CHAPTER 2

Introduction to Probability

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Business decisions are often based on an analysis of uncertainties such as the following:

1. What are the "chances" that sales will decrease if we increase prices?
2. What is the "likelihood" that a new assembly method will increase productivity?
3. How "likely" is it that the project will be completed on time?
4. What are the "odds" that a new investment will be profitable?

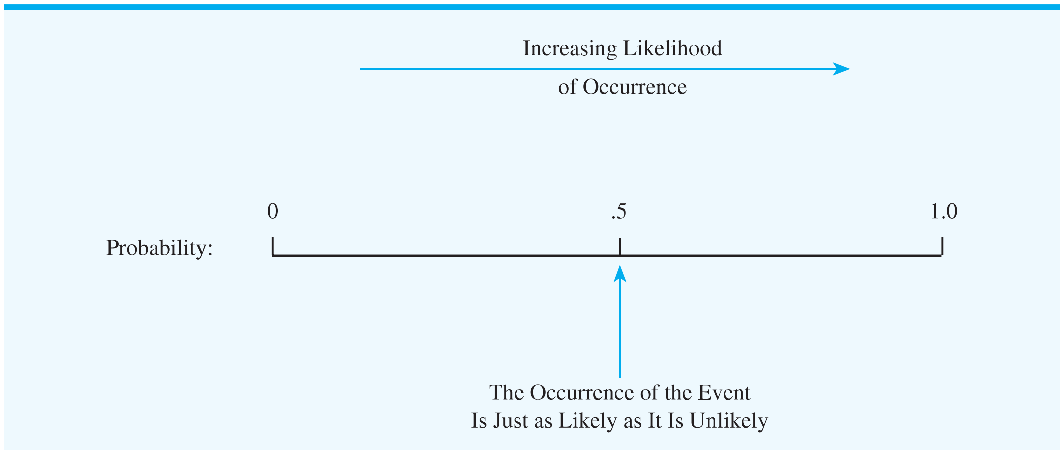
Probability provides a more precise description of uncertainty than expressions such as chances are "pretty good," chances are "fair," and so on.

Probability is a numeric measure of the likelihood that an event will occur. Thus, probabilities could be used as measures of the degree of uncertainty associated with the four events previously listed. If probabilities were available, we could determine the likelihood of each event occurring.

Probability values are always assigned on a scale from 0 to 1. A probability of 0 indicates that an event will not occur (is an **impossible event**); a probability of 1 indicates that an event is certain to occur (is a **certain event**). Other probabilities between 0 and 1 represent varying degrees of likelihood that an event will occur. The closer a probability is to 0, the less likely the associated event is to occur; the closer a probability is to 1, the more likely the associated event is to occur. Figure 2.1 depicts this view of probability.

Probability is important in decision making because it provides a way to measure, express, and analyze the uncertainties associated with future events. The Q.M. in Action, Probability to the Rescue, describes the role that probability played in the efforts to rescue 33 Chilean miners.

FIGURE 2.1 PROBABILITY AS A NUMERIC MEASURE OF THE LIKELIHOOD OF AN EVENT OCCURRING



Q.M. *in* ACTION

*PROBABILITY TO THE RESCUE**

On August 5, 2010, the San José copper and gold mine suffered a cave-in. Thirty-three men were trapped over 2000 feet underground in the Atacama Desert near Copiapó, Chile. While most feared that these men's prospects were grim, several attempts were made to locate the miners and determine if they were still alive. Seventeen days later, rescuers reached the men with a 5½-inch borehole and ascertained that they were still alive.

*The authors are indebted to Dr. Michael Duncan and Clinton Cragg of NASA for providing input for this Q.M. in Action.

(*continued*)

After locating the men and establishing the means to communicate and deliver food, water, and medical supplies to them, the rescue effort could proceed in earnest. While it was important to bring these men safely to the surface as quickly as possible, it was imperative that the rescue effort also proceed cautiously. "The mine is old and there is concern of further collapses," Murray & Roberts Cementation Managing Director Henry Laas said in an interview with the *Santiago Times*. "The rescue methodology therefore has to be carefully designed and implemented."

The Chilean government asked NASA to consult on the rescue operation. In response, NASA sent a four-person team consisting of an engineer (Clinton Cragg), two physicians (Michael Duncan and J. D. Polk), and a psychologist (Al Holland). When asked why a space agency was brought in to consult on the rescue of trapped miners, Duncan stated, "We brought our experience in vehicle design and long duration confinement to our Chilean counterparts."

The probability of failure was prominent in the thoughts of everyone involved. "We were thinking that the rescue vehicle would have to make over forty round trips, so in consideration of the probability of part failures we suggested the rescue team have three rigs and several sets of replacement parts available." said Cragg. "We also tried to increase the probability of success by placing spring loaded rollers on the sides of the cage so the cage itself would not be damaged through direct contact with the rock wall as it moved through the rescue portal."

Duncan added, "While we and the Chileans would have preferred to have precise estimates of various probabilities based on historical data, the uniqueness of the situation made this infeasible. For example, a miner had to stand virtually straight up in the cage on an ascent that was originally estimated to last two to four hours per miner, so we had to be concerned about fainting. All we could do was consider what we thought to be the facts and apply what we had learned from astronauts' experiences in their returns from short and long duration space missions." Duncan continued, "We recommended the miners wear compression hosiery on lower extremities to prevent blood pooling, and that they load up on salty solutions such as chicken consommé prior to their ascents. We used all of this information to develop a subjective estimate of the probability a miner would faint on ascent," Duncan then concluded. "It actually took fifteen minutes to bring the cage up from the bottom of the mine, so our estimates in this case were very conservative. Considering the risk involved, that is exactly what we wanted."

Ultimately the rescue approach designed by the Chileans in consultation with the NASA team was successful. On October 13, 2010, the last of the 33 miners emerged; the 13-foot-long, 924-pound steel Fénix 2 rescue capsule withstood over 40 trips into and out of the mine, and no miner suffered from syncope (i.e., fainting) on his ascent.

The use of subjective probabilities in unique situations is common for NASA. How else could NASA estimate the probability that micrometeoroids or space debris will damage a space vehicle? With the limited space available on a space vehicle, assessing probability of failure for various components and the risks associated with these potential failures becomes critical to how NASA decides which spare components will be included on a space flight. NASA also employs probability to estimate the likelihood of crew health and performance issues arising on space exploration missions. Risk assessment teams then use these estimates in their mission design.

2.1 Experiments and the Sample Space

In discussing probability, we define an **experiment** to be any process that generates well-defined outcomes. On any single repetition of an experiment, *one and only one* of the possible experimental outcomes will occur. Several examples of experiments and their associated outcomes follow.

Experiment	**Experimental Outcomes**
Toss a coin	Head, tail
Select a part for inspection	Defective, nondefective
Conduct a sales call	Purchase, no purchase
Roll a standard die	1, 2, 3, 4, 5, 6
Play a football game	Win, lose, tie

The first step in analyzing a particular experiment is to carefully define the experimental outcomes. When we define *all* possible experimental outcomes, we identify the **sample space** for the experiment; that is, the set of all possible experimental outcomes. Any one particular experimental outcome is also referred to as a **sample point** and is an element of the sample space.

Consider the experiment of tossing a coin. The experimental outcomes are defined by the upward face of the coin—a head or a tail. If we let S denote the sample space, we can use the following notation to describe the sample space and sample points for the coin-tossing experiment:

$$S = \{\text{Head, Tail}\}$$

Using this notation for the second experiment in the preceding table, selecting a part for inspection, provides a sample space with sample points as follows:

$$S = \{\text{Defective, Nondefective}\}$$

Try Problem 1, parts (a) and (b), for practice in listing the experimental outcomes (sample points) for an experiment.

Finally, suppose that we consider the fourth experiment in the table, rolling a standard die. The experimental outcomes are defined as the number of dots appearing on the upward face of the die. In this experiment, the numeric values 1, 2, 3, 4, 5, and 6 represent the possible experimental outcomes or sample points. Thus the sample space is denoted

$$S = \{1, 2, 3, 4, 5, 6\}$$

NOTES AND COMMENTS

1. In probability, the notion of an experiment is somewhat different from the laboratory sciences. In the laboratory sciences, the researcher assumes that each time an experiment is repeated in exactly the same way, the same outcome will occur. For the type of experiment we study in probability, the outcome is determined by chance. Even though the experiment might be repeated in exactly the same way, a different outcome may occur. Because of this difference, the experiments we study in probability are sometimes called random experiments.

2.2 Assigning Probabilities to Experimental Outcomes

With an understanding of an experiment and the sample space, let us now see how probabilities for the experimental outcomes can be determined. The probability of an experimental outcome is a numeric measure of the likelihood that the experimental outcome will occur on a single repetition of the experiment. In assigning probabilities to experimental outcomes, two **basic requirements of probability** must be satisfied:

1. The probability values assigned to each experimental outcome (sample point) must be between 0 and 1. If we let E_i indicate the ith experimental outcome and $P(E_i)$ indicate the probability of this experimental outcome, we must have

$$0 \leq P(E_i) \leq 1 \text{ (for all } i) \qquad \textbf{(2.1)}$$

2. The sum of *all* of the experimental outcome probabilities must be 1. For example, if a sample space has k experimental outcomes, we must have

$$P(E_1) + P(E_2) + \cdots + P(E_k) = 1 \qquad \textbf{(2.2)}$$

Any method of assigning probability values to the experimental outcomes that satisfies these two requirements and results in reasonable numeric measures of the likelihood of the outcomes is acceptable. In practice, the classical method, the relative frequency method, or the subjective method is often used.

Classical Method

To illustrate the classical method of assigning probabilities, let us again consider the experiment of flipping a coin. On any one flip, we will observe one of two experimental outcomes: head or tail. It is reasonable to assume the two possible outcomes are equally likely. Therefore, as one of the two equally likely outcomes is a head, we logically should conclude that the probability of observing a head is $^1/_2$, or 0.50. Similarly, the probability of observing a tail is 0.50. When the assumption of equally likely outcomes is used as a basis for assigning probabilities, the approach is referred to as the **classical method**. If an experiment has n possible outcomes, application of the classical method would lead us to assign a probability of $1/n$ to each experimental outcome.

As another illustration of the classical method, consider again the experiment of rolling a standard die. In Section 2.1 we described the sample space and sample points for this experiment with the notation

$$S = \{1, 2, 3, 4, 5, 6\}$$

A standard die is designed so that the six experimental outcomes are equally likely, and hence each outcome is assigned a probability of $^1/_6$. Thus, if $P(1)$ denotes the probability that one dot appears on the upward face of the die, then $P(1) = {}^1/_6$. Similarly, $P(2) = {}^1/_6$, $P(3) = {}^1/_6$, $P(4) = {}^1/_6$, $P(5) = {}^1/_6$, and $P(6) = {}^1/_6$. Note that this probability assignment satisfies the two basic requirements for assigning probabilities. In fact, requirements (2.1) and (2.2) are automatically satisfied when the classical method is used, because each of the n sample points is assigned a probability of $1/n$.

Try Problem 2, part (b), for practice with the classical method.

The classical method was developed originally to analyze gambling probabilities for which the assumption of equally likely outcomes often is reasonable. In many business problems, however, this assumption is not valid. Hence, alternative methods of assigning probabilities are required.

Relative Frequency Method

Consider a firm that is preparing to market a new product. In order to estimate the probability that a customer will purchase the product, a test market evaluation has been set up wherein salespeople call on potential customers. Each sales call conducted has two possible outcomes: The customer purchases the product, or the customer does not purchase the product. With no reason to assume that the two experimental outcomes are equally likely, the classical method of assigning probabilities is inappropriate.

Try Problem 2, part (c), for practice in assigning probabilities to experimental outcomes using the relative frequency approach.

Suppose that in the test market evaluation of the product, 400 potential customers were contacted; 100 purchased the product, but 300 did not. In effect, we have repeated the experiment of contacting a customer 400 times and have found that the product was purchased 100 times. Thus, we might decide to use the relative frequency of the number of customers that purchased the product as an estimate of the probability of a customer making a purchase. We could assign a probability of $100/400 = 0.25$ to the experimental outcome of purchasing the product. Similarly, $300/400 = 0.75$ could be assigned to the experimental outcome of not purchasing the product. This approach to assigning probabilities is referred to as the **relative frequency method**. It is important to note that if we repeated this experiment 400 additional times, we may not obtain exactly the same probability estimate that we obtained from the first 400 trials of the experiment. The relative frequency approach only generates an empirical estimate of the actual probability of an experimental outcome, and different executions of the relative frequency approach can yield differing estimates of the probability of the same experimental outcome.

Subjective Method

The **subjective method** of assigning probabilities is most appropriate when we cannot realistically assume that the experimental outcomes are equally likely and when little relevant data are available. When the subjective method is used to assign probabilities to the experimental outcomes, we may use any information available, such as our experience or intuition. After considering all available information, a probability value that expresses our *degree of belief* (on a scale from 0 to 1) that the experimental outcome will occur is specified. Because subjective probability expresses a person's degree of belief, it is personal. Using the subjective method, different people may assign different probabilities to the same experimental outcome.

The subjective method requires extra care to ensure that the two basic requirements of equations (2.1) and (2.2) are satisfied. Regardless of a person's degree of belief, the probability value assigned to each experimental outcome must be between 0 and 1, inclusive, and the sum of all the probabilities for the experimental outcomes must equal 1.

Consider the case in which Tom and Judy Elsbernd just made an offer to purchase a house. Two outcomes are possible:

$$E_1 = \text{their offer is accepted}$$
$$E_2 = \text{their offer is rejected}$$

Judy believes that the probability that their offer will be accepted is 0.8; thus, Judy would set $P(E_1) = 0.8$ and $P(E_2) = 0.2$. Tom, however, believes that the probability that their offer will be accepted is 0.6; hence, Tom would set $P(E_1) = 0.6$ and $P(E_2) = 0.4$. Note that Tom's probability estimate for E_1 reflects a greater pessimism that their offer will be accepted.

Bayes' theorem (see Section 2.5) provides a means for combining subjectively determined prior probabilities with probabilities obtained by other means to obtain revised, or posterior, probabilities.

Both Judy and Tom assigned probabilities that satisfy the two basic requirements. The fact that their probability estimates differ reflects the personal nature of the subjective method.

These three approaches for assigning probabilities to experimental outcomes are often used in various combinations. For example, in some business situations managers may combine estimates from the classical or relative frequency approach with subjective probability estimates to obtain improved probability estimates.

2.3 Events and Their Probabilities

An **event** is a collection of sample points (experimental outcomes). For example, in the experiment of rolling a standard die, the sample space has six sample points and is denoted $S = \{1, 2, 3, 4, 5, 6\}$. Now consider the event that the number of dots shown on the upward face of the die is an even number. The three sample points in this event are 2, 4, and 6. Using the letter A to denote this event, we write A as a collection of sample points:

$$A = \{2, 4, 6\}$$

Thus, if the experimental outcome or sample point were 2, 4, or 6, we would say that the event A has occurred.

Much of the focus of probability analysis is involved with computing probabilities for various events that are of interest to a decision maker. If the probabilities of the sample points are defined, the *probability of an event* is equal to the sum of the probabilities of the sample points in the event.

Returning to the experiment of rolling a standard die, we used the classical method to conclude that the probability associated with each sample point is $^1/_6$. Thus, the probability of rolling a 2 is $^1/_6$, the probability of rolling a 4 is $^1/_6$, and the probability of rolling a 6 is $^1/_6$. The probability of event A—an even number of dots on the upward face of the die—is

$$\begin{aligned} P(A) &= P(2) + P(4) + P(6) \\ &= \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{3}{6} = \frac{1}{2} \end{aligned}$$

Try Problem 6 for practice in assigning probabilities to events.

Any time that we can identify all the sample points of an experiment and assign the corresponding sample point probabilities, we can use the preceding approach to compute the probability of an event. However, in many experiments the number of sample points is large, and the identification of the sample points, as well as determining their associated probabilities, becomes extremely cumbersome if not impossible. In the remainder of this chapter we present some basic probability relationships that can be used to compute the probability of an event without knowing all the individual sample point probabilities. These probability relationships require a knowledge of the probabilities for some events in the experiment. Probabilities of other events are then computed from these known probabilities using one or more of the probability relationships.

NOTES AND COMMENTS

1. The sample space, S, is itself an event. It contains all the experimental outcomes, so it has a probability of 1; that is, $P(S) = 1$.
2. When the classical method is used to assign probabilities, the assumption is that the experimental outcomes are equally likely. In such cases, the probability of an event can be computed by counting the number of experimental outcomes in the event and dividing the result by the total number of experimental outcomes.

2.4 Some Basic Relationships of Probability

In this section we present several relationships that will be helpful in computing probabilities. The relationships are the complement of an event, the addition law, conditional probability, and the multiplication law.

Complement of an Event

For an event A, the **complement of event A** is the event consisting of all sample points in sample space S that are *not* in A. The complement of A is denoted by A^c. Figure 2.2 provides a diagram, known as a **Venn diagram**, that illustrates the concept of a complement. The rectangular area represents the sample space for the experiment and as such contains all possible sample points. The circle represents event A and contains only the sample points that belong to A. The remainder of the rectangle contains all sample points not in event A, which by definition is the complement of A.

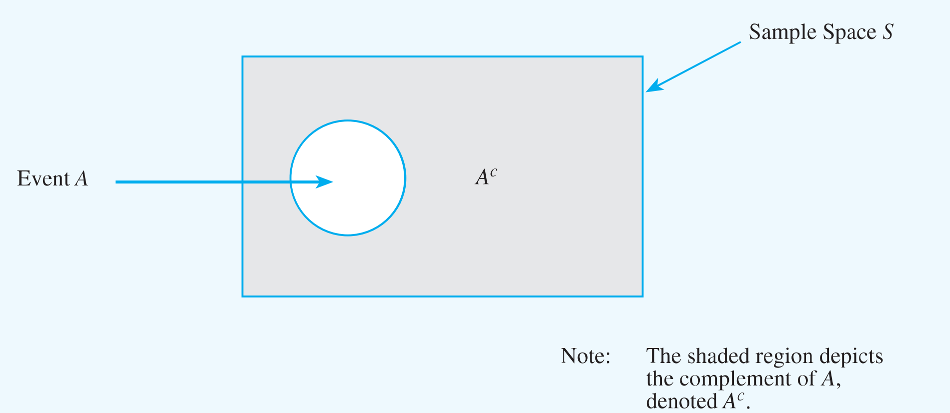
In any probability application, event A and its complement A^c must satisfy the condition

$$P(A) + P(A^c) = 1$$

Solving for $P(A)$, we have

$$P(A) = 1 - P(A^c) \tag{2.3}$$

FIGURE 2.2 COMPLEMENT OF EVENT A



Equation (2.3) shows that the probability of an event A can be computed by subtraction if the probability of its complement, $P(A^c)$, is known. Similarly, subtraction can be used to compute the probability of the compliment A^c of an event A if the probability of the event $P(A)$, is known.

Consider the case of a sales manager who, after reviewing sales reports, states that 80% of new customer contacts result in no sale. By letting A denote the event of a sale and A^c denote the event of no sale, the manager is stating that $P(A^c) = 0.80$. Using equation (2.3), we see that

$$P(A) = 1 - P(A^c) = 1 - 0.80 = 0.20$$

which shows that there is a 0.20 probability that a sale will be made on a new customer contact.

In another case, a purchasing agent states a 0.90 probability that a supplier will send a shipment that is free of defective parts. Using the complement, we can conclude a $1 - 0.90 = 0.10$ probability that the shipment will contain some defective parts.

Addition Law

The addition law is a useful relationship when we have two events and are interested in knowing the probability that at least one of the events occurs. That is, with events A and B, we are interested in knowing the probability that event A or event B or both will occur. Before we present the addition law, we need to discuss two concepts concerning combinations of events: the *union* of events and the *intersection* of events.

Key words for the union of events ($A \cup B$) are "either A or B occurs" or "at least one of the two events occurs." Note that the conjunction "or" commonly indicates a union of events.

Key words for the intersection of events ($A \cap B$) are "both A and B occur." Note that the conjunction "and" commonly indicates an intersection of events.

For two events A and B, the **union of events A and B** is the event containing all sample points belonging to *A or B or both.* The union is denoted $A \cup B$. The Venn diagram shown in Figure 2.3 depicts the union of events A and B; the shaded region contains all the sample points in event A, as well as all the sample points in event B. The fact that the circles overlap (or intersect) indicates that some sample points are contained in both A and B.

For two events A and B, the **intersection of events A and B** is the event containing the sample points belonging to *both A and B.* The intersection is denoted by $A \cap B$. The Venn diagram depicting the intersection of the two events is shown in Figure 2.4. The area where the two circles overlap is the intersection; it contains the sample points that are in both A and B.

FIGURE 2.3 UNION OF EVENTS *A* AND *B* (SHADED REGION)

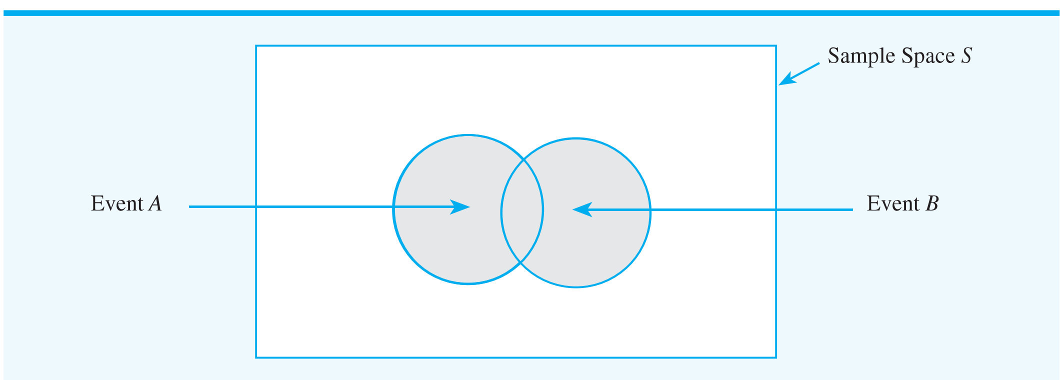
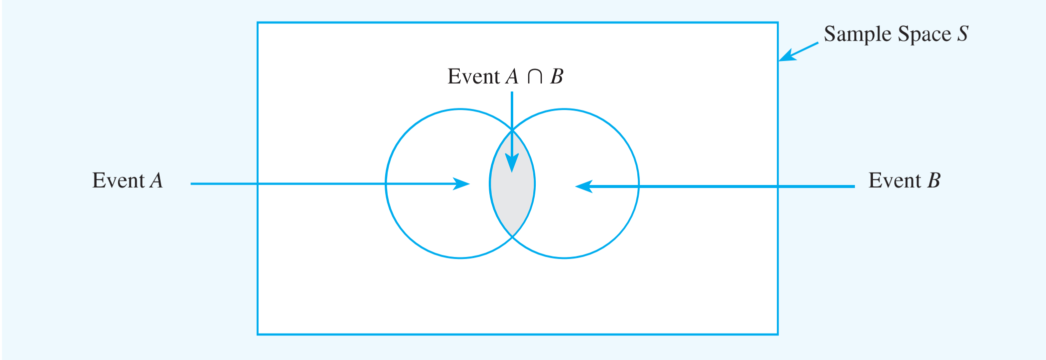


FIGURE 2.4 INTERSECTION OF EVENTS *A* AND *B* (SHADED REGION)



Note that $P(A \cup B) = P(B \cup A)$; that is, the order of events in a union does not affect the probability of the union.

Note that $P(A \cap B) = P(B \cap A)$; that is, the order of events in an intersection does not affect the probability of the intersection.

The addition law provides a way to compute the probability of event A or B or both occurring. In other words, the addition law is used to compute the probability of the union of two events, $A \cup B$. The **addition law** is formally stated as follows:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \tag{2.4}$$

To obtain an intuitive understanding of the addition law, note that the first two terms in the addition law, $P(A) + P(B)$, account for all the sample points in $A \cup B$. However, as the sample points in the intersection $A \cap B$ are in both A and B, when we compute $P(A) + P(B)$, we in effect are counting each of the sample points in $A \cap B$ twice. We correct for this double counting by subtracting $P(A \cap B)$.

To apply the addition law, let us consider the following situations in a college course in quantitative methods for decision making. Of 200 students taking the course, 160 passed the midterm examination and 140 passed the final examination; 124 students passed both exams. Let

$$A = \text{event of passing the midterm exam}$$
$$B = \text{event of passing the final exam}$$

This relative frequency information leads to the following probabilities:

$$P(A) = \frac{160}{200} = 0.80$$
$$P(B) = \frac{140}{200} = 0.70$$
$$P(A \cap B) = \frac{124}{200} = 0.62$$

After reviewing the grades, the instructor decided to give a passing grade to any student who passed at least one of the two exams; note that this implies the instructor will give a passing grade to any student who passed the midterm exam *or* passed the final exam. That is, any student who passed the midterm, any student who passed the final, and any student who passed both exams would receive a passing grade. What is the probability of a student receiving a passing grade in this course?

Your first reaction may be to try to count how many of the 200 students passed at least one exam, but note that the probability question is about the union of the events A and B. That is, we want to know the probability that a student passed the midterm (A), passed the final (B), or passed both. Thus we want to know $P(A \cup B)$. Using the addition law (2.4) for the events A and B, we have

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Knowing the three probabilities on the right-hand side of this equation, we obtain

$$P(A \cup B) = 0.80 + 0.70 - 0.62 = 0.88$$

This result indicates an 88% chance of a student passing the course because of the 0.88 probability of passing at least one of the exams.

Now consider a study involving the television-viewing habits of married couples. It was reported that 30% of the husbands and 20% of the wives were regular viewers of a particular Friday evening program. For 12% of the couples in the study, both husband and wife were regular viewers of the program. What is the probability that at least one member of a married couple is a regular viewer of the program?

Let

$$H = \text{husband is a regular viewer}$$
$$W = \text{wife is a regular viewer}$$

We have $P(H) = 0.30$, $P(W) = 0.20$, and $P(H \cap W) = 0.12$; thus, the addition law yields

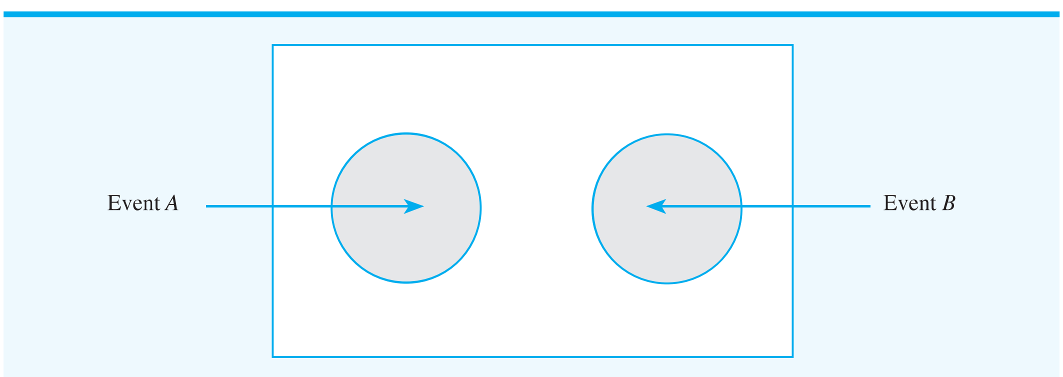
$$P(H \cup W) = P(H) + P(W) - P(H \cap W) = 0.30 + 0.20 - 0.12 = 0.38$$

An event and its complement are mutually exclusive and their union is the entire sample space.

This result shows a 0.38 probability that at least one member of a married couple is a regular viewer of the program.

Before proceeding, let us consider how the addition law is applied to **mutually exclusive events**. Two or more events are said to be mutually exclusive if the events do not have any sample points in common—that is, there are no sample points in the intersection of the events. For two events A and B to be mutually exclusive, $P(A \cap B) = 0$. Figure 2.5 provides

FIGURE 2.5 MUTUALLY EXCLUSIVE EVENTS



For practice, try solving Problem 7.

a Venn diagram depicting two mutually exclusive events. Because $P(A \cap B) = 0$ for the *special case of mutually exclusive events,* the addition law becomes

$$P(A \cup B) = P(A) + P(B) \tag{2.5}$$

To compute the probability of the union of two mutually exclusive events, we simply add the corresponding probabilities.

Conditional Probability

In many probability situations, being able to determine the probability of one event when another related event is known to have occurred is important. Suppose that we have an event A with probability $P(A)$ and that we obtain new information or learn that another event, denoted B, has occurred. If A is related to B, we will want to take advantage of this information in computing a new or revised probability for event A.

For a conditional probability such as $P(A \mid B) = 0.25$, the probability value of 0.25 refers only to the probability of event A. No information is provided about the probability of event B.

This new probability of event A is written $P(A \mid B)$. The "|" denotes the fact that we are considering the probability of event A *given the condition that event B has occurred.* Thus, the notation $P(A \mid B)$ is read "the probability of A given B."

With two events A and B, the general definitions of **conditional probability** for A given B and for B given A are as follows:

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} \tag{2.6}$$

$$P(B \mid A) = \frac{P(A \cap B)}{P(A)} \tag{2.7}$$

Note that for these expressions to have meaning, $P(B)$ cannot equal 0 in equation (2.6) and $P(A)$ cannot equal 0 in equation (2.7). *Also note that $P(A \mid B) \neq P(B \mid A)$, unless $P(A) = P(B)$.*

To obtain an intuitive understanding of the use of equation (2.6), consider the Venn diagram in Figure 2.6. The shaded region (both light gray and dark gray) denotes that

FIGURE 2.6 CONDITIONAL PROBABILITY $P(A \mid B) = P(A \cap B)/P(B)$

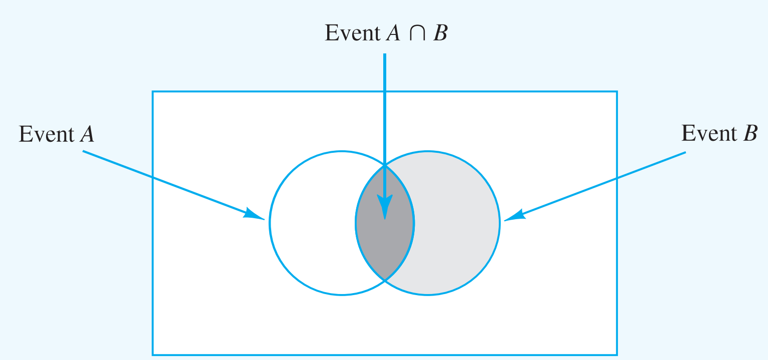


TABLE 2.1 CONTINGENCY TABLE FOR PROMOTIONAL STATUS OF POLICE OFFICERS DURING THE PAST TWO YEARS

	Promoted	Not Promoted	Total
Men	288	672	960
Women	36	204	240
Total	324	876	1200

event B has occurred; the dark gray shaded region denotes the event $(A \cap B)$. We know that once B has occurred, the only way that we can also observe event A is for event $(A \cap B)$ to occur. Thus, the ratio $P(A \cap B)/P(B)$ provides the probability that we will observe event A when event B has already occurred.

Try Problem 12 for practice computing conditional probabilities.

We can apply conditional probability to the promotional status of male and female officers of a major metropolitan police force. The force consists of 1200 officers: 960 men and 240 women. Over the past two years, 324 officers have been promoted. Table 2.1 shows the specific breakdown of promotions for male and female officers. Such a table is often called a *contingency table* or a *crosstabulation.*

After reviewing the promotional record, a committee of female officers filed a discrimination case on the basis that only 36 female officers had received promotions during the past two years. The police administration argued that the relatively low number of promotions for female officers is due not to discrimination but to the fact that few female officers are on the force. We use conditional probability to evaluate the discrimination charge.

Let

$$M = \text{event an officer is a man}$$
$$W = \text{event an officer is a woman}$$
$$B = \text{event an officer is promoted}$$

Dividing the data values in Table 2.1 by the total of 1200 officers permits us to summarize the available information as follows:

$$P(M \cap B) = \frac{288}{1200} = 0.24$$ probability that an officer is a man *and* is promoted

$$P(M \cap B^c) = \frac{672}{1200} = 0.56$$ probability that an officer is a man *and* is not promoted

$$P(W \cap B) = \frac{36}{1200} = 0.03$$ probability that an officer is a woman *and* is promoted

$$P(W \cap B^c) = \frac{204}{1200} = 0.17$$ probability that an officer is a woman *and* is not promoted

Because each of these values gives the probability of the intersection of two events, these probabilities are called **joint probabilities**. Table 2.2, which provides a summary of the probability information for the police officer promotion situation, is referred to as a **joint probability table**.

TABLE 2.2 JOINT PROBABILITY TABLE FOR POLICE OFFICER PROMOTIONS

	Promoted	Not Promoted	Total
Men	0.24	0.56	0.80
Women	0.03	0.17	0.20
Total	0.27	0.73	1.00

Joint probabilities appear in the body of the table.

Marginal probabilities appear in the margins of the table.

The values in the margins of the joint probability table provide the probabilities of each single event separately: $P(M) = 0.80$, $P(W) = 0.20$, $P(B) = 0.27$, and $P(B^c) = 0.73$, which indicate that 80% of the force is male, 20% of the force is female, 27% of all officers received promotions, and 73% were not promoted. These probabilities are referred to as **marginal probabilities** because of their location in the margins of the joint probability table. Returning to the issue of discrimination against the female officers, we see that the probability of promotion of an officer is $P(B) = 0.27$ (regardless of whether that officer is male or female). However, the critical issue in the discrimination case involves the two conditional probabilities $P(B \mid M)$ and $P(B \mid W)$; that is, what is the probability of a promotion *given* that the officer is a man and what is the probability of a promotion *given* that the officer is a woman? If these two probabilities are equal, the discrimination case has no basis because the chances of a promotion are the same for male and female officers. However, different conditional probabilities will support the position that male and female officers are treated differently in terms of promotion.

Using equation (2.7), the conditional probability relationship, we obtain

$$P(B \mid M) = \frac{P(B \cap M)}{P(M)} = \frac{0.24}{0.80} = 0.30 \left(= \frac{288/1200}{960/1200} = \frac{288}{960} \right)$$

$$P(B \mid W) = \frac{P(B \cap W)}{P(W)} = \frac{0.03}{0.20} = 0.15 \left(= \frac{36/1200}{240/1200} = \frac{36}{240} \right)$$

What conclusions do you draw? The probability of a promotion for a male officer is 0.30, which is twice the 0.15 probability of a promotion for a female officer. Although the use of conditional probability does not in itself prove that discrimination exists in this case, the conditional probability values strongly support the argument presented by the female officers.

In this illustration, $P(B) = 0.27$, $P(B \mid M) = 0.30$, and $P(B \mid W) = 0.15$. Clearly, the probability of promotion (event B) differs by gender. In particular, as $P(B \mid M) \neq P(B)$, events B and M are **dependent events**. The probability of event B (promotion) is higher when M (the officer is male) occurs. Similarly, with $P(B \mid W) \neq P(B)$, events B and W are dependent events. But, if the probability of event B was not changed by the existence of

For practice, try solving Problem 13.

event M—that is, $P(B \mid M) = P(B)$—events B and M would be **independent events**. Two events A and B are *independent* if

$$P(B \mid A) = P(B)$$

or

$$P(A \mid B) = P(A)$$

Otherwise, the events are *dependent.*

The Q.M. in Action, Product Testing for Quality Control at Morton International, describes how a subsidiary of Morton International used conditional probability to help decide to implement a quality control test.

Q.M. *in* ACTION

*PRODUCT TESTING FOR QUALITY CONTROL AT MORTON INTERNATIONAL**

Morton International is a company with businesses in salt, household products, rocket motors, and specialty chemicals. Carstab Corporation, a subsidiary of Morton, produces a variety of specialty chemical products designed to meet the unique specifications of its customers. For one particular customer, Carstab produced an expensive catalyst used in chemical processing. Some, but not all, of the product produced by Carstab met the customer's specifications.

Carstab's customer agreed to test each lot after receiving it to determine whether the catalyst would perform the desired function. Lots that did not pass the customer's test would be returned to Carstab. Over time, Carstab found that the customer was accepting 60% of the lots and returning 40%. In probability terms, each Carstab shipment to the customer had a 0.60 probability of being accepted and a 0.40 probability of being returned.

Neither Carstab nor its customer was pleased with these results. In an effort to improve service, Carstab explored the possibility of duplicating the customer's test prior to shipment. However, the high cost of the special testing equipment made that alternative infeasible. Carstab's chemists then proposed a new, relatively low-cost test designed to indicate whether a lot would pass the customer's test. The probability question of interest was: What is the probability that a lot will pass the customer's test given that it passed the new Carstab test?

A sample of lots was tested under both the customer's procedure and Carstab's proposed procedure. Results were that 55% of the lots passed Carstab's test, and 50% of the lots passed both the customer's and Carstab's test. In probability notation, we have

A = the event the lot passes the customer's test
B = the event the lot passes Carstab's test

where

$$P(B) = 0.55 \text{ and } P(A \cap B) = 0.50$$

The probability information sought was the conditional probability $P(A \mid B)$ given by

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{0.50}{0.55} = 0.909$$

Prior to Carstab's new test, the probability that a lot would pass the customer's test was 0.60. However, the new results showed that given that a lot passed Carstab's new test, it had a 0.909 probability of passing the customer's test. This result was good supporting evidence for the use of the test prior to shipment. Based on this probability analysis, the preshipment testing procedure was implemented at the company. Immediate results showed an improved level of customer service. A few lots were still being returned; however, the percentage was greatly reduced. The customer was more satisfied and return shipping costs were reduced.

*Based on information provided by Michael Haskell of Morton International.

Multiplication Law

The **multiplication law** can be used to find the probability of an intersection of two events. The multiplication law is derived from the definition of conditional probability. Using equations (2.6) and (2.7) and solving for $P(A \cap B)$, we obtain the multiplication law:

$$P(A \cap B) = P(A \mid B)P(B) \quad \textbf{(2.8)}$$

$$P(A \cap B) = P(B \mid A)P(A) \quad \textbf{(2.9)}$$

The multiplication law is useful in situations for which probabilities such as $P(A)$, $P(B)$, $P(A \mid B)$, and/or $P(B \mid A)$ are known but $P(A \cap B)$ is not. For example, suppose that a newspaper circulation department knows that 84% of its customers subscribe to the daily edition of the paper. Let D denote the event that a customer subscribes to the daily edition; hence, $P(D) = 0.84$. In addition, the department knows that the conditional probability that a customer who already holds a daily subscription also subscribes to the Sunday edition (event S) is 0.75; that is, $P(S \mid D) = 0.75$. What is the probability that a customer subscribes to both the daily and Sunday editions of the newspaper? Using equation (2.9), we compute $P(D \cap S)$:

$$P(D \cap S) = P(S \mid D)P(D) = 0.75(0.84) = 0.63$$

This result tells us that 63% of the newspaper's customers subscribe to both the daily and Sunday editions.

Before concluding this section, let us consider the special case of the multiplication law when the events involved are independent. Recall that independent events exist whenever $P(B \mid A) = P(B)$ or $P(A \mid B) = P(A)$. Returning to the multiplication law, equations (2.8) and (2.9), we can substitute $P(A)$ for $P(A \mid B)$ and $P(B)$ for $P(B \mid A)$. Hence, for the *special case of independent events,* the multiplication law becomes

$$P(A \cap B) = P(A)P(B) \quad \textbf{(2.10)}$$

Thus, to compute the probability of the intersection of two independent events, we simply multiply the corresponding probabilities. For example, a service station manager knows from past experience that 40% of her customers use a credit card when purchasing gasoline. What is the probability that the next two customers purchasing gasoline will both use a credit card? If we let

A = the event that the first customer uses a credit card
B = the event that the second customer uses a credit card

the event of interest is $A \cap B$. With no other information, it is reasonable to assume A and B are independent events. Thus,

$$P(A \cap B) = P(A)P(B) = (.40)(.40) = 0.16$$

NOTES AND COMMENTS

1. Do not confuse mutually exclusive events with independent events. Two events with nonzero probabilities cannot be both mutually exclusive and independent. If one mutually exclusive event is known to occur, the probability of the other occurring is reduced to zero. Thus, they cannot be independent.

2.5 Bayes' Theorem

In the discussion of conditional probability, we indicated that revising probabilities when new information is obtained is an important phase of probability analysis. Often, we begin an analysis with initial or **prior probability** estimates for specific events of interest; these initial estimates are generally developed using either the relative frequency approach (applied to historical data) or the subjective approach. Then, from sources such as a sample, a special report, or a product test, we obtain some additional information about the events. With this new information, we update the prior probability values by calculating revised probabilities, referred to as **posterior probabilities**. **Bayes' theorem** provides a means for making these probability revisions. The steps in this probability revision process are shown in Figure 2.7.

We can apply Bayes' theorem to a manufacturing firm that receives shipments of parts from two different suppliers. Let A_1 denote the event that a part is from supplier 1 and A_2 denote the event that a part is from supplier 2. Currently, 65% of the parts purchased by the company are from supplier 1, and the remaining 35% are from supplier 2. Thus, if a part is selected at random, we would assign the prior probabilities $P(A_1) = 0.65$ and $P(A_2) = 0.35$.

The quality of the purchased parts varies with the source of supply. Based on historical data, the conditional probabilities of receiving good and bad parts from the two suppliers are shown in Table 2.3. Thus, if we let G denote the event that a part is good and B denote the event that a part is bad, the information in Table 2.3 provides the following conditional probability values:

$$P(G \mid A_1) = 0.98 \quad P(B \mid A_1) = 0.02$$
$$P(G \mid A_2) = 0.95 \quad P(B \mid A_2) = 0.05$$

The tree diagram shown in Figure 2.8 depicts the process of the firm receiving a part from one of the two suppliers and then discovering that the part is good or bad as a two-step experiment. Of the four possible experimental outcomes, two correspond to the part being good, and two correspond to the part being bad.

Each of the experimental outcomes is the intersection of two events, so we can use the multiplication rule to compute the probabilities. For instance,

$$P(A_1 \cap G) = P(A_1)P(G \mid A_1)$$

FIGURE 2.7 PROBABILITY REVISION USING BAYES' THEOREM

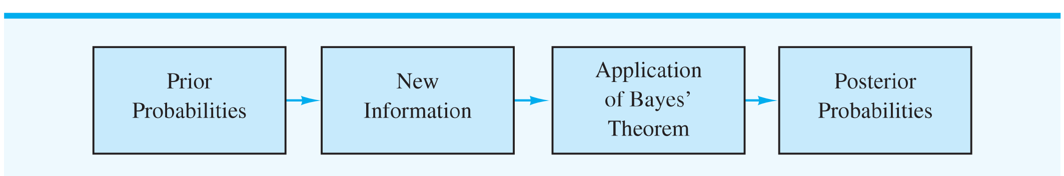
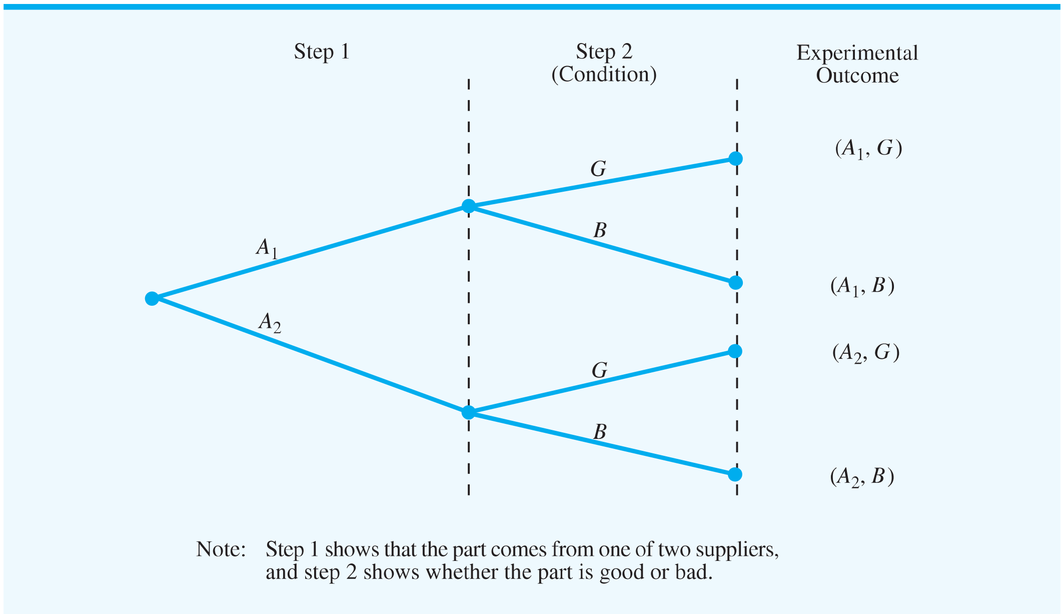


TABLE 2.3 CONDITIONAL PROBABILITIES OF RECEIVING GOOD AND BAD PARTS FROM TWO SUPPLIERS

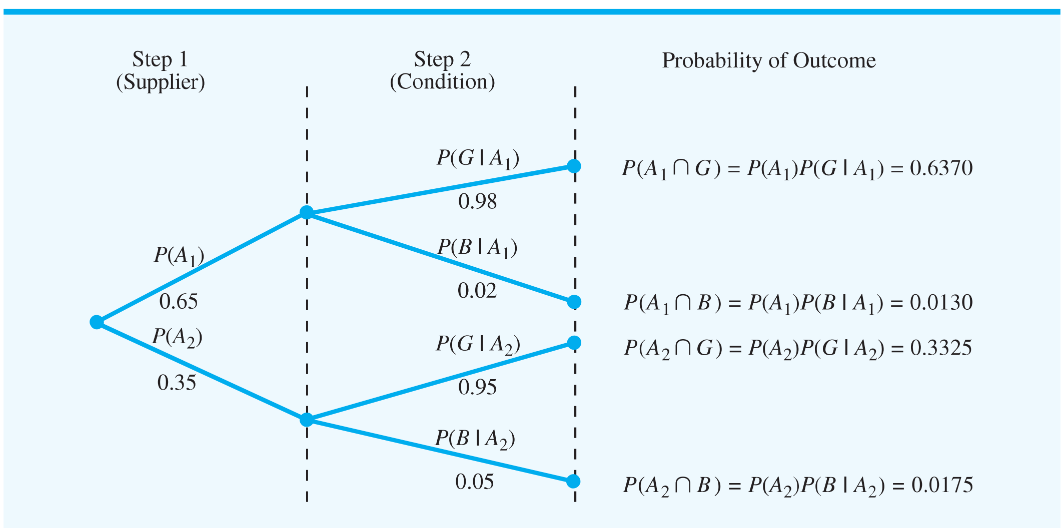
	Good Parts	Bad Parts	
Supplier 1	0.98	0.02	← $P(B \mid A_1)$
Supplier 2	0.95	0.05	

FIGURE 2.8 TWO-STEP TREE DIAGRAM



The process of computing these joint probabilities can be depicted in what is sometimes called a *probability tree,* as shown in Figure 2.9. From left to right in the tree, the probabilities for each of the branches at step 1 are the prior probabilities, and the probabilities for each branch at step 2 are conditional probabilities. To find the probabilities of each experimental outcome, we simply multiply the probabilities on the branches leading to the outcome. Each of these joint probabilities is shown in Figure 2.9, along with the known probabilities for each branch. Note that the probabilities of the four experimental outcomes sum to 1.

FIGURE 2.9 PROBABILITY TREE FOR TWO-SUPPLIER EXAMPLE



Now suppose that the parts from the two suppliers are used in the firm's manufacturing process and that a bad part causes a machine to break down. What is the probability that the bad part came from supplier 1, and what is the probability that it came from supplier 2? With the information in the probability tree (Figure 2.9), we can use Bayes' theorem to answer these questions.

Letting B denote the event the part is bad, we are looking for the posterior probabilities $P(A_1 \mid B)$ and $P(A_2 \mid B)$. From the definition of conditional probability, we know that

$$P(A_1 \mid B) = \frac{P(A_1 \cap B)}{P(B)} \tag{2.11}$$

Referring to the probability tree, we see that

$$P(A_1 \cap B) = P(A_1)P(B \mid A_1) \tag{2.12}$$

The Reverend Thomas Bayes (1702–1761), a Presbyterian minister, is credited with the original work leading to the version of Bayes' theorem in use today.

To find $P(B)$, we note that event B can occur in only two ways: $(A_1 \cap B)$ and $(A_2 \cap B)$. Therefore, we have

$$\begin{aligned} P(B) &= P(A_1 \cap B) + P(A_2 \cap B) \\ &= P(A_1)P(B \mid A_1) + P(A_2)P(B \mid A_2) \end{aligned} \tag{2.13}$$

Substituting from equations (2.12) and (2.13) into equation (2.11) and writing a similar result for $P(A_2 \mid B)$, we obtain Bayes' theorem for the case of two events.

$$P(A_1 \mid B) = \frac{P(A_1)P(B \mid A_1)}{P(A_1)P(B \mid A_1) + P(A_2)P(B \mid A_2)} \tag{2.14}$$

$$P(A_2 \mid B) = \frac{P(A_2)P(B \mid A_2)}{P(A_1)P(B \mid A_1) + P(A_2)P(B \mid A_2)} \tag{2.15}$$

Using equation (2.14) and the probability values provided in our example, we have

$$\begin{aligned} P(A_1 \mid B) &= \frac{P(A_1)P(B \mid A_1)}{P(A_1)P(B \mid A_1) + P(A_2)P(B \mid A_2)} \\ &= \frac{(0.65)(0.02)}{(0.65)(0.02) + (0.35)(0.05)} = \frac{0.0130}{0.0130 + 0.0175} \\ &= \frac{0.0130}{0.0305} = 0.4262 \end{aligned}$$

In addition, using equation (2.15), we obtain $P(A_2 \mid B)$:

$$\begin{aligned} P(A_2 \mid B) &= \frac{(0.35)(0.05)}{(0.65)(0.02) + (0.35)(0.05)} \\ &= \frac{0.0175}{0.0130 + 0.0175} = \frac{0.0175}{0.0305} = 0.5738 \end{aligned}$$

TABLE 2.4 SUMMARY OF BAYES' THEOREM CALCULATIONS FOR THE TWO-SUPPLIER PROBLEM

(1) Events A_i	(2) Prior Probabilities $P(A_i)$	(3) Conditional Probabilities $P(B \mid A_i)$	(4) Joint Probabilities $P(A_i \cap B)$	(5) Posterior Probabilities $P(A_i \mid B)$
A_1	0.65	0.02	0.0130	0.0130/0.0305 = 0.4262
A_2	0.35	0.05	0.0175	0.0175/0.0305 = 0.5738
	1.00		$P(B)$ = 0.0305	1.0000

Note that in this application we initially started with a probability of 0.65 that a part selected at random was from supplier 1. However, given information that the part is bad, we determine the probability that the part is from supplier 1 drops to 0.4262. If the part is bad, the probability that the part was from supplier 2 increases from 0.35 to 0.5738; that is, $P(A_2 \mid B) = 0.5738$.

Bayes' theorem is applicable when the events for which we want to compute posterior probabilities are mutually exclusive and their union is the entire sample space.[1] Bayes' theorem can be extended to the case of n mutually exclusive events $A_1, A_2, \ldots, A_n$, whose union is the entire sample space. In such a case Bayes' theorem for the computation of any posterior probability $P(A_i \mid B)$ becomes

$$P(A_i \mid B) = \frac{P(A_i)P(B \mid A_i)}{P(A_1)P(B \mid A_1) + P(A_2)P(B \mid A_2) + \cdots + P(A_n)P(B \mid A_n)} \tag{2.16}$$

Try Problem 20 for practice using Bayes' theorem to compute posterior probabilities.

With prior probabilities $P(A_1), P(A_2), \ldots, P(A_n)$ and the appropriate conditional probabilities $P(B \mid A_1), P(B \mid A_2), \ldots, P(B \mid A_n)$, equation (2.16) can be used to compute the posterior probability of the events $A_1, A_2, \ldots, A_n$.

The Tabular Approach

The tabular approach is helpful in conducting the Bayes' theorem calculations simultaneously for all events A_i. Such an approach is shown in Table 2.4. The computations shown there involve the following steps.

Step 1. Prepare three columns:
- Column 1—The mutually exclusive events for which posterior probabilities are desired
- Column 2—The prior probabilities for the events
- Column 3—The conditional probabilities of the new information given each event

Step 2. In column 4, compute the joint probabilities for each event and the new information B by using the multiplication law. To get these joint probabilities, multiply the prior probabilities in column 2 by the corresponding conditional probabilities in column 3—that is, $P(A_i \cap B) = P(A_i)P(B \mid A_i)$.

Step 3. Sum the joint probabilities in column 4 to obtain the probability of the new information, $P(B)$. In the example there is a 0.0130 probability that a part is from

[1] If the union of events is the entire sample space, the events are often called *collectively exhausted.*

supplier 1 and is bad and a 0.0175 probability that a part is from supplier 2 and is bad. These are the only two ways by which a bad part can be obtained, so the sum 0.0130 + 0.0175 shows an overall probability of 0.0305 of finding a bad part from the combined shipments of both suppliers.

Step 4. In column 5, compute the posterior probabilities by using the basic relationship of conditional probability:

$$P(A_i \mid B) = \frac{P(A_i \cap B)}{P(B)}$$

Try Problem 25 for an application of Bayes' theorem involving the tabular approach.

Note that the joint probabilities $P(A_i \cap B)$ appear in column 4, whereas $P(B)$ is the sum of the column 4 values.

NOTES AND COMMENTS

1. Bayes' theorem is used in decision analysis (see Chapter 4). The prior probabilities often are subjective estimates provided by a decision maker. Sample information is obtained and posterior probabilities are computed for use in developing a decision strategy.
2. An event and its complement are mutually exclusive, and their union is the entire sample space. Thus, Bayes' theorem is always applicable for computing posterior probabilities of an event and its complement.

2.6 Simpson's Paradox

As we have discussed in Sections 2.4 and 2.5, the concept of conditional probability quantifies the effect of an event B on the likelihood of another event A. It is possible for the apparent association between two events to be reversed upon considering a third event; this phenomenon is referred to as **Simpson's paradox**. Simpson's paradox often occurs when data from two or more subsets of a population are combined or aggregated to produce a summary crosstabulation in an attempt to show how two events are related. In such cases, we must be careful in drawing conclusions from the aggregated data about the relationship between the two events because the conclusions based upon the aggregated data can be completely reversed if we look at the disaggregated data. To provide an illustration of Simpson's paradox, we consider an example involving the analysis of verdicts for two judges.

Judges Ron Luckett and Dennis Kendall presided over cases in Common Pleas Court and Municipal Court during the past three years. Some of the verdicts they rendered were appealed. In most of these cases the appeals court upheld the original verdicts, but in some cases those verdicts were reversed. Table 2.5 illustrates the crosstabulation of cases broken down by verdict (upheld or reversed) and judge (Luckett or Kendall). This crosstabulation shows the number of appeals for which the verdict was upheld and the number of appeals for which the verdict was reversed for each judge (the column percentages in parentheses next to each value).

A review of the column percentages shows that 14% of the verdicts were reversed for Judge Luckett, but only 12% of the verdicts were reversed for Judge Kendall. That is, $P(\text{reversed} \mid \text{Luckett}) = 0.14$ and $P(\text{reversed} \mid \text{Kendall}) = 0.12$. Based on this data, we might conclude that Judge Kendall is doing a better job because a lower percentage of his verdicts are being reversed upon appeal.

TABLE 2.5 SUMMARY OF VERDICTS UPHELD AND REVERSED FOR JUDGES LUCKETT AND KENDALL

	Judge		
Verdict	**Luckett**	**Kendall**	**Total**
Upheld	129 (86%)	110 (88%)	239
Reversed	21 (14%)	15 (12%)	36
Total (%)	150 (100%)	125 (100%)	275

A closer look at the data, however, suggests there may be a problem with the conclusion that Judge Kendall's verdicts are reversed less often. If we further break down the verdict data by the type of court (Common Pleas or Municipal) in which the cases were heard, we obtain the crosstabulations in Table 2.6; column percentages are also shown in parentheses next to each value.

From the crosstabulation and column percentages for Luckett, we see that his verdicts were reversed in 9% of the Common Pleas Court cases and in 15% of the Municipal Court cases. That is, $P(\text{reversed} \mid \text{Luckett and Common Pleas Court}) = 0.09$ and $P(\text{reversed} \mid \text{Luckett and Municipal Court}) = 0.15$. From the crosstabulation and column percentages for Kendall, we see that his verdicts were reversed in 10% of the Common Pleas Court cases and in 20% of the Municipal Court cases. That is, $P(\text{reversed} \mid \text{Kendall and Common Pleas Court}) = 0.10$ and $P(\text{reversed} \mid \text{Kendall and Municipal Court}) = 0.20$. Since $P(\text{reversed} \mid \text{Luckett and Common Pleas Court}) < P(\text{reversed} \mid \text{Kendall and Common Pleas Court})$ and $P(\text{reversed} \mid \text{Luckett and Municipal Court}) < P(\text{reversed} \mid \text{Kendall and Municipal Court})$, we see that Judge Luckett demonstrates a better record than Judge Kendall in each court. This result contradicts the conclusion we reached when we aggregated the data across both courts for the original crosstabulation. It appeared then that Judge Kendall had the better record. This example illustrates Simpson's paradox.

TABLE 2.6 SUMMARY OF VERDICTS UPHELD AND REVERSED IN COMMON PLEAS AND MUNICIPAL COURT FOR JUDGES LUCKETT AND KENDALL

Judge Luckett

Verdict	**Common Pleas**	**Municipal Court**	**Total**
Upheld	29 (91%)	100 (85%)	129
Reversed	3 (9%)	18 (15%)	21
Total (%)	32 (100%)	118 (100%)	150

Judge Kendall

Verdict	**Common Pleas**	**Municipal Court**	**Total**
Upheld	90 (90%)	20 (80%)	110
Reversed	10 (10%)	5 (20%)	15
Total (%)	100 (100%)	25 (100%)	125

The original crosstabulation was obtained by aggregating the data in the separate crosstabulations for the two courts. Note that for both judges the percentage of appeals that resulted in reversals was much higher in Municipal Court than in Common Pleas Court. Because Judge Luckett tried a much higher percentage of his cases in Municipal Court, the aggregated data favored Judge Kendall. When we look at the crosstabulations for the two courts separately, however, Judge Luckett clearly shows the better record. Thus, for the original crosstabulation, we see that the *type of court* is a hidden variable that cannot be ignored when evaluating the records of the two judges.

Because of Simpson's paradox, we need to be particularly careful when drawing conclusions using aggregated data. Before drawing any conclusions about the relationship between two events shown for a crosstabulation involving aggregated data, you should investigate whether any hidden variables could affect the results. The QM in Action, Accurate Comparison of Unemployment Rates, provides an interesting and real example of Simpson's paradox.

NOTES AND COMMENTS

1. While Simpson's paradox is named for Edward Simpson (who described the paradox in 1951), the phenomenon was previously documented in papers by Pearson, Lee, and Bramley-Moore in 1899 and again by Yule in 1903. For this reason some refer to Simpson's paradox as the reversal paradox, the amalgamation paradox, or the Yule–Simpson effect.

Q.M. *in* ACTION

*ACCURATE COMPARISON OF UNEMPLOYMENT RATES—WHICH RECESSION WAS MORE SEVERE?**

In a December 2009 article in *The Wall Street Journal,* Cari Tuna asked, "Is the current economic slump worse than the recession of the early 1980s?" Tuna observed that if one uses the unemployment rate as the standard, on the surface the answer to this question appears to be no; the jobless rate was 10.2% in October of 2009, which is substantially below the peak of 10.8% in November and December of 1982.

Tuna then considers the question seperately for each of several education groups. Surprisingly, the unemployment rate among workers in each education group (*high-school dropouts*, *high-school graduates*, *some college*, and *college graduates*) was higher in October of 2009 than it was during the 1980s recession.

How can this be? How could the overall unemployment rate be lower in 2009 than it was during the recession of the early 1980s, while the unemployment rate for each of these groups was higher in 2009 than it was during the early 1980s recession? The anomaly is an example of Simpson's paradox—the phenomenon by which the apparent association between two events is reversed upon consideration of a third event.

As Tuna explains, the 2009 overall unemployment rate appears to be lower because college graduates (who have the lowest unemployment rate among education groups) were a larger proportion of the workforce in 2009 than they were in the early 1980s (they were approximately one-third of the 2009 workforce and 25% in the early 1980s). At the same time, the proportion of high-school dropouts fell from almost 20% in the early 1980s to approximately 10% in 2009. Even though the 2009 unemployment rate was higher among both groups than it was during the recession of the early 1980s, the

(*continued*)

*Based on C. Tuna, "When Combined Data Reveal the Flaw of Averages in a Statistical Anomaly Dubbed Simpson's Paradox: Aggregated Numbers Obscure Trends in Job Market, Medicine and Baseball," *The Wall Street Journal,* December 2, 2009 (http://online.wsj.com/article/SB125970744553071829.html).

discrepancies in contributions to the composition of the labor force by these two groups reverses the result when data from the two groups are aggregated.

In this case the data aggregated across education groups suggests the opposite of what the disaggregated data imply, and consideration of the aggregated data leads to an erroneous conclusion. Examples of Simpson's paradox are common across industries; Tuna cites examples from medicine, sports, education, and air travel. If we are not careful in our analyses, we can draw incorrect conclusions from aggregate data and as a result make poor decisions.

Summary

In this chapter we introduced basic probability concepts and illustrated how probability analysis can provide helpful decision-making information. We described how probability can be interpreted as a numeric measure of the likelihood that an event will occur, and discussed various ways that probabilities can be assigned to events. In addition, we showed that the probability of an event can be computed either by summing the probabilities of the experimental outcomes (sample points) comprising the event or by using the basic relationships of probability. When additional information becomes available, we showed how conditional probability and Bayes' theorem can be used to obtain revised or posterior probabilities. We also demonstrated how an apparent association between two events can be reversed upon consideration of a third event through the phenomenon of Simpson's paradox.

The probability concepts covered will be helpful in future chapters when we describe quantitative methods based on the use of probability information. Specific chapters and quantitative methods that make use of probability are as follows:

- Chapter 3 Probability distributions
- Chapter 4 Decision analysis
- Chapter 5 Utility and game theory
- Chapter 13 Project scheduling: PERT/CPM
- Chapter 14 Inventory models
- Chapter 15 Waiting line models
- Chapter 16 Simulation
- Chapter 17 Markov processes

Glossary

Probability A numeric measure of the likelihood that an event will occur.
Impossible event An event is certain not to occur. The probability of an impossible event is 0.
Certain event An event is certain to occur. The probability of a certain event is 1.
Experiment Any process that generates well-defined outcomes.
Sample space The set of all sample points (experimental outcomes).
Sample point An experimental outcome and an element of the sample space.
Basic requirements of probability Two requirements that restrict the manner in which probability assignments can be made:

1. For each experimental outcome E_i, $0 \leq P(E_i) \leq 1$.
2. $P(E_1) + P(E_2) + \cdots + P(E_k) = 1$.

Classical method A method of assigning probabilities that is based on the assumption that the experimental outcomes are equally likely.

Relative frequency method A method of assigning probabilities based on experimentation or historical data.
Subjective method A method of assigning probabilities based on judgment.
Event A collection of sample points or experimental outcomes.
Complement of event *A* The event containing all sample points that are not in A.
Venn diagram A graphical device for representing the sample space and operations involving events.
Union of events *A* and *B* The event containing all sample points that are in A, in B, or in both.
Intersection of events *A* and *B* The event containing all sample points that are in both A and B.
Addition law A probability law used to compute the probability of a union: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. For mutually exclusive events, $P(A \cap B) = 0$, and the addition law simplifies to $P(A \cup B) = P(A) + P(B)$.
Mutually exclusive events Events that have no sample points in common; that is, $A \cap B$ is empty and $P(A \cap B) = 0$.
Conditional probability The probability of an event given another event has occurred. The conditional probability of A given B is $P(A \mid B) = P(A \cap B)/P(B)$.
Joint probability The probability of the intersection of two events.
Joint probability table A table used to display joint and marginal probabilities.
Marginal probabilities The values in the margins of the joint probability table, which provide the probability of each event separately.
Dependent events Two events A and B for which $P(A \mid B) \neq P(A)$ or $P(B \mid A) \neq P(B)$; that is, the probability of one event is altered or affected by knowing whether the other event occurs.
Independent events Two events A and B for which $P(A \mid B) = P(A)$ and $P(B \mid A) = P(B)$; that is, the events have no influence on each other.
Multiplication law A probability law used to compute the probability of an intersection: $P(A \cap B) = P(A \mid B)P(B)$ or $P(A \cap B) = P(B \mid A)P(A)$. For independent events, this simplifies to $P(A \cap B) = P(A)P(B)$.
Prior probabilities Initial probabilities of events.
Posterior probabilities Revised probabilities of events based on additional information.
Bayes' theorem A method used to compute posterior probabilities.
Simpson's paradox The phenomenon by which the apparent association between two events is reversed upon consideration of a third event.

Problems

SELF test

1. A study examined waiting times in the X-ray department for a hospital in Jacksonville, Florida. A clerk recorded the number of patients waiting for service at 9:00 A.M. on 20 consecutive days and obtained the following results.

Number Waiting	Number of Days Outcome Occurred
0	2
1	5
2	6
3	4
4	3
Total	20

a. Define the experiment the clerk conducted.
b. List the experimental outcomes.
c. Assign probabilities to the experimental outcomes.
d. What method did you use?

SELF test

2. A company that franchises coffee houses conducted taste tests for a new coffee product. The company prepared four blends and randomly chose individuals to do a taste test and state which of the four blends they liked best. Results of the taste test for 100 individuals are given.

Blend	Taste-Testers' Preference
1	20
2	30
3	35
4	15

a. Define the experiment being conducted. How many times was it repeated?
b. Prior to conducting the experiment, it is reasonable to assume preferences for the four blends are equal. What probabilities would you assign to the experimental outcomes prior to conducting the taste test? What method did you use?
c. After conducting the taste test, what probabilities would you assign to the experimental outcomes? What method did you use?

3. A company that manufactures toothpaste is studying five different package designs. Assuming that one design is just as likely to be selected by a consumer as any other design, what selection probability would you assign to each of the package designs? In an actual experiment, 100 consumers were asked to pick the design they preferred. The following data were obtained. Do the data confirm the belief that one design is just as likely to be selected as another? Explain.

Design	Number of Times Preferred
1	5
2	15
3	30
4	40
5	10

4. In 2006 the U.S. Internal Revenue Service (IRS) received 132,275,830 individual tax returns (*The 2008 New York Times Almanac*). The actual number of each type of individual return received by the IRS in 2006 is given below:

Type of Return	Total Returns Filed
1040A, Income Under $25,000	31,675,935
Non 1040A, Income Under $25,000	20,295,694
Income $25,000–$50,000	30,828,932
Income $50,000–$100,000	26,463,973
Income $100,000 & Over	12,893,802
Schedule C, Receipts Under $25,000	3,376,943
Schedule C, Receipts $25,000–$100,000	3,867,743
Schedule C, Receipts $100,000 & Over	2,288,550
Schedule F, Receipts Under $100,000	318,646
Schedule F, Receipts $100,000 & Over	265,612

Suppose an IRS auditor must randomly select and examine an individual return.

a. What is the probability that the auditor will select an individual return from the 1040A, Income Under $25,000 category?
b. What is the probability that the selected return did *not* use Schedule C?
c. What is the probability that the selected return reported income or reciepts of $100,000 & Over?
d. In 2006 the IRS examined 1% of all individual returns. Assuming the examined returns were evenly distributed across the ten categories in the above table, how many returns from the Non 1040A, Income $50,000–$100,000 category were examined?
e. When examining 2006 individual income tax returns, IRS auditors found that individual taxpayers still owed $13,045,221,000 in income taxes due to errors the individual taxpayers had made on their 2006 individual income tax returns (this is referred to by the IRS as *recommended individual taxes*). Use this information to estimate the recommended additional taxes for the Schedule C, Receipts $100,000 & Over category.

5. Strom Construction made a bid on two contracts. The owner identified the possible outcomes and subjectively assigned the following probabilities.

Experimental Outcome	Obtain Contract 1	Obtain Contract 2	Probability
1	Yes	Yes	0.15
2	Yes	No	0.15
3	No	Yes	0.30
4	No	No	0.25

a. Are these valid probability assignments? Why or why not?
b. If not, what would have to be done to make the probability assignments valid?

SELF test

6. A sample of 100 customers of Montana Gas and Electric resulted in the following frequency distribution of monthly charges.

Amount ($)	Number
0–49	13
50–99	22
100–149	34
150–199	26
200–249	5

a. Let A be the event that monthly charges are $150 or more. Find $P(A)$.
b. Let B be the event that monthly charges are less than $150. Find $P(B)$.

SELF test

7. Suppose that a sample space has five equally likely experimental outcomes: E_1, E_2, E_3, E_4, E_5. Let

$$A = \{E_1, E_2\}$$
$$B = \{E_3, E_4\}$$
$$C = \{E_2, E_3, E_5\}$$

a. Find $P(A)$, $P(B)$, and $P(C)$.
b. Find $P(A \cup B)$. Are A and B mutually exclusive?
c. Find A^c, C^c, $P(A^c)$, and $P(C^c)$.
d. Find $A \cup B^c$ and $P(A \cup B^c)$.
e. Find $P(B \cup C)$.

8. In a recent article *U.S. News and World Report* rated pediatric hospitals and provided data on several characteristics, including daily inpatient volume and nurse-to-patient ratio, for the top 30 hospitals on its list (*U.S. News and World Report*, June 9, 2008). Suppose we consider a daily inpatient volume of at least 200 to be high and a nurse-to-patient ratio of at least 3.0 to be high. Sixteen hospitals had a daily inpatient volume of at least 200, one-third of the hospitals had a nurse-to-patient ratio of at least 3.0, and seven of the hospitals had both a daily inpatient volume of at least 200 and a nurse-to-patient ratio of at least 3.0.
   a. Find the probability of a hospital having a daily inpatient volume of at least 200, the probability of a hospital having a nurse-to-patient ratio of at least 3.0, and the probability of a hospital having both a daily inpatient volume of at least 200 and a nurse-to-patient ratio of at least 3.0.
   b. What is the probability that a hospital had a daily inpatient volume of at least 200 or a nurse-to-patient ratio of at least 3.0 or both?
   c. What is the probability that a hospital had neither a daily inpatient volume of at least 200 nor a nurse-to-patient ratio of at least 3.0?
9. A pharmaceutical company conducted a study to evaluate the effect of an allergy relief medicine; 250 patients with symptoms that included itchy eyes and a skin rash received the new drug. The results of the study are as follows: 90 of the patients treated experienced eye relief, 135 had their skin rash clear up, and 45 experienced relief of both itchy eyes and the skin rash. What is the probability that a patient who takes the drug will experience relief of at least one of the two symptoms?
10. A quality control specialist has sampled 25 widgets from the production line. A widget can have minor or major defects. Of the 25 sampled widgets, 4 have minor defects and 2 have major defects. What is the probability that a widget has a major defect, given that it has a defect?
11. Let A be an event that a person's primary method of transportation to and from work is an automobile and B be an event that a person's primary method of transportation to and from work is a bus. Suppose that in a large city $P(A) = 0.45$ and $P(B) = 0.35$.
   a. Are events A and B mutually exclusive? What is the probability that a person uses an automobile or a bus in going to and from work?
   b. Find the probability that a person's primary method of transportation is some means other than a bus.

SELF test

12. For two events A and B, $P(A) = 0.5$, $P(B) = 0.60$, and $P(A \cap B) = 0.40$.
   a. Find $P(A \mid B)$.
   b. Find $P(B \mid A)$.
   c. Are A and B independent? Why or why not?

SELF test

13. A survey of MBA students obtained the following data on "Students' first reason for application to the school in which they matriculated."

		Reason for Application			
		School Quality	School Cost or Convenience	Other	Totals
Enrollment Status	Full Time	421	393	76	890
	Part Time	400	593	46	1039
	Totals	821	986	122	1929

   a. Develop a joint probability table using these data.
   b. Use the marginal probabilities of school quality, school cost or convenience, and other to comment on the most important reason for choosing a school.

c. If a student goes full time, what is the probability that school quality will be the first reason for choosing a school?
d. If a student goes part time, what is the probability that school quality will be the first reason for choosing a school?
e. Let *A* be the event that a student is full time and let *B* be the event that the student lists school quality as the first reason for applying. Are events *A* and *B* independent? Justify your answer.

14. The checking accounts of Sun Bank are categorized by the age of account and the account balance. Auditors will select accounts at random from the following 1000 accounts (numbers in the table are the number of accounts in each category):

	Account Balance		
Age of the Account	**0–$499**	**$500–$999**	**$1000 or More**
Less than 2 years	120	240	90
2 years or more	75	275	200

a. What is the probability that an account is less than 2 years old?
b. What is the probability that an account has a balance of $1000 or more?
c. What is the probability that two accounts will both have a balance of $1000 or more?
d. What is the probability that an account has a balance of $500–$999 given that its age is 2 years or more?
e. What is the probability that an account is less than 2 years old and has a balance of $1000 or more?
f. What is the probability that an account is at least 2 years old given that the balance is $500–$999?

15. Hundreds of thousands of drivers dropped their automobile insurance in 2008 as the unemployment rate rose (*Wall Street Journal,* December 17, 2008). Sample data representative of the national automobile insurance coverage for individuals 18 years of age and older are shown here.

		Automobile Insurance	
		Yes	**No**
Age	**18 to 34**	1500	340
	35 and over	1900	260

a. Develop a joint probability table for these data and use the table to answer the remaining questions.
b. What do the marginal probabilities tell you about the age of the U.S. population?
c. What is the probability that a randomly selected individual does not have automobile insurance coverage?
d. If the individual is between the ages of 18 and 34, what is the probability that the individual does not have automobile insurance coverage?
e. If the individual is age 35 or over, what is the probability that the individual does not have automobile insurance coverage?
f. If the individual does not have automobile insurance, what is the probability that the individual is in the 18–34 age group?
g. What does the probability information tell you about automobile insurance coverage in the United States?

16. A purchasing agent placed a rush order for a particular raw material with two different suppliers, *A* and *B*. If neither order arrives in four days, the production process must be shut

down until at least one of the orders arrives. The probability that supplier A can deliver the material in four days is 0.55. The probability that supplier B can deliver the material in four days is 0.35.

a. What is the probability that both suppliers deliver the material in four days? Because two separate suppliers are involved, assume independence.
b. What is the probability that at least one supplier delivers the material in four days?
c. What is the probability that the production process is shut down in four days because of a shortage of raw material (that is, both orders are late)?

17. Interested in learning more about its fans, the marketing office of the Arena Football League (AFL) conducted a survey at one of its games. The survey had 989 respondents, 759 males and 230 females. Out of the 989 total respondents, 196 stated that they had attended multiple AFL games. Of these 196 fans that had attended multiple games, 177 were male. Using this survey information, answer the following questions.
    a. What is the probability that a randomly selected fan has attended multiple games?
    b. Given that a randomly selected fan has attended multiple games, what is the probability of this person being male?
    c. What is the probability of a randomly selected fan being male and having attended multiple games?
    d. Given that a randomly selected fan is male, what is the probability that this person has attended multiple games?
    e. What is the probability that a randomly selected fan is male or has attended multiple games?

18. In the evaluation of a sales training program, a firm discovered that of 50 salespeople receiving a bonus last year, 20 had attended a special sales training program. The firm employs 200 salespeople. Let B = the event that a salesperson makes a bonus and S = the event that a salesperson attends the sales training program.
    a. Find $P(B)$, $P(S \mid B)$, and $P(S \cap B)$.
    b. Assume that 40% of the salespeople attended the training program. What is the probability that a salesperson makes a bonus given that the salesperson attended the sales training program, $P(B \mid S)$?
    c. If the firm evaluates the training program in terms of its effect on the probability of a salesperson's receiving a bonus, what is your evaluation of the training program? Comment on whether B and S are dependent or independent events.

19. A company studied the number of lost-time accidents occurring at its Brownsville, Texas, plant. Historical records show that 6% of the employees had lost-time accidents last year. Management believes that a special safety program will reduce the accidents to 5% during the current year. In addition, it estimates that 15% of those employees having had lost-time accidents last year will have a lost-time accident during the current year.
    a. What percentage of the employees will have lost-time accidents in both years?
    b. What percentage of the employees will have at least one lost-time accident over the two-year period?

SELF test

20. The prior probabilities for events A_1, A_2, and A_3 are $P(A_1) = 0.20$, $P(A_2) = 0.50$, and $P(A_3) = 0.30$. The conditional probabilities of event B given A_1, A_2, and A_3 are $P(B \mid A_1) = 0.50$, $P(B \mid A_2) = 0.40$, and $P(B \mid A_3) = 0.30$.
    a. Compute $P(B \cap A_1)$, $P(B \cap A_2)$, and $P(B \cap A_3)$.
    b. Apply Bayes' theorem, equation (2.16), to compute the posterior probability $P(A_2 \mid B)$.
    c. Use the tabular approach to applying Bayes' theorem to compute $P(A_1 \mid B)$, $P(A_2 \mid B)$, and $P(A_3 \mid B)$.

21. A consulting firm submitted a bid for a large research project. The firm's management initially felt there was a 50/50 chance of getting the bid. However, the agency to which the bid was submitted subsequently requested additional information on the bid. Experience

indicates that on 75% of the successful bids and 40% of the unsuccessful bids the agency requested additional information.

a. What is the prior probability that the bid will be successful (i.e., prior to receiving the request for additional information)?
b. What is the conditional probability of a request for additional information given that the bid will ultimately be successful?
c. Compute a posterior probability that the bid will be successful given that a request for additional information has been received.

22. Companies that do business over the Internet can often obtain probability information about website visitors from previous websites visited. For instance, Par Fore created a website to market golf equipment and apparel, and the organization has collected data from its website visitors. Management would like a certain offer to appear for female visitors and a different offer to appear for male visitors. A sample of past website visits indicates that 60% of the visitors to *ParFore.com* are male and 40% are female.
    a. What is your prior probability that the next visitor to the website will be female?
    b. Suppose you know that the current visitor previously visited the Par Fore website and that women are three times as likely to visit this website as men. What is your revised probability that the visitor is female? Should you display the offer that has more appeal to female visitors or the one that has more appeal to male visitors?

23. An oil company purchased an option on land in Alaska. Preliminary geologic studies assigned the following prior probabilities.

$$P(\text{high quality oil}) = 0.50$$
$$P(\text{medium quality oil}) = 0.20$$
$$P(\text{no oil}) = 0.30$$

    a. What is the probability of finding oil?
    b. After 200 feet of drilling on the first well, a soil test is made. The probabilities of finding the particular type of soil identified by the test are

$$P(\text{soil} \mid \text{high quality oil}) = 0.20$$
$$P(\text{soil} \mid \text{medium quality oil}) = 0.20$$
$$P(\text{soil} \mid \text{no oil}) = 0.30$$

How should the firm interpret the soil test? What are the revised probabilities, and what is the new probability of finding oil?

SELF test

24. During a recent year, speeding was reported in 12.9% of all automobile accidents in the United States (*The World Almanac 2010*). Assume the probability that speeding is reported in an accident is 0.129, the probability of an accident in which speeding is reported leading to a fatality is 0.196, and the probability of an accident in which speeding is reported that does not lead to a fatality is 0.05. Suppose you learn of an accident involving a fatality. What is the probability that speeding was reported?

SELF test

25. The Wayne Manufacturing Company purchases a certain part from suppliers *A*, *B*, and *C*. Supplier *A* supplies 60% of the parts, *B* 30%, and *C* 10%. The quality of parts varies among the suppliers, with *A*, *B*, and *C* parts having 0.25%, 1%, and 2% defective rates, respectively. The parts are used in one of the company's major products.
    a. What is the probability that the company's major product is assembled with a defective part? Use the tabular approach to Bayes' theorem to solve.
    b. When a defective part is found, which supplier is the likely source?

26. Bayes' theorem and conditional probability can be used in medical diagnosis. Prior probabilities of diseases are based on the physician's assessment of factors such as geographic location, seasonal influence, and occurrence of epidemics. Assume that a patient is

believed to have one of two diseases, denoted D_1 and D_2, with $P(D_1) = 0.60$ and $P(D_2) = 0.40$, and that medical research shows a probability associated with each symptom that may accompany the diseases. Suppose that, given diseases D_1 and D_2, the probabilities that a patient will have symptoms S_1, S_2, or S_3 are as follows:

		Symptoms		
		S_1	S_2	S_3
Disease	D_1	0.15	0.10	0.15
	D_2	0.80	0.15	0.03

$P(S_3 \mid D_1)$

After finding that a certain symptom is present, the medical diagnosis may be aided by finding the revised probabilities that the patient has each particular disease. Compute the posterior probabilities of each disease for the following medical findings.

a. The patient has symptom S_1.
b. The patient has symptom S_2.
c. The patient has symptom S_3.
d. For the patient with symptom S_1 in part (a), suppose that symptom S_2 also is present. What are the revised probabilities of D_1 and D_2?

27. Recently, management at Oak Tree Golf Course received a few complaints about the condition of the greens. Several players complained that the greens are too fast. Rather than react to the comments of just a few, the Golf Association conducted a survey of 100 male and 100 female golfers. The survey results are summarized here.

	Male Golfers			**Female Golfers**	
	Greens Condition			**Greens Condition**	
Handicap	**Too Fast**	**Acceptable**	**Handicap**	**Too Fast**	**Acceptable**
Under 15	10	40	**Under 15**	1	9
15 or more	25	25	**15 or more**	39	51

a. Combine these two crosstabulations into a single crosstabulation, with Male and Female as the row labels and Too Fast and Acceptable as the column labels. Which group shows the highest proportion saying that the greens are too fast?
b. Refer to the initial crosstabulations. For those players with low handicaps (better players), which group (male or female) shows the highest proportion saying the greens are too fast?
c. Refer to the initial crosstabulations. For those players with higher handicaps, which group (male or female) shows the highest proportion saying the greens are too fast?
d. What conclusions can you draw about the preferences of men and women concerning the speed of the greens? Are the conclusions you draw from part (a) as compared with parts (b) and (c) consistent? Explain any apparent inconsistencies.

28. A small private midwestern university has been accused of favoring male applicants in its admissions process. Prior to the current academic year, 44% of male applicants were admitted by the university while only 33% of female applicants were admitted. The numbers

of applicants accepted and denied for each gender are provided in the following table for the College of Engineering and the College of Business.

	College of Engineering		College of Business	
	Male Applicants	**Female Applicants**	**Male Applicants**	**Female Applicants**
Accept	60	20	10	20
Deny	60	20	30	60

a. Combine these two crosstabulations into a single crosstabulation, with Accept and Deny as the row labels and Male and Female as the column labels. Use these data to confirm the reported rates of acceptance across the university by gender (44% of male applicants were admitted and 33% of female applicants were admitted).

b. Refer to the initial crosstabulations. What are your conclusions about the possible gender bias in the admission process? Is this consistent with the conclusions you reach when assessing the aggregated data in part (a) of this question?

Case Problem Hamilton County Judges

Hamilton County judges try thousands of cases per year. In an overwhelming majority of the cases disposed, the verdict stands as rendered. However, some cases are appealed, and of those appealed, some of the cases are reversed. Kristen DelGuzzi of the *Cincinnati Enquirer* conducted a study of cases handled by Hamilton County judges over a three-year period (*Cincinnati Enquirer,* January 11, 1998). Shown in Table 2.7 are the results for 182,908 cases handled (disposed) by 38 judges in Common Pleas Court, Domestic Relations Court, and Municipal Court. Two of the judges (Dinkelacker and Hogan) did not serve in the same court for the entire three-year period.

The purpose of the newspaper's study was to evaluate the performance of the judges. Appeals are often the result of mistakes made by judges, and the newspaper wanted to know which judges were doing a good job and which were making too many mistakes. You have been called in to assist in the data analysis. Use your knowledge of probability and conditional probability to help with the ranking of the judges. You also may be able to analyze the likelihood of cases handled by the different courts being appealed and reversed.

Managerial Report

Prepare a report with your rankings of the judges. Also include an analysis of the likelihood of appeal and case reversal in the three courts. At a minimum, your report should include the following:

1. The probability of cases being appealed and reversed in the three different courts.
2. The probability of a case being appealed for each judge.
3. The probability of a case being reversed for each judge.
4. The probability of reversal given an appeal for each judge.
5. Rank the judges within each court. State the criteria you used and provide a rationale for your choice.

TABLE 2.7 CASES DISPOSED, APPEALED, AND REVERSED IN HAMILTON COUNTY COURTS

Common Pleas Court

Judge	Total Cases Disposed	Appealed Cases	Reversed Cases
Fred Cartolano	3037	137	12
Thomas Crush	3372	119	10
Patrick Dinkelacker	1258	44	8
Timothy Hogan	1954	60	7
Robert Kraft	3138	127	7
William Mathews	2264	91	18
William Morrissey	3032	121	22
Norbert Nadel	2959	131	20
Arthur Ney, Jr.	3219	125	14
Richard Niehaus	3353	137	16
Thomas Nurre	3000	121	6
John O'Connor	2969	129	12
Robert Ruehlman	3205	145	18
J. Howard Sundermann	955	60	10
Ann Marie Tracey	3141	127	13
Ralph Winkler	3089	88	6
Total	43,945	1762	199

Domestic Relations Court

Judge	Total Cases Disposed	Appealed Cases	Reversed Cases
Penelope Cunningham	2729	7	1
Patrick Dinkelacker	6001	19	4
Deborah Gaines	8799	48	9
Ronald Panioto	12,970	32	3
Total	30,499	106	17

Municipal Court

Judge	Total Cases Disposed	Appealed Cases	Reversed Cases
Mike Allen	6149	43	4
Nadine Allen	7812	34	6
Timothy Black	7954	41	6
David Davis	7736	43	5
Leslie Isaiah Gaines	5282	35	13
Karla Grady	5253	6	0
Deidra Hair	2532	5	0
Dennis Helmick	7900	29	5
Timothy Hogan	2308	13	2
James Patrick Kenney	2798	6	1
Joseph Luebbers	4698	25	8
William Mallory	8277	38	9
Melba Marsh	8219	34	7
Beth Mattingly	2971	13	1
Albert Mestemaker	4975	28	9
Mark Painter	2239	7	3
Jack Rosen	7790	41	13
Mark Schweikert	5403	33	6
David Stockdale	5371	22	4
John A. West	2797	4	2
Total	108,464	500	104

Case Problem College Softball Recruiting

College softball programs have a limited number of scholarships to offer promising high school seniors, so the programs invest a great deal of effort in evaluating these players. One measure of performance the programs commonly use to evaluate recruits is the *batting average*—the proportion of at-bats (excluding times when the player is walked or hit by a pitch) in which the player gets a hit. For example, a player who gets 50 hits in 150 at-bats has a batting average of

$$\frac{50}{150} = 0.333$$

A college softball program is considering two players, Fran Hayes and Millie Marshall, who have recently completed their senior years of high school. Their respective statistics for their junior and senior years are as shown in Table 2.8.

Managerial Report

The Athletic Director and Coach of the women's softball team at a large public university are trying to decide to which of these two players they will offer an athletic scholarship (i.e., an opportunity to attend the university for free in exchange for playing on the university's softball team). Take the following steps to determine which player had the better batting average over the two-year period provided in the table, and use your results to advise the Athletic Director and Coach on their decision.

1. Calculate the batting average of each player for her junior year; then also calculate the batting average of each player for her senior year. Which player would this analysis lead you to choose?
2. Calculate the batting average of each player for her combined junior and senior years. Which player would this analysis lead you to choose?
3. After considering both of your analyses, which player would you choose? Why?
4. Prepare a report on your findings for the atheletic director and coach of the college program. Focus on clearly explaining the discrepancy in your two analyses.

TABLE 2.8 SUMMARY OF BATTING PERFORMANCES IN JUNIOR AND SENIOR YEARS BY HAYES AND MARSHALL

	Junior Year		Senior Year	
	At-Bats	**Hits**	**At-Bats**	**Hits**
Fran Hayes	200	70	40	15
Millie Marshall	196	67	205	76