0.99$ of the area in the left tail of the normal probability distribution occurs at $z = 2.33$ standard deviations above the mean. Therefore, for the assumed normal probability distribution with $\mu = 250$ and $\sigma = 45$, the replenishment level is determined by

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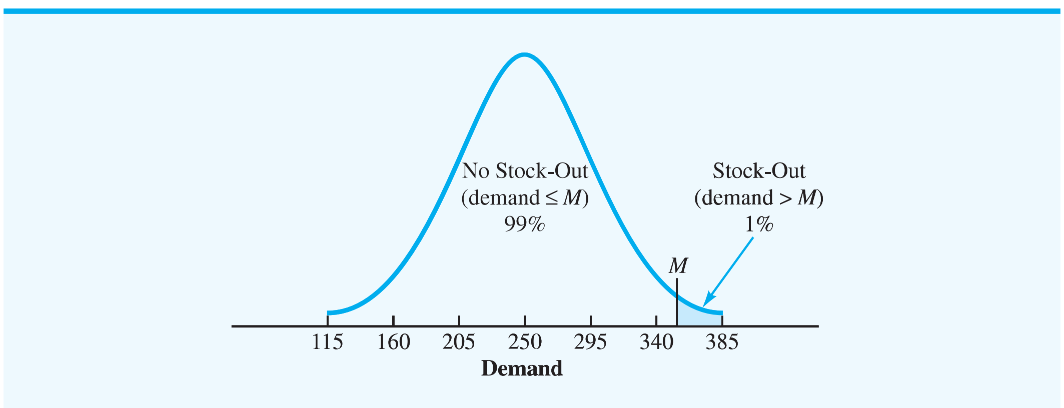
Periodic

$$M = 250 + 2.33(45) = 355$$

FIGURE 14.15 REPLENISHMENT LEVEL M THAT ALLOWS A 1% CHANCE OF A STOCK-OUT FOR THE DOLLAR DISCOUNTS PROBLEM



Problem 33 gives you practice in computing the replenishment level for a periodic review model with probabilistic demand.

Although other probability distributions can be used to express the demand during the review period plus the lead-time period, if the normal probability distribution is used, the general expression for M is

$$M = \mu + z\sigma \tag{14.39}$$

Periodic review systems provide advantages of coordinated orders for multiple items. However, periodic review systems require larger safety-stock levels than corresponding continuous review systems.

where z is the number of standard deviations necessary to obtain the acceptable stock-out probability.

If demand had been deterministic rather than probabilistic, the replenishment level would have been the demand during the review period plus the demand during the lead-time period. In this case, the replenishment level would have been 250 units, and no stock-out would have occurred. However, with the probabilistic demand, we have seen that higher inventory is necessary to allow for uncertain demand and to control the probability of a stock-out. In the Dollar Discounts problem, $355 - 250 = 105$ is the safety stock that is necessary to absorb any higher-than-usual demand during the review period plus the demand during the lead-time period. This safety stock limits the probability of a stock-out to 1%.

More Complex Periodic Review Models

The periodic review model just discussed is one approach to determining a replenishment level for the periodic review inventory system with probabilistic demand. More complex versions of the periodic review model incorporate a reorder point as another decision variable; that is, instead of ordering at every periodic review, a reorder point is established. If the inventory on hand at the periodic review is at or below the reorder point, a decision is made to order up to the replenishment level. However, if the inventory on hand at the periodic review is greater than the reorder level, such an order is not placed, and the system continues until the next periodic review. In this case, the cost of ordering is a relevant cost and can be included in a cost model along with holding and stock-out costs. Optimal policies can be reached based on minimizing the expected total cost. Situations with lead times longer than the review period add to the complexity of the model. The mathematical level required to treat these more extensive periodic review models is beyond the scope of this text.

NOTES AND COMMENTS

1. The periodic review model presented in this section is based on the assumption that the lead time for an order is less than the periodic review period. Most periodic review systems operate under this condition. However, the case in which the lead time is longer than the review period can be handled by defining H in equation (14.38) as the inventory position, where H includes the inventory on hand plus the inventory on order. In this case, the order quantity at any review period is the amount needed for the inventory on hand plus all outstanding orders needed to reach the replenishment level.
2. In the order-quantity, reorder point model discussed in Section 14.6, a continuous review was used to initiate an order whenever the reorder point was reached. The safety stock for this model was based on the probabilistic demand during the lead time. The periodic review model presented in this section also determined a recommended safety stock. However, because the inventory review was only periodic, the safety stock was based on the probabilistic demand during the review period plus the lead-time period. This longer period for the safety stock computation means that periodic review systems tend to require a larger safety stock than do continuous review systems.

Summary

In this chapter we presented some of the approaches used to assist managers in establishing low-cost inventory policies. We first considered cases in which the demand rate for the product is constant. In analyzing these inventory systems, total cost models were developed, which included ordering costs, holding costs, and, in some cases, backorder costs. Then minimum cost formulas for the order quantity Q were presented. A reorder point r can be established by considering the lead-time demand.

In addition, we discussed inventory models in which a deterministic and constant rate could not be assumed, and thus demand was described by a probability distribution. A critical issue with these probabilistic inventory models is obtaining a probability distribution that most realistically approximates the demand distribution. We first described a single-period model where only one order is placed for the product and, at the end of the period, either the product has sold out or a surplus remains of unsold products that will be sold for a salvage value. Solution procedures were then presented for multiperiod models based on either an order-quantity, reorder point, continuous review system or a replenishment-level, periodic review system.

In closing this chapter, we reemphasize that inventory and inventory systems can be an expensive phase of a firm's operation. It is important for managers to be aware of the cost of inventory systems and to make the best possible operating policy decisions for the inventory system. Inventory models, as presented in this chapter, can help managers to develop good inventory policies. The Q.M. in Action, Multistage Inventory Planning at Deere & Company, provides another example of how computer-based inventory models can be used to provide optimal inventory policies and cost reductions.

Q.M. *in* ACTION

*MULTISTAGE INVENTORY PLANNING AT DEERE & COMPANY**

Deere & Company's Commercial & Consumer Equipment (C&CE) Division, located in Raleigh, North Carolina, produces seasonal products such as lawn mowers and snow blowers. The seasonal aspect of demand requires the products to be built in advance. Because many of the products involve impulse purchases, the products must be available at dealerships when a customer walks in. Historically, high inventory levels have resulted in high inventory costs and an unacceptable return on assets. Management has therefore concluded that C&CE needed an inventory planning system that would reduce the average finished goods inventory levels in company warehouses and dealer locations while simultaneously ensuring that stock-outs would not cause a negative impact on sales.

In order to optimize inventory levels, Deere moved from an aggregate inventory planning model to a series of individual product inventory models. This approach enabled Deere to determine optimal inventory levels for each product at each dealer, as well as optimal levels for each product at each plant and warehouse. The computerized system Deere developed, known as SmartOps Multistage Inventory Planning and Optimization (MIPO), manages inventory for four C&CE Division plants, 21 dealers, and 150 products. Easily updated, MIPO provides target inventory levels for each product on a weekly basis. In addition, the system provides information about how optimal inventory levels are affected by lead times, forecast errors, and target service levels.

The inventory optimization system enabled the C&CE Division to meet its inventory reduction goals. C&CE management estimates that the company will continue to achieve annual cost savings from lower inventory carrying costs. Meanwhile, the dealers also benefit from lower warehouse expenses, as well as lower interest and insurance costs.

*Based on "Deere's New Software Achieves Inventory Reduction Goals," *Inventory Management Report* (March 2003): 2.

Glossary

Economic order quantity (EOQ) The order quantity that minimizes the annual holding cost plus the annual ordering cost.

Constant demand rate An assumption of many inventory models that states that the same number of units are taken from inventory each period of time.

Holding cost The cost associated with maintaining an inventory investment, including the cost of the capital investment in the inventory, insurance, taxes, warehouse overhead, and so on. This cost may be stated as a percentage of the inventory investment or as a cost per unit.

Cost of capital The cost a firm incurs to obtain capital for investment. It may be stated as an annual percentage rate, and it is part of the holding cost associated with maintaining inventory.

Ordering cost The fixed cost (salaries, paper, transportation, etc.) associated with placing an order for an item.

Inventory position The inventory on hand plus the inventory on order.

Reorder point The inventory position at which a new order should be placed.

Lead time The time between the placing of an order and its receipt in the inventory system.

Lead-time demand The number of units demanded during the lead-time period.

Cycle time The length of time between the placing of two consecutive orders.

Constant supply rate A situation in which the inventory is built up at a constant rate over a period of time.

Lot size The order quantity in the production inventory model.

Setup cost The fixed cost (labor, materials, lost production) associated with preparing for a new production run.

Shortage or stock-out Occurrence when demand cannot be supplied from inventory.

Backorder The receipt of an order for a product when no units are in inventory. These backorders are eventually satisfied when a new supply of the product becomes available.

Goodwill cost A cost associated with a backorder, a lost sale, or any form of stock-out or unsatisfied demand. This cost may be used to reflect the loss of future profits because a customer experienced an unsatisfied demand.

Quantity discounts Discounts or lower unit costs offered by the manufacturer when a customer purchases larger quantities of the product.

Deterministic inventory model A model where demand is considered known and not subject to uncertainty.

Probabilistic inventory model A model where demand is not known exactly; probabilities must be associated with the possible values for demand.

Single-period inventory model An inventory model in which only one order is placed for the product, and at the end of the period either the item has sold out, or a surplus of unsold items will be sold for a salvage value.

Incremental analysis A method used to determine an optimal order quantity by comparing the cost of ordering an additional unit with the cost of not ordering an additional unit.

Lead-time demand distribution The distribution of demand that occurs during the lead-time period.

Safety stock Inventory maintained in order to reduce the number of stock-outs resulting from higher-than-expected demand.

Continuous review inventory system A system in which the inventory position is monitored or reviewed on a continuous basis so that a new order can be placed as soon as the reorder point is reached.

Periodic review inventory system A system in which the inventory position is checked or reviewed at predetermined periodic points in time. Reorders are placed only at periodic review points.

Problems

SELF test

1. Suppose that the R&B Beverage Company has a soft drink product that shows a constant annual demand rate of 3600 cases. A case of the soft drink costs R&B $3. Ordering costs are $20 per order and holding costs are 25% of the value of the inventory. R&B has 250 working days per year, and the lead time is 5 days. Identify the following aspects of the inventory policy:
   a. Economic order quantity
   b. Reorder point
   c. Cycle time
   d. Total annual cost
2. A general property of the EOQ inventory model is that total inventory holding and total ordering costs are equal at the optimal solution. Use the data in Problem 1 to show that this result is true. Use equations (14.1), (14.2), and (14.3) to show that, in general, total holding costs and total ordering costs are equal whenever Q^* is used.
3. The reorder point [see equation (14.6)] is defined as the lead-time demand for an item. In cases of long lead times, the lead-time demand and thus the reorder point may exceed the economic order quantity Q^*. In such cases, the inventory position will not equal the inventory on hand when an order is placed, and the reorder point may be expressed in terms of either the inventory position or the inventory on hand. Consider the economic order quantity model with $D = 5000$, $C_o = \$32$, $C_h = \$2$, and 250 working days per year. Identify the reorder point in terms of the inventory position and in terms of the inventory on hand for each of the following lead times:
   a. 5 days
   b. 15 days
   c. 25 days
   d. 45 days
4. Westside Auto purchases a component used in the manufacture of automobile generators directly from the supplier. Westside's generator production operation, which is operated at a constant rate, will require 1000 components per month throughout the year (12,000 units annually). Assume that the ordering costs are $25 per order, the unit cost is $2.50 per component, and annual holding costs are 20% of the value of the inventory. Westside has 250 working days per year and a lead time of 5 days. Answer the following inventory policy questions:
   a. What is the EOQ for this component?
   b. What is the reorder point?
   c. What is the cycle time?
   d. What are the total annual holding and ordering costs associated with your recommended EOQ?
5. The Metropolitan Bus Company (MBC) purchases diesel fuel from American Petroleum Supply. In addition to the fuel cost, American Petroleum Supply charges MBC $250 per order to cover the expenses of delivering and transferring the fuel to MBC's storage tanks.

The lead time for a new shipment from American Petroleum is 10 days; the cost of holding a gallon of fuel in the storage tanks is \$0.04 per month, or \$0.48 per year; and annual fuel usage is 150,000 gallons. MBC buses operate 300 days a year.

a. What is the optimal order quantity for MBC?
b. How frequently should MBC order to replenish the gasoline supply?
c. The MBC storage tanks have a capacity of 15,000 gallons. Should MBC consider expanding the capacity of its storage tanks?
d. What is the reorder point?

6. The manager at a local university bookstore wishes to apply the EOQ model to determine the respective order quantities for two products, ballpoint pens and mechanical pencils. The annual demand for pens and pencils is 1500 and 400, respectively. The ordering cost for each product is \$20 per order and the wholesale price of a pen and pencil is \$1.50 and \$4, respectively. Assume the bookstore's annual holding rate is 10% and that the bookstore operates 240 days per year.
   a. Determine the optimal order quantity and the order cycle time for each product. What is the total cost (summed over both products)?
   b. The bookstore orders the pens and pencils from the same supplier. If these two products had the same cycle time, the corresponding shipment consolidation would reduce the ordering cost to \$15. How much money does the bookstore save by consolidating the orders for these two products? (*Hint:* By setting the cycle times equal, we have $Q_{\text{pens}}/(1500/240) = Q_{\text{pencils}}/(400/240)$ or $Q_{\text{pens}} = 3.75Q_{\text{pencils}}$). Make this substitution into the combined cost equation so that it is a function only of Q_{pencils} and apply equation (14.5) with the appropriate values to determine Q_{pencils} (and subsequently Q_{pens}).

7. A large distributor of oil-well drilling equipment operated over the past two years with EOQ policies based on an annual holding cost rate of 22%. Under the EOQ policy, a particular product has been ordered with a $Q^* = 80$. A recent evaluation of holding costs shows that because of an increase in the interest rate associated with bank loans, the annual holding cost rate should be 27%.
   a. What is the new economic order quantity for the product?
   b. Develop a general expression showing how the economic order quantity changes when the annual holding cost rate is changed from I to I'.

8. Nation-Wide Bus Lines is proud of its six-week bus driver–training program that it conducts for all new Nation-Wide drivers. As long as the class size remains less than or equal to 35, a six-week training program costs Nation-Wide \$22,000 for instructors, equipment, and so on. The Nation-Wide training program must provide the company with approximately five new drivers per month. After completing the training program, new drivers are paid \$1600 per month but do not work until a full-time driver position is open. Nation-Wide views the \$1600 per month paid to each idle new driver as a holding cost necessary to maintain a supply of newly trained drivers available for immediate service. Viewing new drivers as inventory-type units, how large should the training classes be to minimize Nation-Wide's total annual training and new driver idle-time costs? How many training classes should the company hold each year? What is the total annual cost associated with your recommendation?

9. Cress Electronic Products manufactures components used in the automotive industry. Cress purchases parts for use in its manufacturing operation from a variety of different suppliers. One particular supplier provides a part where the assumptions of the EOQ model are realistic. The annual demand is 5000 units, the ordering cost is \$80 per order, and the annual holding cost rate is 25%.
   a. If the cost of the part is \$20 per unit, what is the economic order quantity?
   b. Assume 250 days of operation per year. If the lead time for an order is 12 days, what is the reorder point?

c. If the lead time for the part is seven weeks (35 days), what is the reorder point?
d. What is the reorder point for part (c) if the reorder point is expressed in terms of the inventory on hand rather than the inventory position?

10. All-Star Bat Manufacturing, Inc., supplies baseball bats to major and minor league baseball teams. After an initial order in January, demand over the six-month baseball season is approximately constant at 1000 bats per month. Assuming that the bat production process can handle up to 4000 bats per month, the bat production setup costs are $150 per setup, the production cost is $10 per bat, and the holding costs have a monthly rate of 2%, what production lot size would you recommend to meet the demand during the baseball season? If All-Star operates 20 days per month, how often will the production process operate, and what is the length of a production run?

11. Assume that a production line operates such that the production lot size model of Section 14.2 is applicable. Given $D = 6400$ units per year, $C_o = \$100$, and $C_h = \$2$ per unit per year, compute the minimum cost production lot size for each of the following production rates:
a. 8000 units per year
b. 10,000 units per year
c. 32,000 units per year
d. 100,000 units per year

Compute the EOQ recommended lot size using equation (14.5). What two observations can you make about the relationship between the EOQ model and the production lot size model?

12. EL Computer produces its multimedia notebook computer on a production line that has an annual capacity of 16,000 units. EL Computer estimates the annual demand for this model at 6000 units. The cost to set up the production line is $2345, and the annual holding cost is $20 per unit. Current practice calls for production runs of 500 notebook computers each month.
a. What is the optimal production lot size?
b. How many production runs should be made each year? What is the recommended cycle time?
c. Would you recommend changing the current production lot size policy from the monthly 500-unit production runs? Why or why not? What is the projected savings of your recommendation?

SELF test

13. Wilson Publishing Company produces books for the retail market. Demand for a current book is expected to occur at a constant annual rate of 7200 copies. The cost of one copy of the book is $14.50. The holding cost is based on an 18% annual rate, and production setup costs are $150 per setup. The equipment on which the book is produced has an annual production volume of 25,000 copies. Wilson has 250 working days per year, and the lead time for a production run is 15 days. Use the production lot size model to compute the following values:
a. Minimum cost production lot size
b. Number of production runs per year
c. Cycle time
d. Length of a production run
e. Maximum inventory
f. Total annual cost
g. Reorder point

14. A well-known manufacturer of several brands of toothpaste uses the production lot size model to determine production quantities for its various products. The product known as Extra White is currently being produced in production lot sizes of 5000 units. The

length of the production run for this quantity is 10 days. Because of a recent shortage of a particular raw material, the supplier of the material announced that a cost increase will be passed along to the manufacturer of Extra White. Current estimates are that the new raw material cost will increase the manufacturing cost of the toothpaste products by 23% per unit. What will be the effect of this price increase on the production lot sizes for Extra White?

SELF test

15. Suppose that Westside Auto of Problem 4, with $D = 12{,}000$ units per year, $C_h = (2.50)(0.20) = \$0.50$, and $C_o = \$25$, decided to operate with a backorder inventory policy. Backorder costs are estimated to be \$5 per unit per year. Identify the following:
    a. Minimum cost order quantity
    b. Maximum number of backorders
    c. Maximum inventory
    d. Cycle time
    e. Total annual cost
16. Assuming 250 days of operation per year and a lead time of five days, what is the reorder point for Westside Auto in Problem 15? Show the general formula for the reorder point for the EOQ model with backorders. In general, is the reorder point when backorders are allowed greater than or less than the reorder point when backorders are not allowed? Explain.
17. A manager of an inventory system believes that inventory models are important decision-making aids. The manager has experience with the EOQ policy, but has never considered a backorder model because of the assumption that backorders were "bad" and should be avoided. However, with upper management's continued pressure for cost reduction, you have been asked to analyze the economics of a backorder policy for some products that can possibly be backordered. For a specific product with $D = 800$ units per year, $C_o = \$150$, $C_h = \$3$, and $C_b = \$20$, what is the difference in total annual cost between the EOQ model and the planned shortage or backorder model? If the manager adds constraints that no more than 25% of the units can be backordered and that no customer will have to wait more than 15 days for an order, should the backorder inventory policy be adopted? Assume 250 working days per year.
18. If the lead time for new orders is 20 days for the inventory system discussed in Problem 17, find the reorder point for both the EOQ and the backorder models.
19. The A&M Hobby Shop carries a line of radio-controlled model racing cars. Demand for the cars is assumed to be constant at a rate of 40 cars per month. The cars cost \$60 each, and ordering costs are approximately \$15 per order, regardless of the order size. The annual holding cost rate is 20%.
    a. Determine the economic order quantity and total annual cost under the assumption that no backorders are permitted.
    b. Using a \$45 per-unit per-year backorder cost, determine the minimum cost inventory policy and total annual cost for the model racing cars.
    c. What is the maximum number of days a customer would have to wait for a backorder under the policy in part (b)? Assume that the Hobby Shop is open for business 300 days per year.
    d. Would you recommend a no-backorder or a backorder inventory policy for this product? Explain.
    e. If the lead time is six days, what is the reorder point for both the no-backorder and backorder inventory policies?
20. Assume that the following quantity discount schedule is appropriate. If annual demand is 120 units, ordering costs are \$20 per order, and the annual holding cost rate is 25%, what order quantity would you recommend?

Order Size	Discount (%)	Unit Cost
0 to 49	0	\$30.00
50 to 99	5	\$28.50
100 or more	10	\$27.00

SELF test

21. Apply the EOQ model to the following quantity discount situation in which $D = 500$ units per year, $C_o = \$40$, and the annual holding cost rate is 20%. What order quantity do you recommend?

Discount Category	Order Size	Discount (%)	Unit Cost
1	0 to 99	0	\$10.00
2	100 or more	3	\$ 9.70

22. Keith Shoe Stores carries a basic black dress shoe for men that sells at an approximately constant rate of 500 pairs of shoes every three months. Keith's current buying policy is to order 500 pairs each time an order is placed. It costs Keith \$30 to place an order. The annual holding cost rate is 20%. With the order quantity of 500, Keith obtains the shoes at the lowest possible unit cost of \$28 per pair. Other quantity discounts offered by the manufacturer are as follows. What is the minimum cost order quantity for the shoes? What are the annual savings of your inventory policy over the policy currently being used by Keith?

Order Quantity	Price per Pair
0–99	\$36
100–199	\$32
200–299	\$30
300 or more	\$28

23. In the EOQ model with quantity discounts, we stated that if the Q^* for a price category is larger than necessary to qualify for the category price, the category cannot be optimal. Use the two discount categories in Problem 21 to show that this statement is true. That is, plot total cost curves for the two categories and show that if the category 2 minimum cost Q is an acceptable solution, we do not have to consider category 1.

24. University of Iowa Sports Information (UISI) procures its game-day football magazines from a publishing company at a price of \$9.00 per magazine. UISI sells the magazines on the day of the corresponding football game at a retail price of \$10.00. To sell these magazines, UISI hires vendors and pays them \$0.50 for each program that they sell. For the first game of the season, UISI has determined that demand for the game-day football magazines is normally distributed with a mean of 9000 magazines and a standard deviation of 400 magazines. Any magazines that are not sold on the day of the game are worthless and UISI recycles them.
    a. What is UISI's optimal order quantity of game-day football magazines for the first game of the season?

b. Instead of recycling the unsold programs, suppose the publisher offers to buy back any unsold programs for \$8.00. Under this scenario, what is UISI's optimal order quantity?

SELF test

25. The Gilbert Air-Conditioning Company is considering the purchase of a special shipment of portable air conditioners manufactured in Japan. Each unit will cost Gilbert \$80, and it will be sold for \$125. Gilbert does not want to carry surplus air conditioners over until the following year. Thus, all surplus air conditioners will be sold to a wholesaler for \$50 per unit. Assume that the air conditioner demand follows a normal probability distribution with $\mu = 20$ and $\sigma = 8$.
   a. What is the recommended order quantity?
   b. What is the probability that Gilbert will sell all units it orders?

26. The Bridgeport city manager and the chief of police agreed on the size of the police force necessary for normal daily operations. However, they need assistance in determining the number of additional police officers needed to cover daily absences due to injuries, sickness, vacations, and personal leave. Records over the past three years show that the daily demand for additional police officers is normally distributed with a mean of 50 officers and a standard deviation of 10 officers. The cost of an additional police officer is based on the average pay rate of \$150 per day. If the daily demand for additional police officers exceeds the number of additional officers available, the excess demand will be covered by overtime at the pay rate of \$240 per day for each overtime officer.
   a. If the number of additional police officers available is greater than demand, the city will have to pay for more additional police officers than needed. What is the cost of overestimating demand?
   b. If the number of additional police officers available is less than demand, the city will have to use overtime to meet the demand. What is the cost of underestimating demand?
   c. What is the optimal number of additional police officers that should be included in the police force?
   d. On a typical day, what is the probability that overtime will be necessary?

27. A perishable dairy product is ordered daily at a particular supermarket. The product, which costs \$1.19 per unit, sells for \$1.65 per unit. If units are unsold at the end of the day, the supplier takes them back at a rebate of \$1 per unit. Assume that daily demand is approximately normally distributed with $\mu = 150$ and $\sigma = 30$.
   a. What is your recommended daily order quantity for the supermarket?
   b. What is the probability that the supermarket will sell all the units it orders?
   c. In problems such as these, why would the supplier offer a rebate as high as \$1? For example, why not offer a nominal rebate of, say, 25¢ per unit? What happens to the supermarket order quantity as the rebate is reduced?

28. A retail outlet sells holiday candy for \$10 per bag. The cost of the product is \$8 per bag. All units not sold during the selling season prior to the holiday are sold for half the retail price in a postholiday clearance sale. Assume that demand for bags of holiday candy during the selling season is uniformly distributed between 200 and 800.
   a. What is the recommended order quantity?
   b. What is the probability that at least some customers will ask to purchase the product after the outlet is sold out? That is, what is the probability of a stock-out using your order quantity in part (a)?
   c. To keep customers happy and returning to the store later, the owner feels that stock-outs should be avoided if at all possible. What is your recommended order quantity if the owner is willing to tolerate a 0.15 probability of a stock-out?
   d. Using your answer to part (c), what is the goodwill cost you are assigning to a stock-out?

SELF test

29. Floyd Distributors, Inc., provides a variety of auto parts to small local garages. Floyd purchases parts from manufacturers according to the EOQ model and then ships the parts from a regional warehouse direct to its customers. For a particular type of muffler, Floyd's EOQ analysis recommends orders with $Q^* = 25$ to satisfy an annual demand of 200 mufflers. Floyd's has 250 working days per year, and the lead time averages 15 days.
    a. What is the reorder point if Floyd assumes a constant demand rate?
    b. Suppose that an analysis of Floyd's muffler demand shows that the lead-time demand follows a normal probability distribution with $\mu = 12$ and $\sigma = 2.5$. If Floyd's management can tolerate one stock-out per year, what is the revised reorder point?
    c. What is the safety stock for part (b)? If $C_h = \$5$/unit/year, what is the extra cost due to the uncertainty of demand?
30. To serve "to-go" orders, Terrapin Coffeehouse faces normally distributed weekly demand with an average of 300 paper cups and a standard deviation of 75 cups per week. Terrapin orders cups by the box. Each box costs $10 and contains 100 cups. For each order placed, Terrapin pays a fixed $15 shipping fee (regardless of the number of boxes ordered) and the order arrives one week after Terrapin places it with the cup supplier. Terrapin estimates that holding costs are 15% per dollar per year. Due to the importance of cups to business, Terrapin wants no more than a 1% chance of a stock-out during the one-week lead time for cup replenishment. Assume that there are 52 weeks in a year.
    a. What is the optimal order quantity (in terms of number of boxes)?
    b. What is the optimal reorder point (in terms of number of cups)?
31. A product with an annual demand of 1000 units has $C_o = \$25.50$ and $C_h = \$8$. The demand exhibits some variability such that the lead-time demand follows a normal probability distribution with $\mu = 25$ and $\sigma = 5$.
    a. What is the recommended order quantity?
    b. What are the reorder point and safety stock if the firm desires at most a 2% probability of stock-out on any given order cycle?
    c. If a manager sets the reorder point at 30, what is the probability of a stock-out on any given order cycle? How many times would you expect a stock-out during the year if this reorder point were used?
32. The B&S Novelty and Craft Shop in Bennington, Vermont, sells a variety of quality handmade items to tourists. B&S will sell 300 hand-carved miniature replicas of a Colonial soldier each year, but the demand pattern during the year is uncertain. The replicas sell for $20 each, and B&S uses a 15% annual inventory holding cost rate. Ordering costs are $5 per order, and demand during the lead time follows a normal probability distribution with $\mu = 15$ and $\sigma = 6$.
    a. What is the recommended order quantity?
    b. If B&S is willing to accept a stock-out roughly twice a year, what reorder point would you recommend? What is the probability that B&S will have a stock-out in any one order cycle?
    c. What are the safety stock and annual safety stock costs for this product?

SELF test

33. A firm uses a one-week periodic review inventory system. A two-day lead time is needed for any order, and the firm is willing to tolerate an average of one stock-out per year.
    a. Using the firm's service guideline, what is the probability of a stock-out associated with each replenishment decision?
    b. What is the replenishment level if demand during the review period plus lead-time period is normally distributed with a mean of 60 units and a standard deviation of 12 units?
    c. What is the replenishment level if demand during the review period plus lead-time period is uniformly distributed between 35 and 85 units?
34. Foster Drugs, Inc., handles a variety of health and beauty aid products. A particular hair conditioner product costs Foster Drugs $2.95 per unit. The annual holding cost rate is 20%.

An order-quantity, reorder point inventory model recommends an order quantity of 300 units per order.

a. Lead time is one week, and the lead-time demand is normally distributed with a mean of 150 units and a standard deviation of 40 units. What is the reorder point if the firm is willing to tolerate a 1% chance of stock-out on any one cycle?
b. What safety stock and annual safety stock costs are associated with your recommendation in part (a)?
c. The order-quantity, reorder point model requires a continuous review system. Management is considering making a transition to a periodic review system in an attempt to coordinate ordering for many of its products. The demand during the proposed two-week review period and the one-week lead-time period is normally distributed with a mean of 450 units and a standard deviation of 70 units. What is the recommended replenishment level for this periodic review system if the firm is willing to tolerate the same 1% chance of stock-out associated with any replenishment decision?
d. What safety stock and annual safety stock costs are associated with your recommendation in part (c)?
e. Compare your answers to parts (b) and (d). The company is seriously considering the periodic review system. Would you support this decision? Explain.
f. Would you tend to favor the continuous review system for more expensive items? For example, assume that the product in the preceding example sold for $295 per unit. Explain.

35. Statewide Auto Parts uses a four-week periodic review system to reorder parts for its inventory stock. A one-week lead time is required to fill the order. Demand for one particular part during the five-week replenishment period is normally distributed with a mean of 18 units and a standard deviation of 6 units.
    a. At a particular periodic review, 8 units are in inventory. The parts manager places an order for 16 units. What is the probability that this part will have a stock-out before an order that is placed at the next four-week review period arrives?
    b. Assume that the company is willing to tolerate a 2.5% chance of a stock-out associated with a replenishment decision. How many parts should the manager have ordered in part (a)? What is the replenishment level for the four-week periodic review system?

36. Rose Office Supplies, Inc., which is open six days a week, uses a two-week periodic review for its store inventory. On alternating Monday mornings, the store manager fills out an order sheet requiring a shipment of various items from the company's warehouse. A particular three-ring notebook sells at an average rate of 16 notebooks per week. The standard deviation in sales is 5 notebooks per week. The lead time for a new shipment is three days. The mean lead-time demand is 8 notebooks with a standard deviation of 3.5.
    a. What is the mean or expected demand during the review period plus the lead-time period?
    b. Under the assumption of independent demand from week to week, the variances in demands are additive. Thus, the variance of the demand during the review period plus the lead-time period is equal to the variance of demand during the first week plus the variance of demand during the second week plus the variance of demand during the lead-time period. What is the variance of demand during the review period plus the lead-time period? What is the standard deviation of demand during the review period plus the lead-time period?
    c. Assuming that demand has a normal probability distribution, what is the replenishment level that will provide an expected stock-out rate of one per year?
    d. On Monday, March 22, 18 notebooks remain in inventory at the store. How many notebooks should the store manager order?

Case Problem 1 Wagner Fabricating Company

Managers at Wagner Fabricating Company are reviewing the economic feasibility of manufacturing a part that the company currently purchases from a supplier. Forecasted annual demand for the part is 3200 units. Wagner operates 250 days per year.

Wagner's financial analysts established a cost of capital of 14% for the use of funds for investments within the company. In addition, over the past year $600,000 was the average investment in the company's inventory. Accounting information shows that a total of $24,000 was spent on taxes and insurance related to the company's inventory. In addition, an estimated $9000 was lost due to inventory shrinkage, which included damaged goods as well as pilferage. A remaining $15,000 was spent on warehouse overhead, including utility expenses for heating and lighting.

An analysis of the purchasing operation shows that approximately two hours are required to process and coordinate an order for the part regardless of the quantity ordered. Purchasing salaries average $28 per hour, including employee benefits. In addition, a detailed analysis of 125 orders showed that $2375 was spent on telephone, paper, and postage directly related to the ordering process.

A one-week lead time is required to obtain the part from the supplier. An analysis of demand during the lead time shows it is approximately normally distributed with a mean of 64 units and a standard deviation of 10 units. Service level guidelines indicate that one stock-out per year is acceptable.

Currently, the company has a contract to purchase the part from a supplier at a cost of $18 per unit. However, over the past few months, the company's production capacity has been expanded. As a result, excess capacity is now available in certain production departments, and the company is considering the alternative of producing the parts itself.

Forecasted utilization of equipment shows that production capacity will be available for the part being considered. The production capacity is available at the rate of 1000 units per month, with up to five months of production time available. Management believes that with a two-week lead time, schedules can be arranged so that the part can be produced whenever needed. The demand during the two-week lead time is approximately normally distributed, with a mean of 128 units and a standard deviation of 20 units. Production costs are expected to be $17 per part.

A concern of management is that setup costs will be significant. The total cost of labor and lost production time is estimated to be $50 per hour, and a full eight-hour shift will be needed to set up the equipment for producing the part.

Managerial Report

Develop a report for management of Wagner Fabricating that will address the question of whether the company should continue to purchase the part from the supplier or begin to produce the part itself. Include the following factors in your report:

1. An analysis of the holding costs, including the appropriate annual holding cost rate
2. An analysis of ordering costs, including the appropriate cost per order from the supplier
3. An analysis of setup costs for the production operation
4. A development of the inventory policy for the following two alternatives:
   a. Ordering a fixed quantity Q from the supplier
   b. Ordering a fixed quantity Q from in-plant production

5. Include the following in the policies of parts 4(a) and 4(b):
   a. Optimal quantity Q^*
   b. Number of order or production runs per year
   c. Cycle time
   d. Reorder point
   e. Amount of safety stock
   f. Expected maximum inventory
   g. Average inventory
   h. Annual holding cost
   i. Annual ordering cost
   j. Annual cost of the units purchased or manufactured
   k. Total annual cost of the purchase policy and the total annual cost of the production policy
6. Make a recommendation as to whether the company should purchase or manufacture the part. What savings are associated with your recommendation as compared with the other alternative?

Case Problem 2 River City Fire Department

The River City Fire Department (RCFD) fights fires and provides a variety of rescue operations in the River City metropolitan area. The RCFD staffs 13 ladder companies, 26 pumper companies, and several rescue units and ambulances. Normal staffing requires 186 firefighters to be on duty every day.

RCFD is organized with three firefighting units. Each unit works a full 24-hour day and then has two days (48 hours) off. For example, Unit 1 covers Monday, Unit 2 covers Tuesday, and Unit 3 covers Wednesday. Then Unit 1 returns on Thursday, and so on. Over a three-week (21-day) scheduling period, each unit will be scheduled for seven days. On a rotational basis, firefighters within each unit are given one of the seven regularly scheduled days off. This day off is referred to as a Kelley day. Thus, over a three-week scheduling period, each firefighter in a unit works six of the seven scheduled unit days and gets one Kelley day off.

Determining the number of firefighters to be assigned to each unit includes the 186 firefighters who must be on duty plus the number of firefighters in the unit who are off for a Kelley day. Furthermore, each unit needs additional staffing to cover firefighter absences due to injury, sick leave, vacations, or personal time. This additional staffing involves finding the best mix of adding full-time firefighters to each unit and the selective use of overtime. If the number of absences on a particular day brings the number of available firefighters below the required 186, firefighters who are currently off (e.g., on a Kelley day) must be scheduled to work overtime. Overtime is compensated at 1.55 times the regular pay rate.

Analysis of the records maintained over the last several years concerning the number of daily absences shows a normal probability distribution. A mean of 20 and a standard deviation of 5 provides a good approximation of the probability distribution for the number of daily absences.

Managerial Report

Develop a report that will enable Fire Chief O. E. Smith to determine the necessary numbers for the Fire Department. Include, at a minimum, the following items in your report:

1. Assuming no daily absences and taking into account the need to staff Kelley days, determine the base number of firefighters needed by each unit.

2. Using a minimum cost criterion, how many additional firefighters should be added to each unit in order to cover the daily absences? These extra daily needs will be filled by the additional firefighters and, when necessary, the more expensive use of overtime by off-duty firefighters.
3. On a given day, what is the probability that Kelley-day firefighters will be called in to work overtime?
4. Based on the three-unit organization, how many firefighters should be assigned to each unit? What is the total number of full-time firefighters required for the River City Fire Department?

Appendix 14.1 Development of the Optimal Order Quantity (Q) Formula for the EOQ Model

Given equation (14.4) as the total annual cost for the EOQ model,

$$TC = \frac{1}{2}QC_h + \frac{D}{Q}C_o \quad \textbf{(14.4)}$$

we can find the order quantity Q that minimizes the total cost by setting the derivative, dTC/dQ, equal to zero and solving for Q^*.

$$\frac{dTC}{dQ} = \frac{1}{2}C_h - \frac{D}{Q^2}C_o = 0$$

$$\frac{1}{2}C_h = \frac{D}{Q^2}C_o$$

$$C_hQ^2 = 2DC_o$$

$$Q^2 = \frac{2DC_o}{C_h}$$

Hence,

$$Q^* = \sqrt{\frac{2DC_o}{C_h}} \quad \textbf{(14.5)}$$

The second derivative is

$$\frac{d^2TC}{dQ^2} = \frac{2D}{Q^3}C_o$$

Because the value of the second derivative is greater than zero, Q^* from equation (14.5) is the minimum-cost solution.

Appendix 14.2 Development of the Optimal Lot Size (Q^*) Formula for the Production Lot Size Model

Given equation (14.15) as the total annual cost for the production lot size model,

$$TC = \frac{1}{2}\left(1 - \frac{D}{P}\right)QC_h + \frac{D}{Q}C_o \quad \textbf{(14.15)}$$

we can find the order quantity Q that minimizes the total cost by setting the derivative, dTC/dQ, equal to zero and solving for Q^*.

$$\frac{dTC}{dQ} = \frac{1}{2}\left(1 - \frac{D}{P}\right)C_h - \frac{D}{Q^2}C_o = 0$$

Solving for Q^*, we have

$$\frac{1}{2}\left(1 - \frac{D}{P}\right)C_h = \frac{D}{Q^2}C_o$$

$$\left(1 - \frac{D}{P}\right)C_hQ^2 = 2DC_o$$

$$Q^2 = \frac{2DC_o}{(1 - D/P)C_h}$$

Hence,

$$Q^* = \sqrt{\frac{2DC_o}{(1 - D/P)C_h}} \quad \textbf{(14.16)}$$

The second derivative is

$$\frac{d^2TC}{dQ^2} = \frac{2DC_o}{Q^3}$$

Because the value of the second derivative is greater than zero, Q^* from equation (14.16) is a minimum-cost solution.