CHAPTER 4

Decision Analysis

CONTENTS

Decision analysis can be used to develop an optimal strategy when a decision maker is faced with several decision alternatives and an uncertain or risk-filled pattern of future events. For example, Ohio Edison used decision analysis to choose the best type of particulate control equipment for coal-fired generating units when it faced future uncertainties concerning sulfur content restrictions, construction costs, and so on. The State of North Carolina used decision analysis in evaluating whether to implement a medical screening test to detect metabolic disorders in newborns. The Q.M. in Action, Phytopharm's New Product Research and Development, discusses the use of decision analysis to manage Phytopharm's pipeline of pharmaceutical products, which have long development times and relatively high levels of uncertainty.

Even when a careful decision analysis has been conducted, the uncertain future events make the final consequence uncertain. In some cases, the selected decision alternative may provide good or excellent results. In other cases, a relatively unlikely future event may occur, causing the selected decision alternative to provide only fair or even poor results. The risk associated with any decision alternative is a direct result of the uncertainty associated with the final consequence. A good decision analysis includes careful consideration of risk. Through risk analysis the decision maker is provided with probability information about the favorable as well as the unfavorable consequences that may occur.

We begin the study of decision analysis by considering problems that involve reasonably few decision alternatives and reasonably few possible future events. Influence diagrams and payoff tables are introduced to provide a structure for the decision problem and

Q.M. *in* ACTION

*PHYTOPHARM'S NEW PRODUCT RESEARCH AND DEVELOPMENT**

As a pharmaceutical development and functional food company, Phytopharm's primary revenue streams come from licensing agreements with larger companies. After Phytopharm establishes proof of principle for a new product by successfully completing early clinical trials, it seeks to reduce its risk by licensing the product to a large pharmaceutical or nutrition company that will further develop and market the product.

There is substantial uncertainty regarding the future sales potential of early stage products, as only one in ten of such products makes it to market and only 30% of these yield a healthy return. Phytopharm and its licensing partners would often initially propose very different terms of the licensing agreement. Therefore, Phytopharm employed a team of researchers to develop a flexible methodology for appraising a product's potential and subsequently supporting the negotiation of the lump-sum payments for development milestones and royalties on eventual sales that comprise the licensing agreement.

Using computer simulation, the resulting decision analysis model allows Phytopharm to perform sensitivity analysis on estimates of development cost, the probability of successful Food & Drug Administration clearance, launch date, market size, market share, and patent expiry. In particular, a decision tree model allows Phytopharm and its licensing partner to mutually agree upon the number of development milestones. Depending on the status of the project at a milestone, the licensing partner can opt to abandon the project or continue development. Laying out these sequential decisions in a decision tree allows Phytopharm to negotiate milestone payments and royalties that equitably split the project's value between Phytopharm and its potential licensee.

*Pascale Crama, Bert De Ryck, Zeger Degraeve, and Wang Chong, "Research and Development Project Valuation and Licensing Negotiations at Phytopharm plc," *Interfaces,* 37 no. 5: 472–487.

to illustrate the fundamentals of decision analysis. We then introduce decision trees to show the sequential nature of decision problems. Decision trees are used to analyze more complex problems and to identify an optimal sequence of decisions, referred to as an optimal decision strategy. Sensitivity analysis shows how changes in various aspects of the problem affect the recommended decision alternative.

4.1 Problem Formulation

The first step in the decision analysis process is problem formulation. We begin with a verbal statement of the problem. We then identify the **decision alternatives**; the uncertain future events, referred to as **chance events**; and the **consequences** associated with each combination of decision alternative and chance event outcome. Let us begin by considering a construction project of the Pittsburgh Development Corporation.

Pittsburgh Development Corporation (PDC) purchased land that will be the site of a new luxury condominium complex. The location provides a spectacular view of downtown Pittsburgh and the Golden Triangle, where the Allegheny and Monongahela Rivers meet to form the Ohio River. PDC plans to price the individual condominium units between $300,000 and $1,400,000.

PDC commissioned preliminary architectural drawings for three different projects: one with 30 condominiums, one with 60 condominiums, and one with 90 condominiums. The financial success of the project depends upon the size of the condominium complex and the chance event concerning the demand for the condominiums. The statement of the PDC decision problem is to select the size of the new luxury condominium project that will lead to the largest profit given the uncertainty concerning the demand for the condominiums.

Given the statement of the problem, it is clear that the decision is to select the best size for the condominium complex. PDC has the following three decision alternatives:

d_1 = a small complex with 30 condominiums
d_2 = a medium complex with 60 condominiums
d_3 = a large complex with 90 condominiums

A factor in selecting the best decision alternative is the uncertainty associated with the chance event concerning the demand for the condominiums. When asked about the possible demand for the condominiums, PDC's president acknowledged a wide range of possibilities but decided that it would be adequate to consider two possible chance event outcomes: a strong demand and a weak demand.

In decision analysis, the possible outcomes for a chance event are referred to as the **states of nature**. The states of nature are defined so they are mutually exclusive (no more than one can occur) and collectively exhaustive (at least one must occur); thus one and only one of the possible states of nature will occur. For the PDC problem, the chance event concerning the demand for the condominiums has two states of nature:

s_1 = strong demand for the condominiums
s_2 = weak demand for the condominiums

Management must first select a decision alternative (complex size); then a state of nature follows (demand for the condominiums) and finally a consequence will occur. In this case, the consequence is PDC's profit.

Influence Diagrams

An **influence diagram** is a graphical device that shows the relationships among the decisions, the chance events, and the consequences for a decision problem. The **nodes** in an influence diagram represent the decisions, chance events, and consequences. Rectangles or squares depict **decision nodes**, circles or ovals depict **chance nodes**, and diamonds depict **consequence nodes**. The lines connecting the nodes, referred to as *arcs,* show the direction of influence that the nodes have on one another. Figure 4.1 shows the influence diagram for the PDC problem. The complex size is the decision node, demand is the chance node, and profit is the consequence node. The arcs connecting the nodes show that both the complex size and the demand influence PDC's profit.

Payoff Tables

Given the three decision alternatives and the two states of nature, which complex size should PDC choose? To answer this question, PDC will need to know the consequence associated with each decision alternative and each state of nature. In decision analysis, we refer to the consequence resulting from a specific combination of a decision alternative and a state of nature as a **payoff**. A table showing payoffs for all combinations of decision alternatives and states of nature is a **payoff table**.

Payoffs can be expressed in terms of profit, cost, time, distance, or any other measure appropriate for the decision problem being analyzed.

Because PDC wants to select the complex size that provides the largest profit, profit is used as the consequence. The payoff table with profits expressed in millions of dollars is shown in Table 4.1. Note, for example, that if a medium complex is built and demand turns out to be strong, a profit of $14 million will be realized. We will use the notation V_{ij} to denote the payoff associated with decision alternative *i* and state of nature *j*. Using Table 4.1, $V_{31} = 20$ indicates a payoff of $20 million occurs if the decision is to build a large complex (d_3) and the strong demand state of nature (s_1) occurs. Similarly, $V_{32} = -9$ indicates a loss of $9 million if the decision is to build a large complex (d_3) and the weak demand state of nature (s_2) occurs.

FIGURE 4.1 INFLUENCE DIAGRAM FOR THE PDC PROJECT

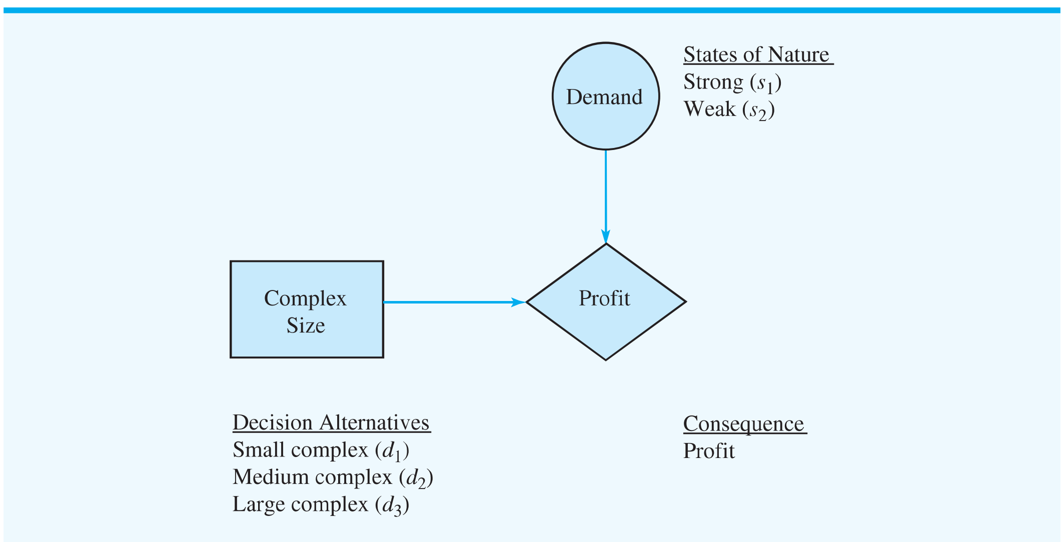


TABLE 4.1 PAYOFF TABLE FOR THE PDC CONDOMINIUM PROJECT (PAYOFFS IN $ MILLIONS)

	State of Nature	
Decision Alternative	**Strong Demand s_1**	**Weak Demand s_2**
Small complex, d_1	8	7
Medium complex, d_2	14	5
Large complex, d_3	20	−9

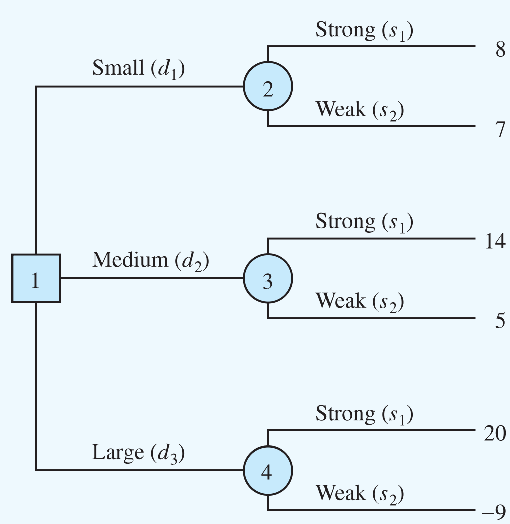
Decision Trees

A **decision tree** provides a graphical representation of the decision-making process. Figure 4.2 presents a decision tree for the PDC problem. Note that the decision tree shows the natural or logical progression that will occur over time. First, PDC must make a decision regarding the size of the condominium complex (d_1, d_2, or d_3). Then, after the decision is implemented, either state of nature s_1 or s_2 will occur. The number at each endpoint of the tree indicates the payoff associated with a particular sequence. For example, the topmost payoff of 8 indicates that an $8 million profit is anticipated if PDC constructs a small condominium complex (d_1) and demand turns out to be strong (s_1). The next payoff of 7 indicates an anticipated profit of $7 million if PDC constructs a small condominium complex (d_1) and demand turns out to be weak (s_2). Thus, the decision tree provides a graphical depiction of the sequences of decision alternatives and states of nature that provide the six possible payoffs for PDC.

If you have a payoff table, you can develop a decision tree. Try Problem 1, part (a).

The decision tree in Figure 4.2 shows four nodes, numbered 1–4. Squares are used to depict decision nodes and circles are used to depict chance nodes. Thus, node 1 is a decision node, and nodes 2, 3, and 4 are chance nodes. The **branches** connect the nodes; those

FIGURE 4.2 DECISION TREE FOR THE PDC CONDOMINIUM PROJECT (PAYOFFS IN $ MILLIONS)



leaving the decision node correspond to the decision alternatives. The branches leaving each chance node correspond to the states of nature. The payoffs are shown at the end of the states-of-nature branches. We now turn to the question: How can the decision maker use the information in the payoff table or the decision tree to select the best decision alternative? Several approaches may be used.

NOTES AND COMMENTS

1. The first step in solving a complex problem is to decompose the problem into a series of smaller subproblems. Decision trees provide a useful way to decompose a problem and illustrate the sequential nature of the decision process.
2. People often view the same problem from different perspectives. Thus, the discussion regarding the development of a decision tree may provide additional insight about the problem.

4.2 Decision Making Without Probabilities

In this section we consider approaches to decision making that do not require knowledge of the probabilities of the states of nature. These approaches are appropriate in situations in which the decision maker has little confidence in his or her ability to assess the probabilities, or in which a simple best-case and worst-case analysis is desirable. Because different approaches sometimes lead to different decision recommendations, the decision maker must understand the approaches available and then select the specific approach that, according to the judgment of the decision maker, is the most appropriate.

Many people think of a good decision as one in which the consequence is good. However, in some instances, a good, well-thought-out decision may still lead to a bad or undesirable consequence while a poor, ill-conceived decision may still lead to a good or desirable consequence.

Optimistic Approach

The **optimistic approach** evaluates each decision alternative in terms of the *best* payoff that can occur. The decision alternative that is recommended is the one that provides the best possible payoff. For a problem in which maximum profit is desired, as in the PDC problem, the optimistic approach would lead the decision maker to choose the alternative corresponding to the largest profit. For problems involving minimization, this approach leads to choosing the alternative with the smallest payoff.

For a maximization problem, the optimistic approach often is referred to as the maximax approach; for a minimization problem, the corresponding terminology is minimin.

To illustrate the optimistic approach, we use it to develop a recommendation for the PDC problem. First, we determine the maximum payoff for each decision alternative; then we select the decision alternative that provides the overall maximum payoff. These steps systematically identify the decision alternative that provides the largest possible profit. Table 4.2 illustrates these steps.

TABLE 4.2 MAXIMUM PAYOFF FOR EACH PDC DECISION ALTERNATIVE

Decision Alternative	Maximum Payoff	
Small complex, d_1	8	
Medium complex, d_2	14	
Large complex, d_3	20	← Maximum of the maximum payoff values

Because 20, corresponding to d_3, is the largest payoff, the decision to construct the large condominium complex is the recommended decision alternative using the optimistic approach.

Conservative Approach

For a maximization problem, the conservative approach is often referred to as the maximin approach; for a minimization problem, the corresponding terminology is minimax.

The **conservative approach** evaluates each decision alternative in terms of the *worst* payoff that can occur. The decision alternative recommended is the one that provides the best of the worst possible payoffs. For a problem in which the output measure is profit, as in the PDC problem, the conservative approach would lead the decision maker to choose the alternative that maximizes the minimum possible profit that could be obtained. For problems involving minimization, this approach identifies the alternative that will minimize the maximum payoff.

To illustrate the conservative approach, we use it to develop a recommendation for the PDC problem. First, we identify the minimum payoff for each of the decision alternatives; then we select the decision alternative that maximizes the minimum payoff. Table 4.3 illustrates these steps for the PDC problem.

Because 7, corresponding to d_1, yields the maximum of the minimum payoffs, the decision alternative of a small condominium complex is recommended. This decision approach is considered conservative because it identifies the worst possible payoffs and then recommends the decision alternative that avoids the possibility of extremely "bad" payoffs. In the conservative approach, PDC is guaranteed a profit of at least $7 million. Although PDC may make more, it *cannot* make less than $7 million.

Minimax Regret Approach

In decision analysis, **regret** is the difference between the payoff associated with a particular decision alternative and the payoff associated with the decision that would yield the most desirable payoff for a given state of nature. Thus, regret represents how much potential payoff one would forgo by selecting a particular decision alternative given that a specific state of nature will occur. This is why regret is often referred to as **opportunity loss**.

As its name implies, under the **minimax regret approach** to decision making one would choose the decision alternative that minimizes the maximum state of regret that could occur over all possible states of nature. This approach is neither purely optimistic nor purely conservative. Let us illustrate the minimax regret approach by showing how it can be used to select a decision alternative for the PDC problem.

Suppose that PDC constructs a small condominium complex (d_1) and demand turns out to be strong (s_1). Table 4.1 showed that the resulting profit for PDC would be $8 million. However, given that the strong demand state of nature (s_1) has occurred, we realize

TABLE 4.3 MINIMUM PAYOFF FOR EACH PDC DECISION ALTERNATIVE

Decision Alternative	Minimum Payoff	
Small complex, d_1	7	← Maximum of the minimum payoff values
Medium complex, d_2	5	
Large complex, d_3	−9	

TABLE 4.4 OPPORTUNITY LOSS, OR REGRET, TABLE FOR THE PDC CONDOMINIUM PROJECT ($ MILLIONS)

	State of Nature	
Decision Alternative	**Strong Demand s_1**	**Weak Demand s_2**
Small complex, d_1	12	0
Medium complex, d_2	6	2
Large complex, d_3	0	16

that the decision to construct a large condominium complex (d_3), yielding a profit of $20 million, would have been the best decision. The difference between the payoff for the best decision alternative ($20 million) and the payoff for the decision to construct a small condominium complex ($8 million) is the regret or opportunity loss associated with decision alternative d_1 when state of nature s_1 occurs; thus, for this case, the opportunity loss or regret is $20 million − $8 million = $12 million. Similarly, if PDC makes the decision to construct a medium condominium complex (d_2) and the strong demand state of nature (s_1) occurs, the opportunity loss, or regret, associated with d_2 would be $20 million − $14 million = $6 million.

In general, the following expression represents the opportunity loss, or regret:

$$R_{ij} = |V_j^* - V_{ij}| \tag{4.1}$$

where

R_{ij} = the regret associated with decision alternative d_i and state of nature s_j
V_j^* = the payoff value[1] corresponding to the best decision for the state of nature s_j
V_{ij} = the payoff corresponding to decision alternative d_i and state of nature s_j

Note the role of the absolute value in equation (4.1). For minimization problems, the best payoff, V_j^*, is the smallest entry in column j. Because this value always is less than or equal to V_{ij}, the absolute value of the difference between V_j^* and V_{ij} ensures that the regret is always the magnitude of the difference.

Using equation (4.1) and the payoffs in Table 4.1, we can compute the regret associated with each combination of decision alternative d_i and state of nature s_j. Because the PDC problem is a maximization problem, V_j^* will be the largest entry in column j of the payoff table. Thus, to compute the regret, we simply subtract each entry in a column from the largest entry in the column. Table 4.4 shows the opportunity loss, or regret, table for the PDC problem.

The next step in applying the minimax regret approach is to list the maximum regret for each decision alternative; Table 4.5 shows the results for the PDC problem. Selecting the decision alternative with the *minimum* of the *maximum* regret values—hence, the name *minimax regret*—yields the minimax regret decision. For the PDC problem, the alternative to construct the medium condominium complex, with a corresponding maximum regret of $6 million, is the recommended minimax regret decision.

[1]In maximization problems, V_j^* will be the largest entry in column j of the payoff table. In minimization problems, V_j^* will be the smallest entry in column j of the payoff table.

TABLE 4.5 MAXIMUM REGRET FOR EACH PDC DECISION ALTERNATIVE

Decision Alternative	Maximum Regret	
Small complex, d_1	12	
Medium complex, d_2	6	← Minimum of the maximum regret
Large complex, d_3	16	

For practice in developing a decision recommendation using the optimistic, conservative, and minimax regret approaches, try Problem 1, part (b).

Note that the three approaches discussed in this section provide different recommendations, which in itself isn't bad. It simply reflects the difference in decision-making philosophies that underlie the various approaches. Ultimately, the decision maker will have to choose the most appropriate approach and then make the final decision accordingly. The main criticism of the approaches discussed in this section is that they do not consider any information about the probabilities of the various states of nature. In the next section we discuss an approach that utilizes probability information in selecting a decision alternative.

4.3 Decision Making With Probabilities

In many decision-making situations, we can obtain probability assessments for the states of nature. When such probabilities are available, we can use the **expected value approach** to identify the best decision alternative. Let us first define the expected value of a decision alternative and then apply it to the PDC problem.

Let

$$N = \text{the number of states of nature}$$
$$P(s_j) = \text{the probability of state of nature } s_j$$

Because one and only one of the N states of nature can occur, the probabilities must satisfy two conditions:

$$P(s_j) \geq 0 \qquad \text{for all states of nature} \tag{4.2}$$

$$\sum_{j=1}^{N} P(s_j) = P(s_1) + P(s_2) + \cdots + P(s_N) = 1 \tag{4.3}$$

The **expected value (EV)** of decision alternative d_i is defined as follows:

$$\text{EV}(d_i) = \sum_{j=1}^{N} P(s_j)V_{ij} \tag{4.4}$$

In words, the expected value of a decision alternative is the sum of weighted payoffs for the decision alternative. The weight for a payoff is the probability of the associated state of nature and therefore the probability that the payoff will occur. Let us return to the PDC problem to see how the expected value approach can be applied.

PDC is optimistic about the potential for the luxury high-rise condominium complex. Suppose that this optimism leads to an initial subjective probability assessment of 0.8 that demand will be strong (s_1) and a corresponding probability of 0.2 that demand will be weak (s_2).

Can you now use the expected value approach to develop a decision recommendation? Try Problem 5.

Thus, $P(s_1) = 0.8$ and $P(s_2) = 0.2$. Using the payoff values in Table 4.1 and equation (4.4), we compute the expected value for each of the three decision alternatives as follows:

$$\text{EV}(d_1) = 0.8(8) + 0.2(7) = 7.8$$
$$\text{EV}(d_2) = 0.8(14) + 0.2(5) = 12.2$$
$$\text{EV}(d_3) = 0.8(20) + 0.2(-9) = 14.2$$

Thus, using the expected value approach, we find that the large condominium complex, with an expected value of \$14.2 million, is the recommended decision.

The calculations required to identify the decision alternative with the best expected value can be conveniently carried out on a decision tree. Figure 4.3 shows the decision tree for the PDC problem with state-of-nature branch probabilities. Working backward through the decision tree, we first compute the expected value at each chance node. That is, at each chance node, we weight each possible payoff by its probability of occurrence. By doing so, we obtain the expected values for nodes 2, 3, and 4, as shown in Figure 4.4.

Computer packages are available to help in constructing more complex decision trees. See Appendix 4.1.

Because the decision maker controls the branch leaving decision node 1 and because we are trying to maximize the expected profit, the best decision alternative at node 1 is d_3. Thus, the decision tree analysis leads to a recommendation of d_3, with an expected value of \$14.2 million. Note that this recommendation is also obtained with the expected value approach in conjunction with the payoff table.

Other decision problems may be substantially more complex than the PDC problem, but if a reasonable number of decision alternatives and states of nature are present, you can use the decision tree approach outlined here. First, draw a decision tree consisting of decision nodes, chance nodes, and branches that describe the sequential nature of the problem. If you use the expected value approach, the next step is to determine the probabilities for each of the states of nature and compute the expected value at each chance node. Then select the decision branch leading to the chance node with the best expected value. The decision alternative associated with this branch is the recommended decision.

The Q.M. in Action, Early Detection of High-Risk Worker Disability Claims, describes how the Workers' Compensation Board of British Columbia used a decision tree and expected cost to help determine whether a short-term disability claim should be considered a high-risk or a low-risk claim.

FIGURE 4.3 PDC DECISION TREE WITH STATE-OF-NATURE BRANCH PROBABILITIES

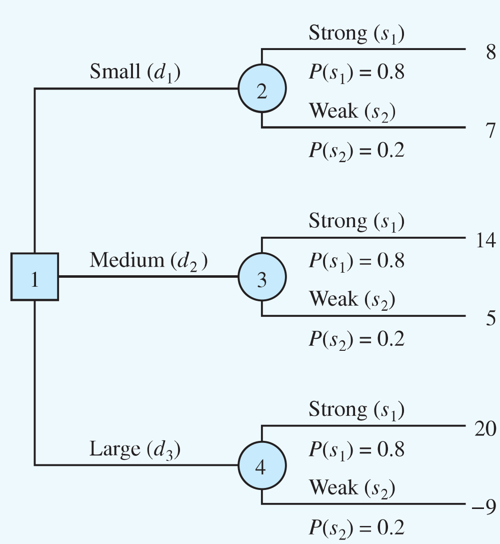
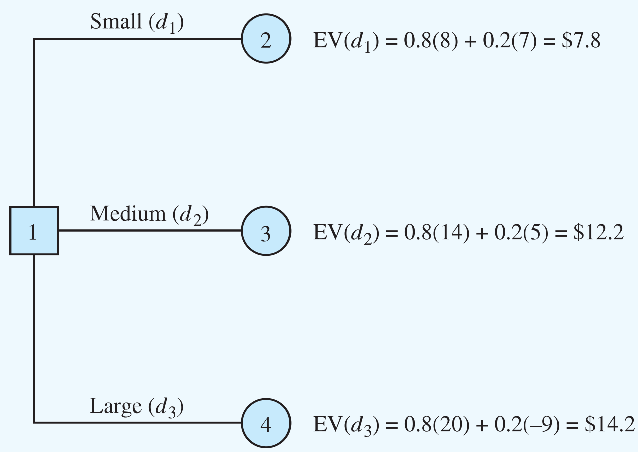


FIGURE 4.4 APPLYING THE EXPECTED VALUE APPROACH USING A DECISION TREE



Q.M. *in* ACTION

*EARLY DETECTION OF HIGH-RISK WORKER DISABILITY CLAIMS**

The Workers' Compensation Board of British Columbia (WCB) helps workers and employers maintain safe workplaces and helps injured workers obtain disability income and return to work safely. The funds used to make the disability compensation payments are obtained from assessments levied on employers. In return, employers receive protection from lawsuits arising from work-related injuries. In recent years, the WCB spent more than $1 billion on worker compensation and rehabilitation.

A short-term disability claim occurs when a worker suffers an injury or illness that results in temporary absence from work. Whenever a worker fails to recover completely from a short-term disability, the claim is reclassified as a long-term disability claim, and more expensive long-term benefits are paid.

The WCB wanted a systematic way to identify short-term disability claims that posed a high financial risk of being converted to the more expensive long-term disability claims. If a short-term disability claim could be classified as high risk early in the process, a WCB management team could intervene and monitor the claim and the recovery process more closely. As a result, WCB could improve the management of the high-risk claims and reduce the cost of any subsequent long-term disability claims.

The WCB used a decision analysis approach to classify each new short-term disability claim as being either a high-risk claim or a low-risk claim. A decision tree consisting of two decision nodes and two states-of-nature nodes was developed. The two decision alternatives were: (1) Classify the new short-term claim as high-risk and intervene; (2) classify the new short-term claim as low-risk and do not intervene. The two states of nature were: (1) The short-term claim converts to a long-term claim; (2) the short-term claim does not convert to a long-term claim. The characteristics of each new short-term claim were used to determine the probabilities for the states of nature. The payoffs were the disability claim costs associated with each decision alternative and each state-of-nature outcome. The objective of minimizing the expected cost determined whether a new short-term claim should be classified as high risk.

Implementation of the decision analysis model improved the practice of claim management for the Workers' Compensation Board. Early intervention on the high-risk claims saved an estimated $4.7 million per year.

*Based on E. Urbanovich, E. Young, M. Puterman, and S. Fattedad, "Early Detection of High-Risk Claims at the Workers' Compensation Board of British Columbia," *Interfaces* (July/August 2003): 15–26.

Expected Value of Perfect Information

Suppose that PDC has the opportunity to conduct a market research study that would help evaluate buyer interest in the condominium project and provide information that management could use to improve the probability assessments for the states of nature. To determine the potential value of this information, we begin by supposing that the study could provide *perfect information* regarding the states of nature; that is, we assume for the moment that PDC could determine with certainty, prior to making a decision, which state of nature is going to occur. To make use of this perfect information, we will develop a decision strategy that PDC should follow once it knows which state of nature will occur. A decision strategy is simply a decision rule that specifies the decision alternative to be selected after new information becomes available.

To help determine the decision strategy for PDC, we reproduced PDC's payoff table as Table 4.6. Note that, if PDC knew for sure that state of nature s_1 would occur, the best decision alternative would be d_3, with a payoff of \$20 million. Similarly, if PDC knew for sure that state of nature s_2 would occur, the best decision alternative would be d_1, with a payoff of \$7 million. Thus, we can state PDC's optimal decision strategy when the perfect information becomes available as follows:

If s_1, select d_3 and receive a payoff of \$20 million.
If s_2, select d_1 and receive a payoff of \$7 million.

What is the expected value for this decision strategy? To compute the expected value with perfect information, we return to the original probabilities for the states of nature: $P(s_1) = 0.8$ and $P(s_2) = 0.2$. Thus, there is a 0.8 probability that the perfect information will indicate state of nature s_1, and the resulting decision alternative d_3 will provide a \$20 million profit. Similarly, with a 0.2 probability for state of nature s_2, the optimal decision alternative d_1 will provide a \$7 million profit. Thus, from equation (4.4) the expected value of the decision strategy that uses perfect information is $0.8(20) + 0.2(7) = 17.4$.

We refer to the expected value of \$17.4 million as the *expected value with perfect information* (EVwPI).

Earlier in this section we showed that the recommended decision using the expected value approach is decision alternative d_3, with an expected value of \$14.2 million. Because this decision recommendation and expected value computation were made without the benefit of perfect information, \$14.2 million is referred to as the *expected value without perfect information* (EVwoPI).

The expected value with perfect information is \$17.4 million, and the expected value without perfect information is \$14.2; therefore, the expected value of the perfect information (EVPI) is \$17.4 − \$14.2 = \$3.2 million. In other words, \$3.2 million represents the additional expected value that can be obtained if perfect information were available about the states of nature.

TABLE 4.6 PAYOFF TABLE FOR THE PDC CONDOMINIUM PROJECT (\$ MILLIONS)

	State of Nature	
Decision Alternative	**Strong Demand s_1**	**Weak Demand s_2**
Small complex, d_1	8	7
Medium complex, d_2	14	5
Large complex, d_3	20	−9

It would be worth $3.2 million for PDC to learn the level of market acceptance before selecting a decision alternative.

Generally speaking, a market research study will not provide "perfect" information; however, if the market research study is a good one, the information gathered might be worth a sizable portion of the $3.2 million. Given the EVPI of $3.2 million, PDC might seriously consider a market survey as a way to obtain more information about the states of nature.

In general, the **expected value of perfect information (EVPI)** is computed as follows:

$$\text{EVPI} = |\text{EVwPI} - \text{EVwoPI}| \tag{4.5}$$

where

EVPI = expected value of perfect information
EVwPI = expected value *with* perfect information about the states of nature
EVwoPI = expected value *without* perfect information about the states of nature

For practice in determining the expected value of perfect information, try Problem 14.

Note the role of the absolute value in equation (4.5). For minimization problems, the expected value with perfect information is always less than or equal to the expected value without perfect information. In this case, EVPI is the magnitude of the difference between EVwPI and EVwoPI, or the absolute value of the difference as shown in equation (4.5).

NOTES AND COMMENTS

1. We restate the *opportunity loss,* or *regret,* table for the PDC problem (see Table 4.4) as follows:

	State of Nature	
Decision	**Strong Demand** s_1	**Weak Demand** s_2
Small complex, d_1	12	0
Medium complex, d_2	6	2
Large complex, d_3	0	16

Using $P(s_1)$, $P(s_2)$, and the opportunity loss values, we can compute the *expected opportunity loss* (EOL) for each decision alternative. With $P(s_1) = 0.8$ and $P(s_2) = 0.2$, the expected opportunity loss for each of the three decision alternatives is

$$\text{EOL}(d_1) = 0.8(12) + 0.2(0) = 9.6$$
$$\text{EOL}(d_2) = 0.8(6) + 0.2(2) = 5.2$$
$$\text{EOL}(d_3) = 0.8(0) + 0.2(16) = 3.2$$

Regardless of whether the decision analysis involves maximization or minimization, the *minimum* expected opportunity loss always provides the best decision alternative. Thus, with $\text{EOL}(d_3) = 3.2$, d_3 is the recommended decision. In addition, the minimum expected opportunity loss always is *equal to the expected value of perfect information.* That is, EOL(best decision) = EVPI; for the PDC problem, this value is $3.2 million.

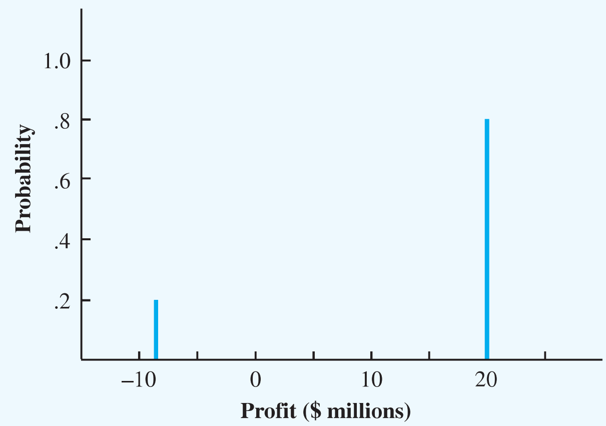
4.4 Risk Analysis and Sensitivity Analysis

Risk analysis helps the decision maker recognize the difference between the expected value of a decision alternative and the payoff that may actually occur. **Sensitivity analysis** also helps the decision maker by describing how changes in the state-of-nature probabilities and/or changes in the payoffs affect the recommended decision alternative.

Risk Analysis

A decision alternative and a state of nature combine to generate the payoff associated with a decision. The **risk profile** for a decision alternative shows the possible payoffs along with their associated probabilities.

FIGURE 4.5 RISK PROFILE FOR THE LARGE COMPLEX DECISION ALTERNATIVE FOR THE PDC CONDOMINIUM PROJECT



Let us demonstrate risk analysis and the construction of a risk profile by returning to the PDC condominium construction project. Using the expected value approach, we identified the large condominium complex (d_3) as the best decision alternative. The expected value of $14.2 million for d_3 is based on a 0.8 probability of obtaining a $20 million profit and a 0.2 probability of obtaining a $9 million loss. The 0.8 probability for the $20 million payoff and the 0.2 probability for the −$9 million payoff provide the risk profile for the large complex decision alternative. This risk profile is shown graphically in Figure 4.5.

Sometimes a review of the risk profile associated with an optimal decision alternative may cause the decision maker to choose another decision alternative even though the expected value of the other decision alternative is not as good. For example, the risk profile for the medium complex decision alternative (d_2) shows a 0.8 probability for a $14 million payoff and a 0.2 probability for a $5 million payoff. Because no probability of a loss is associated with decision alternative d_2, the medium complex decision alternative would be judged less risky than the large complex decision alternative. As a result, a decision maker might prefer the less risky medium complex decision alternative even though it has an expected value of $2 million less than the large complex decision alternative.

Sensitivity Analysis

Sensitivity analysis can be used to determine how changes in the probabilities for the states of nature or changes in the payoffs affect the recommended decision alternative. In many cases, the probabilities for the states of nature and the payoffs are based on subjective assessments. Sensitivity analysis helps the decision maker understand which of these inputs are critical to the choice of the best decision alternative. If a small change in the value of one of the inputs causes a change in the recommended decision alternative, the solution to the decision analysis problem is sensitive to that particular input. Extra effort and care should be taken to make sure the input value is as accurate as possible. On the other hand, if a modest-to-large change in the value of one of the inputs does not cause a change in the recommended decision alternative, the solution to the decision analysis problem is not sensitive to that particular input. No extra time or effort would be needed to refine the estimated input value.

One approach to sensitivity analysis is to select different values for the probabilities of the states of nature and the payoffs and then resolve the decision analysis problem. If the recommended decision alternative changes, we know that the solution is sensitive to the changes made. For example, suppose that in the PDC problem the probability for a strong demand is revised to 0.2 and the probability for a weak demand is revised to 0.8. Would the recommended decision alternative change? Using $P(s_1) = 0.2$, $P(s_2) = 0.8$, and equation (4.4), the revised expected values for the three decision alternatives are

$$\begin{aligned} \text{EV}(d_1) &= 0.2(8) + 0.8(7) = 7.2 \\ \text{EV}(d_2) &= 0.2(14) + 0.8(5) = 6.8 \\ \text{EV}(d_3) &= 0.2(20) + 0.8(-9) = -3.2 \end{aligned}$$

With these probability assessments, the recommended decision alternative is to construct a small condominium complex (d_1), with an expected value of \$7.2 million. The probability of strong demand is only 0.2, so constructing the large condominium complex (d_3) is the least preferred alternative, with an expected value of −\$3.2 million (a loss).

Thus, when the probability of strong demand is large, PDC should build the large complex; when the probability of strong demand is small, PDC should build the small complex. Obviously, we could continue to modify the probabilities of the states of nature and learn even more about how changes in the probabilities affect the recommended decision alternative. The drawback to this approach is the numerous calculations required to evaluate the effect of several possible changes in the state-of-nature probabilities.

Computer software packages for decision analysis make it easy to calculate these revised scenarios.

For the special case of two states of nature, a graphical procedure can be used to determine how changes for the probabilities of the states of nature affect the recommended decision alternative. To demonstrate this procedure, we let p denote the probability of state of nature s_1; that is, $P(s_1) = p$. With only two states of nature in the PDC problem, the probability of state of nature s_2 is

$$P(s_2) = 1 - P(s_1) = 1 - p$$

Using equation (4.4) and the payoff values in Table 4.1, we determine the expected value for decision alternative d_1 as follows:

$$\begin{aligned} \text{EV}(d_1) &= P(s_1)(8) + P(s_2)(7) \\ &= p(8) + (1 - p)(7) \\ &= 8p + 7 - 7p = p + 7 \end{aligned} \quad \textbf{(4.6)}$$

Repeating the expected value computations for decision alternatives d_2 and d_3, we obtain expressions for the expected value of each decision alternative as a function of p:

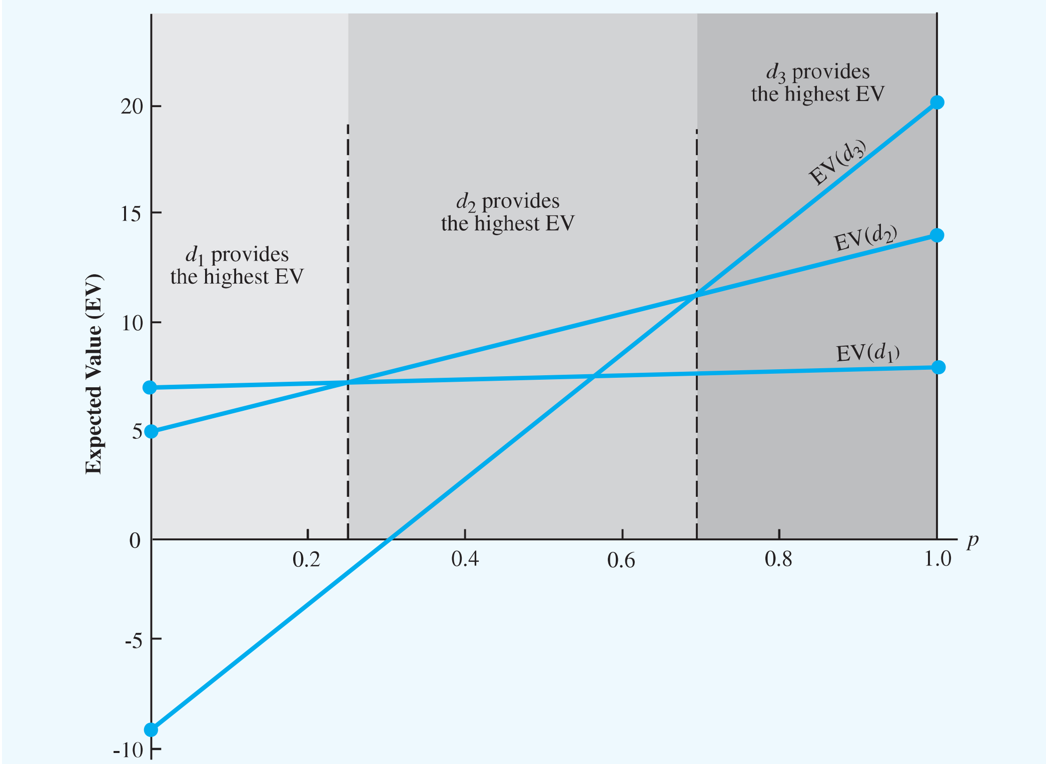
$$\text{EV}(d_2) = 9p + 5 \quad \textbf{(4.7)}$$

$$\text{EV}(d_3) = 29p - 9 \quad \textbf{(4.8)}$$

Thus, we have developed three equations that show the expected value of the three decision alternatives as a function of the probability of state of nature s_1.

We continue by developing a graph with values of p on the horizontal axis and the associated EVs on the vertical axis. Because equations (4.6), (4.7), and (4.8) are linear equations, the graph of each equation is a straight line. For each equation, we can obtain the line

FIGURE 4.6 EXPECTED VALUE FOR THE PDC DECISION ALTERNATIVES AS A FUNCTION OF p



by identifying two points that satisfy the equation and drawing a line through the points. For instance, if we let $p = 0$ in equation (4.6), EV(d_1) = 7. Then, letting $p = 1$, EV(d_1) = 8. Connecting these two points, (0,7) and (1,8), provides the line labeled EV(d_1) in Figure 4.6. Similarly, we obtain the lines labeled EV(d_2) and EV(d_3); these lines are the graphs of equations (4.7) and (4.8), respectively.

Figure 4.6 shows how the recommended decision changes as p, the probability of the strong demand state of nature (s_1), changes. Note that for small values of p, decision alternative d_1 (small complex) provides the largest expected value and is thus the recommended decision. When the value of p increases to a certain point, decision alternative d_2 (medium complex) provides the largest expected value and is the recommended decision. Finally, for large values of p, decision alternative d_3 (large complex) becomes the recommended decision.

The value of p for which the expected values of d_1 and d_2 are equal is the value of p corresponding to the intersection of the EV(d_1) and the EV(d_2) lines. To determine this value, we set EV(d_1) = EV(d_2) and solve for the value of p:

$$\begin{aligned} p + 7 &= 9p + 5 \\ 8p &= 2 \\ p &= \frac{2}{8} = 0.25 \end{aligned}$$

Hence, when $p = 0.25$, decision alternatives d_1 and d_2 provide the same expected value. Repeating this calculation for the value of p corresponding to the intersection of the EV(d_2) and EV(d_3) lines, we obtain $p = 0.70$.

Graphical sensitivity analysis shows how changes in the probabilities for the states of nature affect the recommended decision alternative. Try Problem 8.

Using Figure 4.6, we can conclude that decision alternative d_1 provides the largest expected value for $p \leq 0.25$, decision alternative d_2 provides the largest expected value for $0.25 \leq p \leq 0.70$, and decision alternative d_3 provides the largest expected value for $p \geq 0.70$. Because p is the probability of state of nature s_1 and $(1 - p)$ is the probability of state of nature s_2, we now have the sensitivity analysis information that tells us how changes in the state-of-nature probabilities affect the recommended decision alternative.

Sensitivity analysis calculations can also be made for the values of the payoffs. In the original PDC problem, the expected values for the three decision alternatives were as follows: EV(d_1) = 7.8, EV(d_2) = 12.2, and EV(d_3) = 14.2. Decision alternative d_3 (large complex) was recommended. Note that decision alternative d_2 with EV(d_2) = 12.2 was the second best decision alternative. Decision alternative d_3 will remain the optimal decision alternative as long as EV(d_3) is greater than or equal to the expected value of the second best decision alternative. Thus, decision alternative d_3 will remain the optimal decision alternative as long as

$$\text{EV}(d_3) \geq 12.2 \quad \textbf{(4.9)}$$

Let

S = the payoff of decision alternative d_3 when demand is strong
W = the payoff of decision alternative d_3 when demand is weak

Using $P(s_1) = 0.8$ and $P(s_2) = 0.2$, the general expression for EV(d_3) is

$$\text{EV}(d_3) = 0.8S + 0.2W \quad \textbf{(4.10)}$$

Assuming that the payoff for d_3 stays at its original value of −$9 million when demand is weak, the large complex decision alternative will remain optimal as long as

$$\text{EV}(d_3) = 0.8S + 0.2(-9) \geq 12.2 \quad \textbf{(4.11)}$$

Solving for S, we have

$$0.8S - 1.8 \geq 12.2$$
$$0.8S \geq 14$$
$$S \geq 17.5$$

Recall that when demand is strong, decision alternative d_3 has an estimated payoff of $20 million. The preceding calculation shows that decision alternative d_3 will remain optimal as long as the payoff for d_3 when demand is strong is at least $17.5 million.

Assuming that the payoff for d_3 when demand is strong stays at its original value of $20 million, we can make a similar calculation to learn how sensitive the optimal solution is with regard to the payoff for d_3 when demand is weak. Returning to the expected value calculation of equation (4.10), we know that the large complex decision alternative will remain optimal as long as

$$\text{EV}(d_3) = 0.8(20) + 0.2W \geq 12.2 \quad \textbf{(4.12)}$$

Solving for W, we have

$$16 + 0.2 \geq 12.2$$
$$0.2W \geq -3.8$$
$$W \geq -19$$

Recall that when demand is weak, decision alternative d_3 has an estimated payoff of $-\$9$ million. The preceding calculation shows that decision alternative d_3 will remain optimal as long as the payoff for d_3 when demand is weak is at least $-\$19$ million.

Based on this sensitivity analysis, we conclude that the payoffs for the large complex decision alternative (d_3) could vary considerably, and d_3 would remain the recommended decision alternative. Thus, we conclude that the optimal solution for the PDC decision problem is not particularly sensitive to the payoffs for the large complex decision alternative. We note, however, that this sensitivity analysis has been conducted based on only one change at a time. That is, only one payoff was changed and the probabilities for the states of nature remained $P(s_1) = 0.8$ and $P(s_2) = 0.2$. Note that similar sensitivity analysis calculations can be made for the payoffs associated with the small complex decision alternative d_1 and the medium complex decision alternative d_2. However, in these cases, decision alternative d_3 remains optimal only if the changes in the payoffs for decision alternatives d_1 and d_2 meet the requirements that $\text{EV}(d_1) \leq 14.2$ and $\text{EV}(d_2) \leq 14.2$.

Sensitivity analysis can assist management in deciding whether more time and effort should be spent obtaining better estimates of payoffs and probabilities.

NOTES AND COMMENTS

1. Some decision analysis software automatically provides the risk profiles for the optimal decision alternative. These packages also allow the user to obtain the risk profiles for other decision alternatives. After comparing the risk profiles, a decision maker may decide to select a decision alternative with a good risk profile even though the expected value of the decision alternative is not as good as the optimal decision alternative.
2. A *tornado diagram,* a graphical display, is particularly helpful when several inputs combine to determine the value of the optimal solution. By varying each input over its range of values, we obtain information about how each input affects the value of the optimal solution. To display this information, a bar is constructed for the input, with the width of the bar showing how the input affects the value of the optimal solution. The widest bar corresponds to the input that is most sensitive. The bars are arranged in a graph with the widest bar at the top, resulting in a graph that has the appearance of a tornado.

4.5 Decision Analysis with Sample Information

In applying the expected value approach, we showed how probability information about the states of nature affects the expected value calculations and thus the decision recommendation. Frequently, decision makers have preliminary or **prior probability** assessments for the states of nature that are the best probability values available at that time. However, to make the best possible decision, the decision maker may want to seek additional information about the states of nature. This new information can be used to revise or update the prior probabilities so that the final decision is based on more accurate probabilities for the states of nature. Most often, additional information is obtained through experiments designed to provide **sample information** about the states of nature. Raw material sampling, product testing, and market research studies are examples of experiments (or studies) that may enable management to revise or update the state-of-nature probabilities. These revised probabilities are called **posterior probabilities**.

Let us return to the PDC problem and assume that management is considering a 6-month market research study designed to learn more about potential market acceptance of the PDC condominium project. Management anticipates that the market research study will provide one of the following two results:

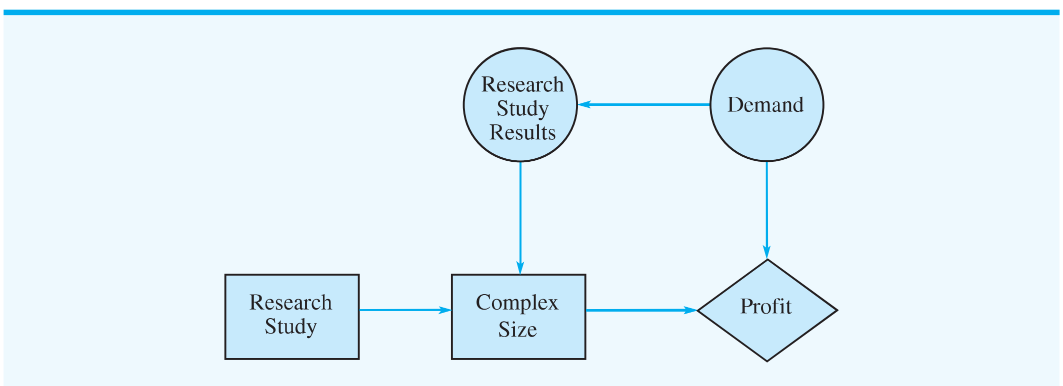
1. Favorable report: A substantial number of the individuals contacted express interest in purchasing a PDC condominium.
2. Unfavorable report: Very few of the individuals contacted express interest in purchasing a PDC condominium.

Influence Diagram

By introducing the possibility of conducting a market research study, the PDC problem becomes more complex. The influence diagram for the expanded PDC problem is shown in Figure 4.7. Note that the two decision nodes correspond to the research study and the complex-size decisions. The two chance nodes correspond to the research study results and demand for the condominiums. Finally, the consequence node is the profit. From the arcs of the influence diagram, we see that demand influences both the research study results and profit. Although demand is currently unknown to PDC, some level of demand for the condominiums already exists in the Pittsburgh area. If existing demand is strong, the research study is likely to find a substantial number of individuals who express an interest in purchasing a condominium. However, if the existing demand is weak, the research study is more likely to find a substantial number of individuals who express little interest in purchasing a condominium. In this sense, existing demand for the condominiums will influence the research study results, and clearly, demand will have an influence upon PDC's profit.

The arc from the research study decision node to the complex-size decision node indicates that the research study decision precedes the complex-size decision. No arc spans from the research study decision node to the research study results node because the decision to conduct the research study does not actually influence the research study results. The decision to conduct the research study makes the research study results available, but it does not influence the results of the research study. Finally, the complex-size node and

FIGURE 4.7 INFLUENCE DIAGRAM FOR THE PDC PROBLEM WITH SAMPLE INFORMATION



the demand node both influence profit. Note that if a stated cost to conduct the research study were given, the decision to conduct the research study would also influence profit. In such a case, we would need to add an arc from the research study decision node to the profit node to show the influence that the research study cost would have on profit.

Decision Tree

The decision tree for the PDC problem with sample information shows the logical sequence for the decisions and the chance events in Figure 4.8.

First, PDC's management must decide whether the market research should be conducted. If it is conducted, PDC's management must be prepared to make a decision about the size of the condominium project if the market research report is favorable and, possibly, a different decision about the size of the condominium project if the market research report is unfavorable. In Figure 4.8, the squares are decision nodes and the circles are chance nodes. At each decision node, the branch of the tree that is taken is based on the decision made. At each chance node, the branch of the tree that is taken is based on probability or chance. For example, decision node 1 shows that PDC must first make the decision of whether to conduct the market research study. If the market research study is undertaken, chance node 2 indicates that both the favorable report branch and the unfavorable report branch are not under PDC's control and will be determined by chance. Node 3 is a decision node, indicating that PDC must make the decision to construct the small, medium, or large complex if the market research report is favorable. Node 4 is a decision node showing that PDC must make the decision to construct the small, medium, or large complex if the market research report is unfavorable. Node 5 is a decision node indicating that PDC must make the decision to construct the small, medium, or large complex if the market research is not undertaken. Nodes 6 to 14 are chance nodes indicating that the strong demand or weak demand state-of-nature branches will be determined by chance.

Analysis of the decision tree and the choice of an optimal strategy require that we know the branch probabilities corresponding to all chance nodes. PDC has developed the following branch probabilities:

If the market research study is undertaken

We explain in Section 4.6 how the branch probabilities for P(Favorable report) and P(Unfavorable report) can be developed.

$$P(\text{Favorable report}) = 0.77$$
$$P(\text{Unfavorable report}) = 0.23$$

If the market research report is favorable

$$P(\text{Strong demand given a favorable report}) = 0.94$$
$$P(\text{Weak demand given a favorable report}) = 0.06$$

If the market research report is unfavorable

$$P(\text{Strong demand given a favorable report}) = 0.35$$
$$P(\text{Weak demand given a favorable report}) = 0.65$$

If the market research report is not undertaken, the prior probabilities are applicable.

$$P(\text{Strong demand}) = 0.80$$
$$P(\text{Weak demand}) = 0.20$$

The branch probabilities are shown on the decision tree in Figure 4.9.

FIGURE 4.8 THE PDC DECISION TREE INCLUDING THE MARKET RESEARCH STUDY

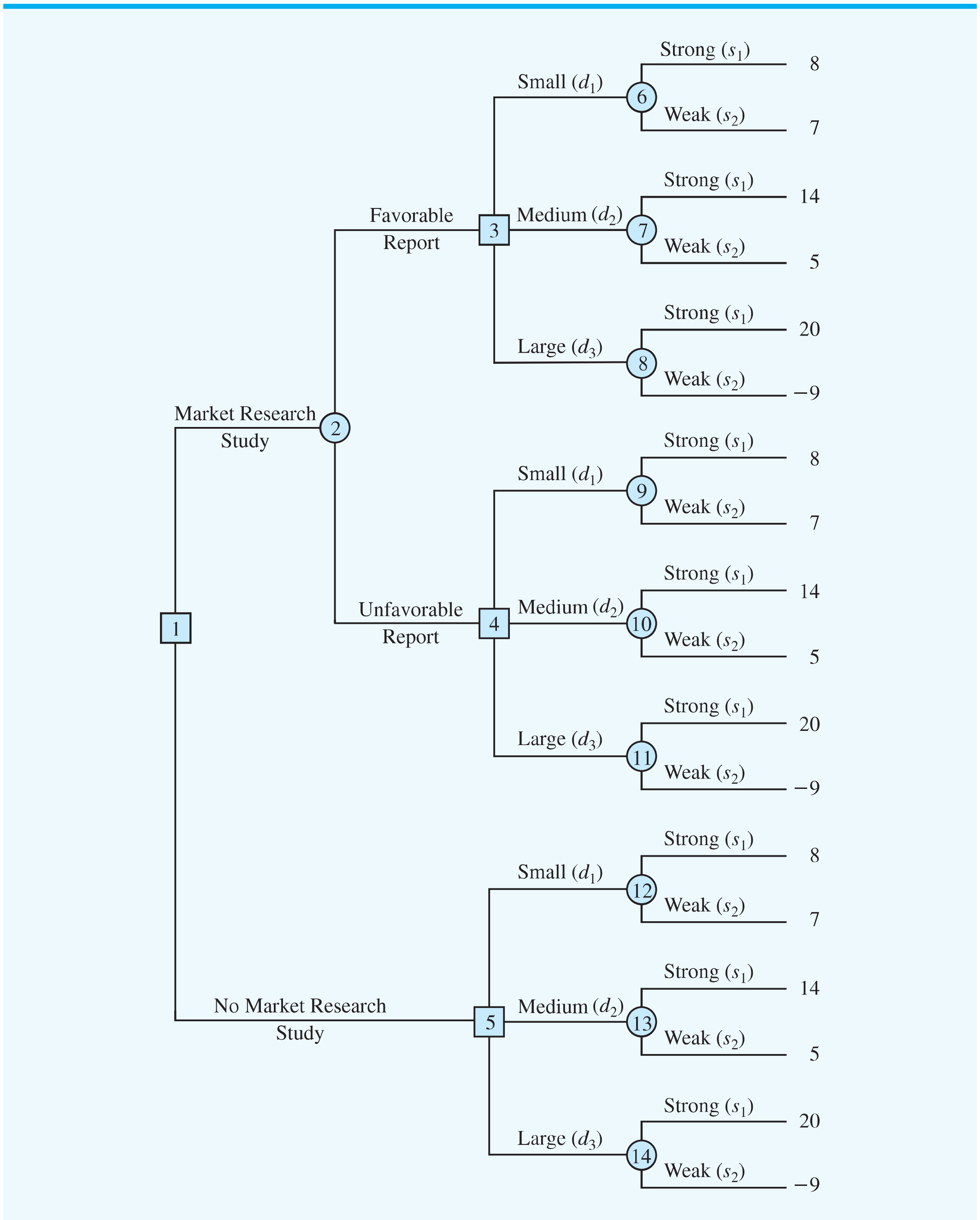
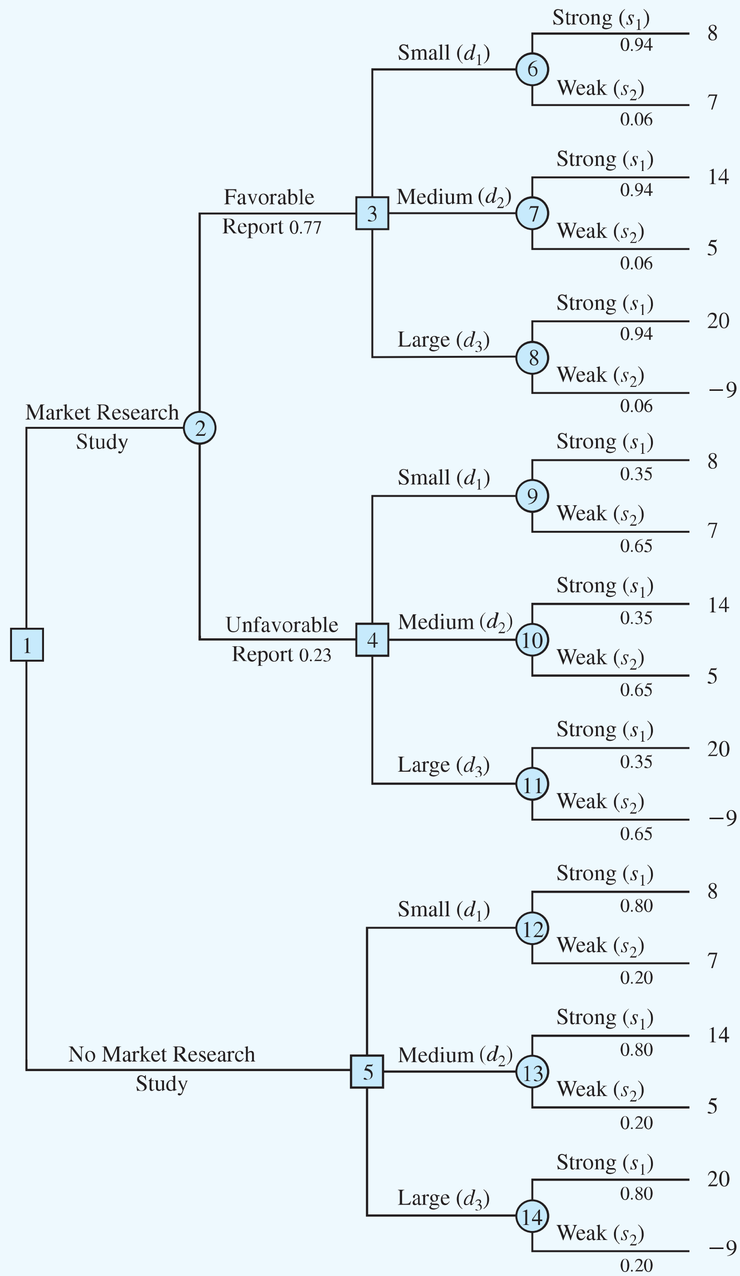


FIGURE 4.9 THE PDC DECISION TREE WITH BRANCH PROBABILITIES



Decision Strategy

A **decision strategy** is a sequence of decisions and chance outcomes where the decisions chosen depend on the yet-to-be-determined outcomes of chance events.

The approach used to determine the optimal decision strategy is based on a backward pass through the decision tree using the following steps:

1. At chance nodes, compute the expected value by multiplying the payoff at the end of each branch by the corresponding branch probabilities.
2. At decision nodes, select the decision branch that leads to the best expected value. This expected value becomes the expected value at the decision node.

Starting the backward pass calculations by computing the expected values at chance nodes 6 to 14 provides the following results:

$$
\begin{aligned}
\text{EV(Node 6)} &= 0.94(8) + 0.06(7) &&= 7.94 \\
\text{EV(Node 7)} &= 0.94(14) + 0.06(5) &&= 13.46 \\
\text{EV(Node 8)} &= 0.94(20) + 0.06(-9) &&= 18.26 \\
\text{EV(Node 9)} &= 0.35(8) + 0.65(7) &&= 7.35 \\
\text{EV(Node 10)} &= 0.35(14) + 0.65(5) &&= 8.15 \\
\text{EV(Node 11)} &= 0.35(20) + 0.65(-9) &&= 1.15 \\
\text{EV(Node 12)} &= 0.80(8) + 0.20(7) &&= 7.80 \\
\text{EV(Node 13)} &= 0.80(14) + 0.20(5) &&= 12.20 \\
\text{EV(Node 14)} &= 0.80(20) + 0.20(-9) &&= 14.20
\end{aligned}
$$

Figure 4.10 shows the reduced decision tree after computing expected values at these chance nodes.

Next, move to decision nodes 3, 4, and 5. For each of these nodes, we select the decision alternative branch that leads to the best expected value. For example, at node 3 we have the choice of the small complex branch with EV(Node 6) = 7.94, the medium complex branch with EV(Node 7) = 13.46, and the large complex branch with EV(Node 8) = 18.26. Thus, we select the large complex decision alternative branch and the expected value at node 3 becomes EV(Node 3) = 18.26.

For node 4, we select the best expected value from nodes 9, 10, and 11. The best decision alternative is the medium complex branch that provides EV(Node 4) = 8.15. For node 5, we select the best expected value from nodes 12, 13, and 14. The best decision alternative is the large complex branch that provides EV(Node 5) = 14.20. Figure 4.11 shows the reduced decision tree after choosing the best decisions at nodes 3, 4, and 5.

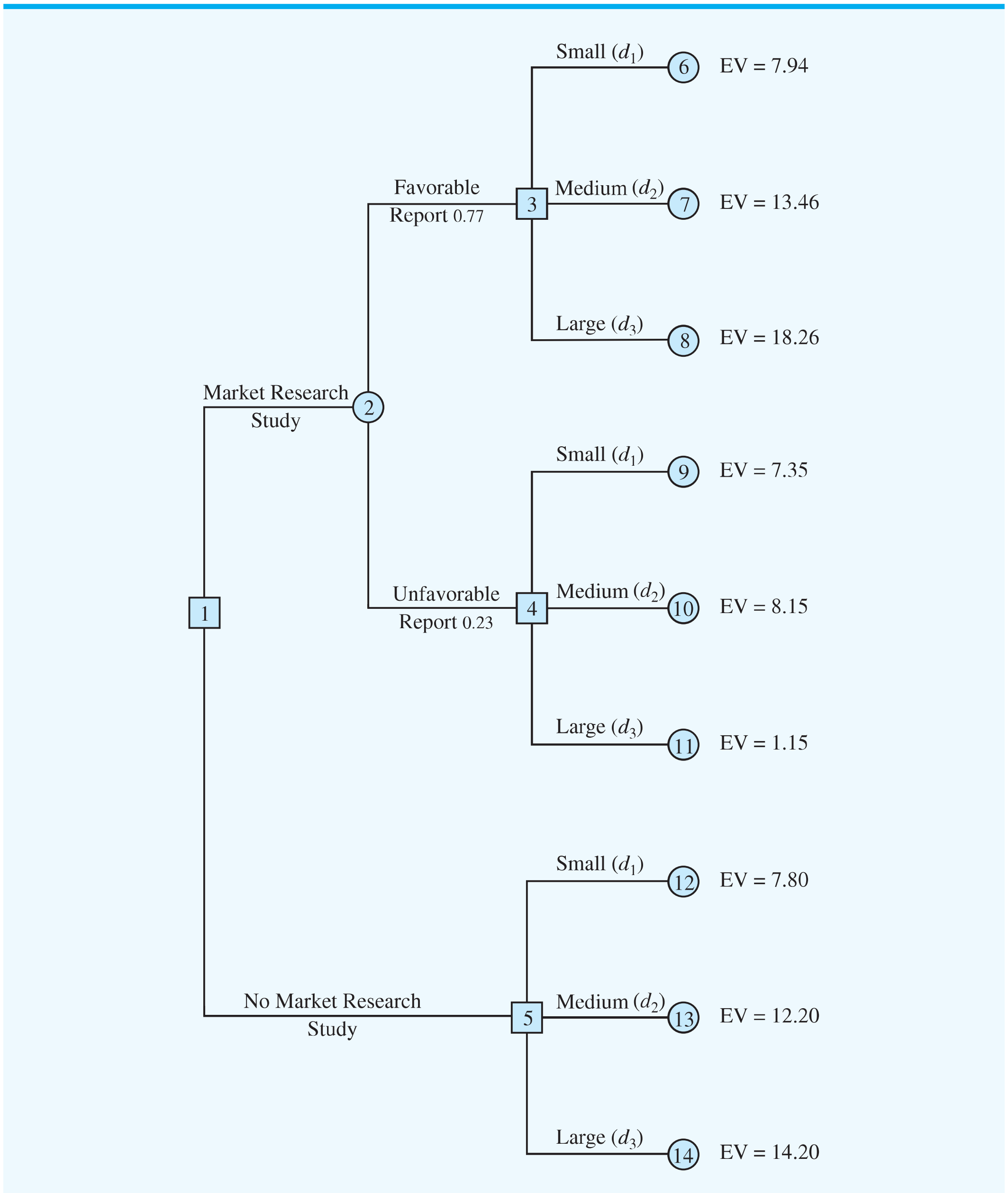
The expected value at chance node 2 can now be computed as follows:

$$
\begin{aligned}
\text{EV(Node 2)} &= 0.77\text{EV(Node 3)} + 0.23\text{EV(Node 4)} \\
&= 0.77(18.26) + 0.23(8.15) = 15.93
\end{aligned}
$$

This calculation reduces the decision tree to one involving only the two decision branches from node 1 (see Figure 4.12).

Finally, the decision can be made at decision node 1 by selecting the best expected values from nodes 2 and 5. This action leads to the decision alternative to conduct the market research study, which provides an overall expected value of 15.93.

FIGURE 4.10 PDC DECISION TREE AFTER COMPUTING EXPECTED VALUES AT CHANCE NODES 6 TO 14



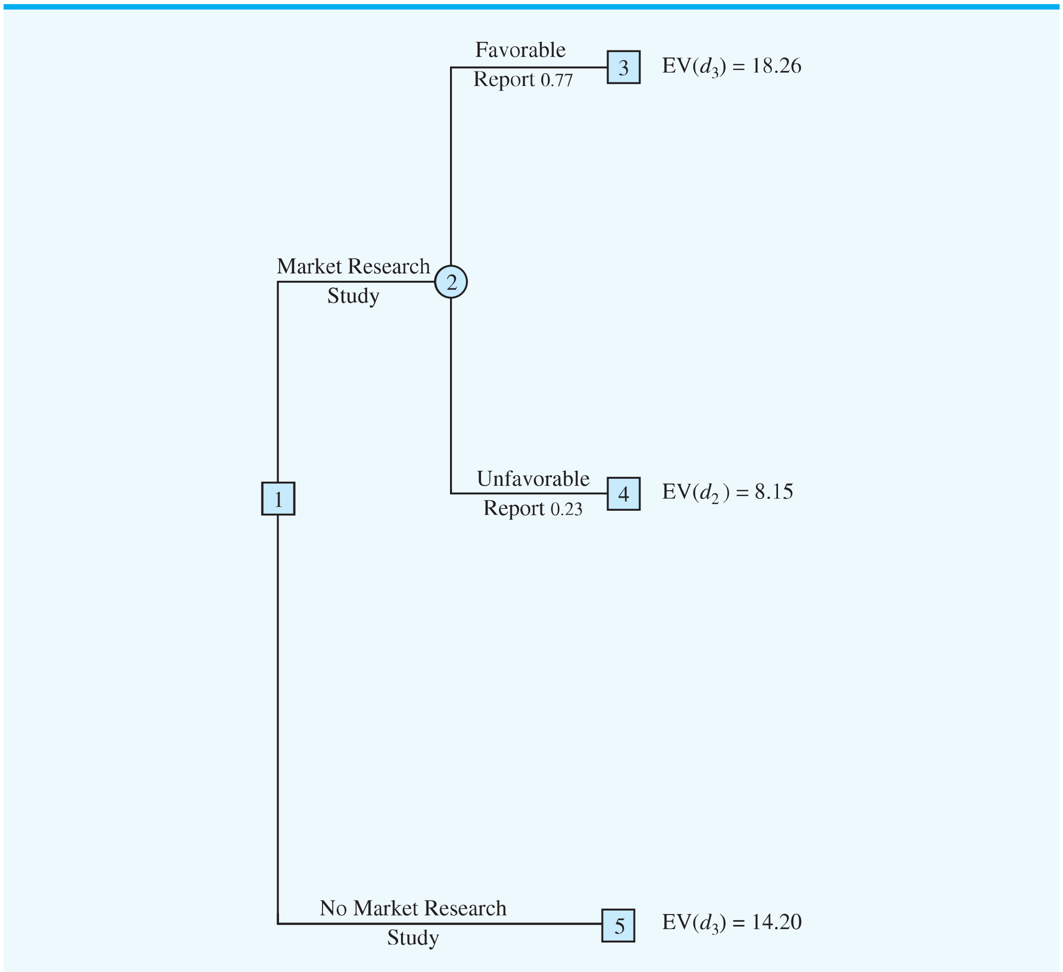
Problem 16 will test your ability to develop an optimal decision strategy.

The optimal decision for PDC is to conduct the market research study and then carry out the following decision strategy:

If the market research is favorable, construct the large condominium complex.

If the market research is unfavorable, construct the medium condominium complex.

FIGURE 4.11 PDC DECISION TREE AFTER CHOOSING BEST DECISIONS AT NODES 3, 4, AND 5



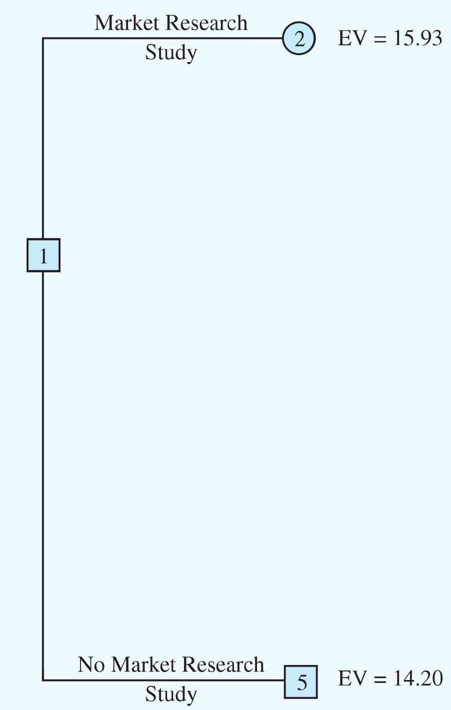
The analysis of the PDC decision tree describes the methods that can be used to analyze more complex sequential decision problems. First, draw a decision tree consisting of decision and chance nodes and branches that describe the sequential nature of the problem. Determine the probabilities for all chance outcomes. Then, by working backward through the tree, compute expected values at all chance nodes and select the best decision branch at all decision nodes. The sequence of optimal decision branches determines the optimal decision strategy for the problem.

The Q.M. in Action, New Drug Decision Analysis at Bayer Pharmaceuticals, describes how an extension of the decision analysis principles presented in this section enabled Bayer to make decisions about the development and marketing of a new drug.

Risk Profile

Figure 4.13 provides a reduced decision tree showing only the sequence of decision alternatives and chance events for the PDC optimal decision strategy. By implementing the

FIGURE 4.12 PDC DECISION TREE REDUCED TO TWO DECISION BRANCHES



Q.M. *in* ACTION

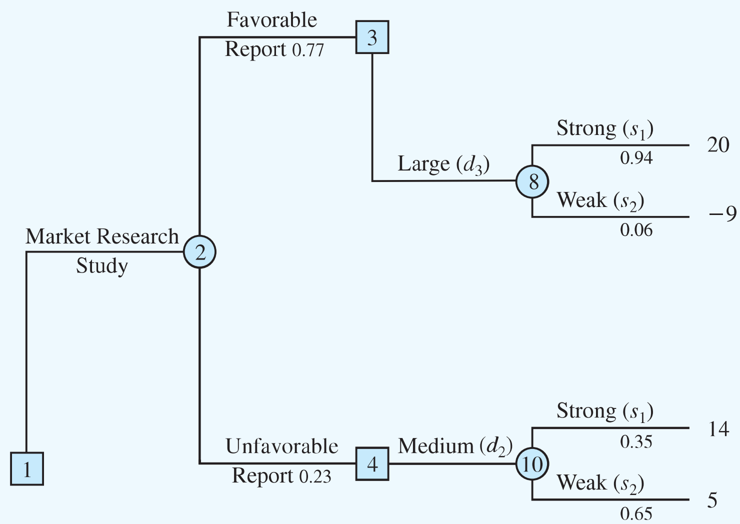
*NEW DRUG DECISION ANALYSIS AT BAYER PHARMACEUTICALS**

Drug development in the United States requires substantial investment and is very risky. It takes nearly 15 years to research and develop a new drug. The Bayer Biological Products (BP) group used decision analysis to evaluate the potential for a new blood-clot busting drug. An influence diagram was used to describe the complex structure of the decision analysis process. Six key yes-or-no decision nodes were identified: (1) begin preclinical development; (2) begin testing in humans; (3) continue development into phase 3; (4) continue development into phase 4; (5) file a license application with the FDA; and (6) launch the new drug into the marketplace. More than 50 chance nodes appeared in the influence diagram. The chance nodes showed how uncertainties—related to factors such as direct labor costs, process development costs, market share, tax rate, and pricing—affected the outcome. Net present value provided the consequence and the decision-making criterion.

Probability assessments were made concerning both the technical risk and market risk at each stage of the process. The resulting sequential decision tree had 1955 possible paths that led to different net present value outcomes. Cost inputs, judgments of potential outcomes, and the assignment of probabilities helped evaluate the project's potential contribution. Sensitivity analysis was used to identify key variables that would require special attention by the project team and management during the drug development process. Application of decision analysis principles allowed Bayer to make good decisions about how to develop and market the new drug.

*Based on Jeffrey S. Stonebraker, "How Bayer Makes Decisions to Develop New Drugs," *Interfaces* no. 6 (November/December 2002): 77–90.

FIGURE 4.13 PDC DECISION TREE SHOWING ONLY BRANCHES ASSOCIATED WITH OPTIMAL DECISION STRATEGY



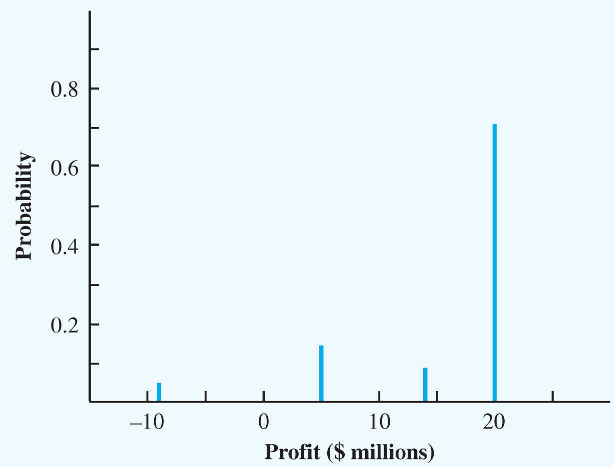
optimal decision strategy, PDC will obtain one of the four payoffs shown at the terminal branches of the decision tree. Recall that a risk profile shows the possible payoffs with their associated probabilities. Thus, in order to construct a risk profile for the optimal decision strategy, we will need to compute the probability for each of the four payoffs.

Note that each payoff results from a sequence of branches leading from node 1 to the payoff. For instance, the payoff of $20 million is obtained by following the upper branch from node 1, the upper branch from node 2, the lower branch from node 3, and the upper branch from node 8. The probability of following that sequence of branches can be found by multiplying the probabilities for the branches from the chance nodes in the sequence. Thus, the probability of the $20 million payoff is (0.77)(0.94) = 0.72. Similarly, the probabilities for each of the other payoffs are obtained by multiplying the probabilities for the branches from the chance nodes leading to the payoffs. By doing so, we find the probability of the −$9 million payoff is (0.77)(0.06) = 0.05; the probability of the $14 million payoff is (0.23)(0.35) = 0.08; and the probability of the $5 million payoff is (0.23)(0.65) = 0.15. The following table showing the probability distribution for the payoffs for the PDC optimal decision strategy is the tabular representation of the risk profile for the optimal decision strategy.

Payoff ($ millions)	Probability
−9	0.05
5	0.15
14	0.08
20	0.72
	1.00

Figure 4.14 provides a graphical representation of the risk profile. Comparing Figures 4.5 and 4.14, we see that the PDC risk profile is changed by the strategy to conduct the

FIGURE 4.14 RISK PROFILE FOR PDC CONDOMINIUM PROJECT WITH SAMPLE INFORMATION SHOWING PAYOFFS ASSOCIATED WITH OPTIMAL DECISION STRATEGY



market research study. In fact, the use of the market research study lowered the probability of the \$9 million loss from 0.20 to 0.05. PDC's management would most likely view that change as a considerable reduction in the risk associated with the condominium project.

Expected Value of Sample Information

In the PDC problem, the market research study is the sample information used to determine the optimal decision strategy. The expected value associated with the market research study is \$15.93. In Section 4.3 we showed that the best expected value if the market research study is *not* undertaken is \$14.20. Thus, we can conclude that the difference, \$15.93 − \$14.20 = \$1.73, is the **expected value of sample information (EVSI)**. In other words, conducting the market research study adds \$1.73 million to the PDC expected value. In general, the expected value of sample information is as follows:

$$\text{EVSI} = |\text{EVwSI} - \text{EVwoSI}| \tag{4.13}$$

where

EVSI = expected value of sample information
EVwSI = expected value *with* sample information about the states of nature
EVwoSI = expected value *without* sample information about the states of nature

The EVSI = \$1.73 million suggests PDC should be willing to pay up to \$1.73 million to conduct the market research study.

Note the role of the absolute value in equation (4.13). For minimization problems, the expected value with sample information is always less than or equal to the expected value without sample information. In this case, EVSI is the magnitude of the difference between EVwSI and EVwoSI; thus, by taking the absolute value of the difference as shown in equation (4.13), we can handle both the maximization and minimization cases with one equation.

Efficiency of Sample Information

In Section 4.3 we showed that the expected value of perfect information (EVPI) for the PDC problem is $3.2 million. We never anticipated that the market research report would obtain perfect information, but we can use an **efficiency** measure to express the value of the market research information. With perfect information having an efficiency rating of 100%, the efficiency rating E for sample information is computed as follows:

$$E = \frac{\text{EVSI}}{\text{EVPI}} \times 100 \tag{4.14}$$

For the PDC problem,

$$E = \frac{1.73}{3.2} \times 100 = 54.1\%$$

In other words, the information from the market research study is 54.1% as efficient as perfect information.

Low efficiency ratings for sample information might lead the decision maker to look for other types of information. However, high efficiency ratings indicate that the sample information is almost as good as perfect information and that additional sources of information would not yield substantially better results.

4.6 Computing Branch Probabilities

In Section 4.5 the branch probabilities for the PDC decision tree chance nodes were specified in the problem description. No computations were required to determine these probabilities. In this section we show how **Bayes' theorem** can be used to compute branch probabilities for decision trees.

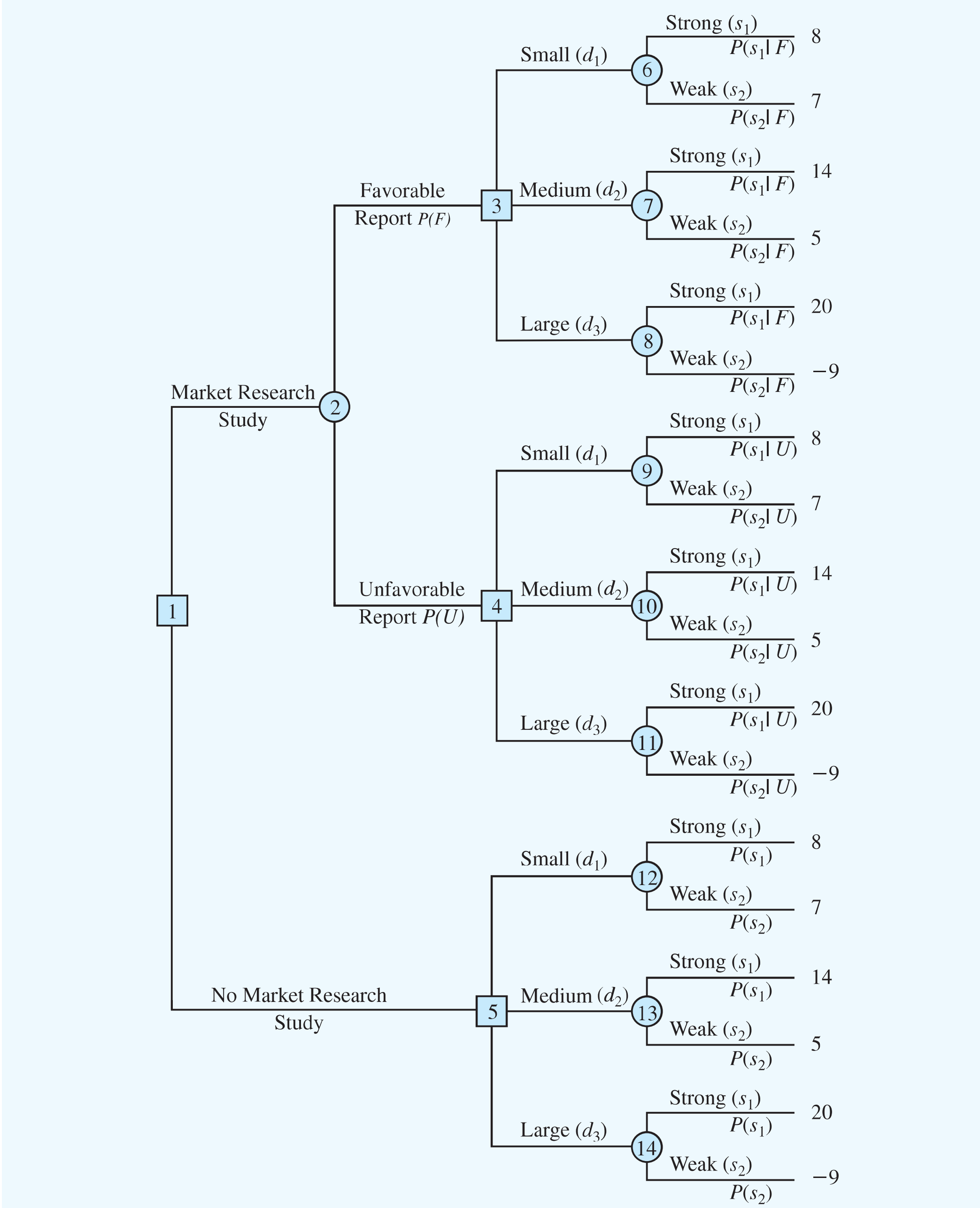
The PDC decision tree is shown again in Figure 4.15. Let

F = Favorable market research report
U = Unfavorable market research report
s_1 = Strong demand (state of nature 1)
s_2 = Weak demand (state of nature 2)

At chance node 2, we need to know the branch probabilities $P(F)$ and $P(U)$. At chance nodes 6, 7, and 8, we need to know the branch probabilities $P(s_1 \mid F)$, the probability of state of nature 1 given a favorable market research report, and $P(s_2 \mid F)$, the probability of state of nature 2 given a favorable market research report. $P(s_1 \mid F)$ and $P(s_2 \mid F)$ are referred to as *posterior probabilities* because they are conditional probabilities based on the outcome of the sample information. At chance nodes 9, 10, and 11, we need to know the branch probabilities $P(s_1 \mid U)$ and $P(s_2 \mid U)$; note that these are also posterior probabilities, denoting the probabilities of the two states of nature *given* that the market research report is unfavorable. Finally, at chance nodes 12, 13, and 14, we need the probabilities for the states of nature, $P(s_1)$ and $P(s_2)$, if the market research study is not undertaken.

In performing the probability computations, we need to know PDC's assessment of the probabilities for the two states of nature, $P(s_1)$ and $P(s_2)$, which are the prior probabilities as discussed earlier. In addition, we must know the **conditional probability** of the market

FIGURE 4.15 THE PDC DECISION TREE



research outcomes (the sample information) *given* each state of nature. For example, we need to know the conditional probability of a favorable market research report given that the state of nature is strong demand for the PDC project; note that this conditional probability of F given state of nature s_1 is written $P(F \mid s_1)$. To carry out the probability calculations, we will need conditional probabilities for all sample outcomes given all states of nature, that is, $P(F \mid s_1)$, $P(F \mid s_2)$, $P(U \mid s_1)$, and $P(U \mid s_2)$. In the PDC problem we assume that the following assessments are available for these conditional probabilities:

	Market Research	
State of Nature	**Favorable, *F***	**Unfavorable, *U***
Strong demand, s_1	$P(F \mid s_1) = 0.90$	$P(U \mid s_1) = 0.10$
Weak demand, s_2	$P(F \mid s_2) = 0.25$	$P(U \mid s_2) = 0.75$

A favorable market research report given that the state of nature is weak demand is often referred to as a "false positive," while the converse (an unfavorable market research report given that the state of nature is strong demand) is referred to as a "false negative."

Note that the preceding probability assessments provide a reasonable degree of confidence in the market research study. If the true state of nature is s_1, the probability of a favorable market research report is 0.90, and the probability of an unfavorable market research report is 0.10. If the true state of nature is s_2, the probability of a favorable market research report is 0.25, and the probability of an unfavorable market research report is 0.75. The reason for a 0.25 probability of a potentially misleading favorable market research report for state of nature s_2 is that when some potential buyers first hear about the new condominium project, their enthusiasm may lead them to overstate their real interest in it. A potential buyer's initial favorable response can change quickly to a "no thank you" when later faced with the reality of signing a purchase contract and making a down payment.

In the following discussion we present a tabular approach as a convenient method for carrying out the probability computations. The computations for the PDC problem based on a favorable market research report (F) are summarized in Table 4.7. The steps used to develop this table are as follows:

Step 1. In column 1 enter the states of nature. In column 2 enter the *prior probabilities* for the states of nature. In column 3 enter the *conditional probabilities* of a favorable market research report (F) given each state of nature.

Step 2. In column 4 compute the **joint probabilities** by multiplying the prior probability values in column 2 by the corresponding conditional probability values in column 3.

Step 3. Sum the joint probabilities in column 4 to obtain the probability of a favorable market research report, $P(F)$.

Step 4. Divide each joint probability in column 4 by $P(F) = 0.77$ to obtain the revised or *posterior probabilities,* $P(s_1 \mid F)$ and $P(s_2 \mid F)$.

Table 4.7 shows that the probability of obtaining a favorable market research report is $P(F) = 0.77$. In addition, $P(s_1 \mid F) = 0.94$ and $P(s_2 \mid F) = 0.06$. In particular, note that a

TABLE 4.7 BRANCH PROBABILITIES FOR THE PDC CONDOMINIUM PROJECT BASED ON A FAVORABLE MARKET RESEARCH REPORT

States of Nature s_j	Prior Probabilities $P(s_j)$	Conditional Probabilities $P(F \mid s_j)$	Joint Probabilities $P(F \cap s_j)$	Posterior Probabilities $P(s_j \mid F)$
s_1	0.8	0.90	0.72	0.94
s_2	0.2	0.25	0.05	0.06
	1.0		$P(F) = 0.77$	1.00

TABLE 4.8 BRANCH PROBABILITIES FOR THE PDC CONDOMINIUM PROJECT BASED ON AN UNFAVORABLE MARKET RESEARCH REPORT

States of Nature s_j	Prior Probabilities $P(s_j)$	Conditional Probabilities $P(U \mid s_j)$	Joint Probabilities $P(U \cup s_j)$	Posterior Probabilities $P(s_j \mid U)$
s_1	0.8	0.10	0.08	0.35
s_2	0.2	0.75	0.15	0.65
	1.0		$P(U) = 0.23$	1.00

favorable market research report will prompt a revised or posterior probability of 0.94 that the market demand of the condominium will be strong, s_1.

The tabular probability computation procedure must be repeated for each possible sample information outcome. Table 4.8 shows the computations of the branch probabilities of the PDC problem based on an unfavorable market research report. Note that the probability of obtaining an unfavorable market research report is $P(U) = 0.23$. If an unfavorable report is obtained, the posterior probability of a strong market demand, s_1, is 0.35 and of a weak market demand, s_2, is 0.65. The branch probabilities from Tables 4.7 and 4.8 were shown on the PDC decision tree in Figure 4.9.

Problem 23 asks you to compute the posterior probabilities.

The discussion in this section shows an underlying relationship between the probabilities on the various branches in a decision tree. To assume different prior probabilities, $P(s_1)$ and $P(s_2)$, without determining how these changes would alter $P(F)$ and $P(U)$, as well as the posterior probabilities $P(s_1 \mid F)$, $P(s_2 \mid F)$, $P(s_1 \mid U)$, and $P(s_2 \mid U)$, would be inappropriate.

The Q.M. in Action, Decision Analysis Helps Treat and Prevent Hepatitis B, discusses how medical researchers use posterior probability information and decision analysis to understand the risks and costs associated with treatment and screening procedures.

Q.M. *in* ACTION

*DECISION ANALYSIS HELPS TREAT AND PREVENT HEPATITIS B**

Hepatitis B is a viral disease that left untreated can lead to fatal liver conditions such as cirrhosis and cancer. The hepatitis B virus can be treated, and there exists a vaccine to prevent it. However, in order to make economically prudent allocations of their limited health care budgets, public health officials require analysis on the cost effectiveness (health benefit per dollar investment) of any potential health program. Unfortunately, since hepatitis B is a slow-progressing condition whose victims are often unaware of their potentially fatal infection, gathering data on the benefits of any public health policy addressing hepatitis B would take decades.

A multidisciplinary team consisting of management science researchers and a liver transplant surgeon from Stanford University applied decision analysis techniques to determine which combination of hepatitis B screening, treatment, and vaccination would be appropriate in the United States. Their decision tree contained the sequential decisions of: (1) whether or not to perform a blood test to screen an individual for a hepatitis B

*David W. Hutton, Margaret L. Brandeau, and Samuel K. So, "Doing Good With Good OR: Supporting Cost-Effective Hepatitis B Interventions," *Interfaces* May/June 2011 41:289–300.

(*continued*)

infection, (2) whether or not to treat infected individuals, and (3) whether or not to vaccinate a noninfected (or nonscreened) individual.

For each policy, composed of a sequence of screening, treatment, and vaccination decisions, the researchers utilized existing infection and treatment knowledge to model future disease progression. Implementing their decision model in an Excel spreadsheet, the researchers concluded that it is cost effective to screen adult Asian and Pacific Islanders so that infected individuals can be treated (these individuals are genetically at a high risk for hepatitis B infection). Although it is not cost effective to universally vaccinate all U.S. adult Asian and Pacific Islanders, it proves to be cost effective to vaccinate people in close contact with infected individuals. Influenced by these findings, the Centers for Disease Control updated its official policy in 2008 to recommend screening all adult Asian and Pacific Islanders and all adults in areas of intermediate (2 to 7%) hepatitis B prevalence.

Summary

Decision analysis can be used to determine a recommended decision alternative or an optimal decision strategy when a decision maker is faced with an uncertain and risk-filled pattern of future events. The goal of decision analysis is to identify the best decision alternative or the optimal decision strategy given information about the uncertain events and the possible consequences or payoffs. The uncertain future events are called chance events, and the outcomes of the chance events are called states of nature.

We showed how influence diagrams, payoff tables, and decision trees could be used to structure a decision problem and describe the relationships among the decisions, the chance events, and the consequences. We presented three approaches to decision making without probabilities: the optimistic approach, the conservative approach, and the minimax regret approach. When probability assessments are provided for the states of nature, the expected value approach can be used to identify the recommended decision alternative or decision strategy.

In cases where sample information about the chance events is available, a sequence of decisions has to be made. First we must decide whether to obtain the sample information. If the answer to this decision is yes, an optimal decision strategy based on the specific sample information must be developed. In this situation, decision trees and the expected value approach can be used to determine the optimal decision strategy.

Even though the expected value approach can be used to obtain a recommended decision alternative or optimal decision strategy, the payoff that actually occurs will usually have a value different from the expected value. A risk profile provides a probability distribution for the possible payoffs and can assist the decision maker in assessing the risks associated with different decision alternatives. Finally, sensitivity analysis can be conducted to determine the effect changes in the probabilities for the states of nature and changes in the values of the payoffs have on the recommended decision alternative.

Glossary

Decision alternatives Options available to the decision maker.

Chance event An uncertain future event affecting the consequence, or payoff, associated with a decision.

Consequence The result obtained when a decision alternative is chosen and a chance event occurs. A measure of the consequence is often called a payoff.

States of nature The possible outcomes for chance events that affect the payoff associated with a decision alternative.

Influence diagram A graphical device that shows the relationship among decisions, chance events, and consequences for a decision problem.

Node An intersection or junction point of an influence diagram or a decision tree.

Decision nodes Nodes indicating points where a decision is made.

Chance nodes Nodes indicating points where an uncertain event will occur.

Consequence nodes Nodes of an influence diagram indicating points where a payoff will occur.

Payoff A measure of the consequence of a decision such as profit, cost, or time. Each combination of a decision alternative and a state of nature has an associated payoff (consequence).

Payoff table A tabular representation of the payoffs for a decision problem.

Decision tree A graphical representation of the decision problem that shows the sequential nature of the decision-making process.

Branch Lines showing the alternatives from decision nodes and the outcomes from chance nodes.

Optimistic approach An approach to choosing a decision alternative without using probabilities. For a maximization problem, it leads to choosing the decision alternative corresponding to the largest payoff; for a minimization problem, it leads to choosing the decision alternative corresponding to the smallest payoff.

Conservative approach An approach to choosing a decision alternative without using probabilities. For a maximization problem, it leads to choosing the decision alternative that maximizes the minimum payoff; for a minimization problem, it leads to choosing the decision alternative that minimizes the maximum payoff.

Opportunity loss, or regret The amount of loss (lower profit or higher cost) from not making the best decision for each state of nature.

Minimax regret approach An approach to choosing a decision alternative without using probabilities. For each alternative, the maximum regret is computed, which leads to choosing the decision alternative that minimizes the maximum regret.

Expected value approach An approach to choosing a decision alternative based on the expected value of each decision alternative. The recommended decision alternative is the one that provides the best expected value.

Expected value (EV) For a chance node, it is the weighted average of the payoffs. The weights are the state-of-nature probabilities.

Expected value of perfect information (EVPI) The expected value of information that would tell the decision maker exactly which state of nature is going to occur (i.e., perfect information).

Risk analysis The study of the possible payoffs and probabilities associated with a decision alternative or a decision strategy.

Sensitivity analysis The study of how changes in the probability assessments for the states of nature or changes in the payoffs affect the recommended decision alternative.

Risk profile The probability distribution of the possible payoffs associated with a decision alternative or decision strategy.

Prior probabilities The probabilities of the states of nature prior to obtaining sample information.

Sample information New information obtained through research or experimentation that enables an updating or revision of the state-of-nature probabilities.

Posterior (revised) probabilities The probabilities of the states of nature after revising the prior probabilities based on sample information.

Decision strategy A strategy involving a sequence of decisions and chance outcomes to provide the optimal solution to a decision problem.
Expected value of sample information (EVSI) The difference between the expected value of an optimal strategy based on sample information and the "best" expected value without any sample information.
Efficiency The ratio of EVSI to EVPI as a percentage; perfect information is 100% efficient.
Bayes' theorem A theorem that enables the use of sample information to revise prior probabilities.
Conditional probabilities The probability of one event given the known outcome of a (possibly) related event.
Joint probabilities The probabilities of both sample information and a particular state of nature occurring simultaneously.

Problems

SELF test

1. The following payoff table shows profit for a decision analysis problem with two decision alternatives and three states of nature:

	State of Nature		
Decision Alternative	s_1	s_2	s_3
d_1	250	100	25
d_2	100	100	75

   a. Construct a decision tree for this problem.
   b. If the decision maker knows nothing about the probabilities of the three states of nature, what is the recommended decision using the optimistic, conservative, and minimax regret approaches?

2. Suppose that a decision maker faced with four decision alternatives and four states of nature develops the following profit payoff table:

	State of Nature			
Decision Alternative	s_1	s_2	s_3	s_4
d_1	14	9	10	5
d_2	11	10	8	7
d_3	9	10	10	11
d_4	8	10	11	13

   a. If the decision maker knows nothing about the probabilities of the four states of nature, what is the recommended decision using the optimistic, conservative, and minimax regret approaches?
   b. Which approach do you prefer? Explain. Is establishing the most appropriate approach before analyzing the problem important for the decision maker? Explain.
   c. Assume that the payoff table provides *cost* rather than profit payoffs. What is the recommended decision using the optimistic, conservative, and minimax regret approaches?

SELF test

3. Southland Corporation's decision to produce a new line of recreational products resulted in the need to construct either a small plant or a large plant. The best selection of plant size depends on how the marketplace reacts to the new product line. To conduct an analysis, marketing management has decided to view the possible long-run demand as low, medium, or high. The following payoff table shows the projected profit in millions of dollars:

	Long-Run Demand		
Plant Size	**Low**	**Medium**	**High**
Small	150	200	200
Large	50	200	500

   a. What is the decision to be made, and what is the chance event for Southland's problem?
   b. Construct an influence diagram.
   c. Construct a decision tree.
   d. Recommend a decision based on the use of the optimistic, conservative, and minimax regret approaches.

4. Amy Lloyd is interested in leasing a new Honda and has contacted three automobile dealers for pricing information. Each dealer offered Amy a closed-end 36-month lease with no down payment due at the time of signing. Each lease includes a monthly charge and a mileage allowance. Additional miles receive a surcharge on a per-mile basis. The monthly lease cost, the mileage allowance, and the cost for additional miles follow:

Dealer	Monthly Cost	Mileage Allowance	Cost per Additional Mile
Hepburn Honda	\$299	36,000	\$0.15
Midtown Motors	\$310	45,000	\$0.20
Hopkins Automotive	\$325	54,000	\$0.15

   Amy decided to choose the lease option that will minimize her total 36-month cost. The difficulty is that Amy is not sure how many miles she will drive over the next three years. For purposes of this decision, she believes it is reasonable to assume that she will drive 12,000 miles per year, 15,000 miles per year, or 18,000 miles per year. With this assumption Amy estimated her total costs for the three lease options. For example, she figures that the Hepburn Honda lease will cost her \$10,764 if she drives 12,000 miles per year, \$12,114 if she drives 15,000 miles per year, or \$13,464 if she drives 18,000 miles per year.

   a. What is the decision, and what is the chance event?
   b. Construct a payoff table for Amy's problem.
   c. If Amy has no idea which of the three mileage assumptions is most appropriate, what is the recommended decision (leasing option) using the optimistic, conservative, and minimax regret approaches?
   d. Suppose that the probabilities that Amy drives 12,000, 15,000, and 18,000 miles per year are 0.5, 0.4, and 0.1, respectively. What option should Amy choose using the expected value approach?
   e. Develop a risk profile for the decision selected in part (d). What is the most likely cost, and what is its probability?
   f. Suppose that after further consideration Amy concludes that the probabilities that she will drive 12,000, 15,000, and 18,000 miles per year are 0.3, 0.4, and 0.3, respectively. What decision should Amy make using the expected value approach?

SELF test

5. The following profit payoff table was presented in Problem 1. Suppose that the decision maker obtained the probability assessments $P(s_1) = 0.65$, $P(s_2) = 0.15$, and $P(s_3) = 0.20$. Use the expected value approach to determine the optimal decision.

	State of Nature		
Decision Alternative	s_1	s_2	s_3
d_1	250	100	25
d_2	100	100	75

6. Investment advisors estimated the stock market returns for four market segments: computers, financial, manufacturing, and pharmaceuticals. Annual return projections vary depending on whether the general economic conditions are improving, stable, or declining. The anticipated annual return percentages for each market segment under each economic condition are as follows:

	Economic Condition		
Market Segment	Improving	Stable	Declining
Computers	10	2	−4
Financial	8	5	−3
Manufacturing	6	4	−2
Pharmaceuticals	6	5	−1

a. Assume that an individual investor wants to select one market segment for a new investment. A forecast shows stable to declining economic conditions with the following probabilities: improving (0.2), stable (0.5), and declining (0.3). What is the preferred market segment for the investor, and what is the expected return percentage?
b. At a later date, a revised forecast shows a potential for an improvement in economic conditions. New probabilities are as follows: improving (0.4), stable (0.4), and declining (0.2). What is the preferred market segment for the investor based on these new probabilities? What is the expected return percentage?

SELF test

7. Hudson Corporation is considering three options for managing its data processing operation: continuing with its own staff, hiring an outside vendor to do the managing (referred to as *outsourcing*), or using a combination of its own staff and an outside vendor. The cost of the operation depends on future demand. The annual cost of each option (in thousands of dollars) depends on demand as follows:

	Demand		
Staffing Options	High	Medium	Low
Own staff	650	650	600
Outside vendor	900	600	300
Combination	800	650	500

a. If the demand probabilities are 0.2, 0.5, and 0.3, which decision alternative will minimize the expected cost of the data processing operation? What is the expected annual cost associated with that recommendation?
b. Construct a risk profile for the optimal decision in part (a). What is the probability of the cost exceeding $700,000?

SELF test

8. The following payoff table shows the profit for a decision problem with two states of nature and two decision alternatives:

Decision Alternative	State of Nature s_1	s_2
d_1	10	1
d_2	4	3

   a. Use graphical sensitivity analysis to determine the range of probabilities of state of nature s_1 for which each of the decision alternatives has the largest expected value.
   b. Suppose $P(s_1) = 0.2$ and $P(s_2) = 0.8$. What is the best decision using the expected value approach?
   c. Perform sensitivity analysis on the payoffs for decision alternative d_1. Assume the probabilities are as given in part (b), and find the range of payoffs under states of nature s_1 and s_2 that will keep the solution found in part (b) optimal. Is the solution more sensitive to the payoff under state of nature s_1 or s_2?

9. Myrtle Air Express decided to offer direct service from Cleveland to Myrtle Beach. Management must decide between a full-price service using the company's new fleet of jet aircraft and a discount service using smaller capacity commuter planes. It is clear that the best choice depends on the market reaction to the service Myrtle Air offers. Management developed estimates of the contribution to profit for each type of service based upon two possible levels of demand for service to Myrtle Beach: strong and weak. The following table shows the estimated quarterly profits (in thousands of dollars):

Service	Demand for Service Strong	Weak
Full price	$960	−$490
Discount	$670	$320

   a. What is the decision to be made, what is the chance event, and what is the consequence for this problem? How many decision alternatives are there? How many outcomes are there for the chance event?
   b. If nothing is known about the probabilities of the chance outcomes, what is the recommended decision using the optimistic, conservative, and minimax regret approaches?
   c. Suppose that management of Myrtle Air Express believes that the probability of strong demand is 0.7 and the probability of weak demand is 0.3. Use the expected value approach to determine an optimal decision.
   d. Suppose that the probability of strong demand is 0.8 and the probability of weak demand is 0.2. What is the optimal decision using the expected value approach?
   e. Use graphical sensitivity analysis to determine the range of demand probabilities for which each of the decision alternatives has the largest expected value.

10. Video Tech is considering marketing one of two new video games for the coming holiday season: Battle Pacific or Space Pirates. Battle Pacific is a unique game and appears to have no competition. Estimated profits (in thousands of dollars) under high, medium, and low demand are as follows:

Battle Pacific	Demand High	Medium	Low
Profit	$1000	$700	$300
Probability	0.2	0.5	0.3

Video Tech is optimistic about its Space Pirates game. However, the concern is that profitability will be affected by a competitor's introduction of a video game viewed as similar to Space Pirates. Estimated profits (in thousands of dollars) with and without competition are as follows:

Space Pirates with Competition	Demand High	Demand Medium	Demand Low
Profit	\$800	\$400	\$200
Probability	0.3	0.4	0.3

Space Pirates without Competition	Demand High	Demand Medium	Demand Low
Profit	\$1600	\$800	\$400
Probability	0.5	0.3	0.2

a. Develop a decision tree for the Video Tech problem.
b. For planning purposes, Video Tech believes there is a 0.6 probability that its competitor will produce a new game similar to Space Pirates. Given this probability of competition, the director of planning recommends marketing the Battle Pacific video game. Using expected value, what is your recommended decision?
c. Show a risk profile for your recommended decision.
d. Use sensitivity analysis to determine what the probability of competition for Space Pirates would have to be for you to change your recommended decision alternative.

11. For the Pittsburgh Development Corporation problem in Section 4.3, the decision alternative to build the large condominium complex was found to be optimal using the expected value approach. In Section 4.4 we conducted a sensitivity analysis for the payoffs associated with this decision alternative. We found that the large complex remained optimal as long as the payoff for the strong demand was greater than or equal to \$17.5 million and as long as the payoff for the weak demand was greater than or equal to −\$19 million.
    a. Consider the medium complex decision. How much could the payoff under strong demand increase and still keep decision alternative d_3 the optimal solution?
    b. Consider the small complex decision. How much could the payoff under strong demand increase and still keep decision alternative d_3 the optimal solution?

12. The distance from Potsdam to larger markets and limited air service have hindered the town in attracting new industry. Air Express, a major overnight delivery service, is considering establishing a regional distribution center in Potsdam. However, Air Express will not establish the center unless the length of the runway at the local airport is increased. Another candidate for new development is Diagnostic Research, Inc. (DRI), a leading producer of medical testing equipment. DRI is considering building a new manufacturing plant. Increasing the length of the runway is not a requirement for DRI, but the planning commission feels that doing so will help convince DRI to locate its new plant in Potsdam. Assuming that the town lengthens the runway, the Potsdam planning commission believes that the probabilities shown in the following table are applicable.

	DRI Plant	No DRI Plant
Air Express Center	0.30	0.10
No Air Express Center	0.40	0.20

For instance, the probability that Air Express will establish a distribution center and DRI will build a plant is 0.30.

The estimated annual revenue to the town, after deducting the cost of lengthening the runway, is as follows:

	DRI Plant	No DRI Plant
Air Express Center	$600,000	$150,000
No Air Express Center	$250,000	−$200,000

If the runway expansion project is not conducted, the planning commission assesses the probability that DRI will locate its new plant in Potsdam at 0.6; in this case, the estimated annual revenue to the town will be $450,000. If the runway expansion project is not conducted and DRI does not locate in Potsdam, the annual revenue will be $0 because no cost will have been incurred and no revenues will be forthcoming.

a. What is the decision to be made, what is the chance event, and what is the consequence?
b. Compute the expected annual revenue associated with the decision alternative to lengthen the runway.
c. Compute the expected annual revenue associated with the decision alternative not to lengthen the runway.
d. Should the town elect to lengthen the runway? Explain.
e. Suppose that the probabilities associated with lengthening the runway were as follows:

	DRI Plant	No DRI Plant
Air Express Center	0.40	0.10
No Air Express Center	0.30	0.20

What effect, if any, would this change in the probabilities have on the recommended decision?

13. Seneca Hill Winery recently purchased land for the purpose of establishing a new vineyard. Management is considering two varieties of white grapes for the new vineyard: Chardonnay and Riesling. The Chardonnay grapes would be used to produce a dry Chardonnay wine, and the Riesling grapes would be used to produce a semidry Riesling wine. It takes approximately four years from the time of planting before new grapes can be harvested. This length of time creates a great deal of uncertainty concerning future demand and makes the decision about the type of grapes to plant difficult. Three possibilities are being considered: Chardonnay grapes only; Riesling grapes only; and both Chardonnay and Riesling grapes. Seneca management decided that for planning purposes it would be adequate to consider only two demand possibilities for each type of wine: strong or weak. With two possibilities for each type of wine, it was necessary to assess four probabilities. With the help of some forecasts in industry publications, management made the following probability assessments:

	Riesling Demand	
Chardonnay Demand	Weak	Strong
Weak	0.05	0.50
Strong	0.25	0.20

Revenue projections show an annual contribution to profit of $20,000 if Seneca Hill only plants Chardonnay grapes and demand is weak for Chardonnay wine, and $70,000 if Seneca only plants Chardonnay grapes and demand is strong for Chardonnay wine. If Seneca only plants Riesling grapes, the annual profit projection is $25,000 if demand is weak for Riesling grapes and $45,000 if demand is strong for Riesling grapes. If Seneca plants both types of grapes, the annual profit projections are shown in the following table:

	Riesling Demand	
Chardonnay Demand	**Weak**	**Strong**
Weak	$22,000	$40,000
Strong	$26,000	$60,000

a. What is the decision to be made, what is the chance event, and what is the consequence? Identify the alternatives for the decisions and the possible outcomes for the chance events.
b. Develop a decision tree.
c. Use the expected value approach to recommend which alternative Seneca Hill Winery should follow in order to maximize expected annual profit.
d. Suppose management is concerned about the probability assessments when demand for Chardonnay wine is strong. Some believe it is likely for Riesling demand to also be strong in this case. Suppose the probability of strong demand for Chardonnay and weak demand for Riesling is 0.05 and that the probability of strong demand for Chardonnay and strong demand for Riesling is 0.40. How does this change the recommended decision? Assume that the probabilities when Chardonnay demand is weak are still 0.05 and 0.50.
e. Other members of the management team expect the Chardonnay market to become saturated at some point in the future, causing a fall in prices. Suppose that the annual profit projections fall to $50,000 when demand for Chardonnay is strong and Chardonnay grapes only are planted. Using the original probability assessments, determine how this change would affect the optimal decision.

SELF test

14. The following profit payoff table was presented in Problem 1:

	State of Nature		
Decision Alternative	s_1	s_2	s_3
d_1	250	100	25
d_2	100	100	75

The probabilities for the states of nature are $P(s_1) = 0.65$, $P(s_2) = 0.15$, and $P(s_3) = 0.20$.
a. What is the optimal decision strategy if perfect information were available?
b. What is the expected value for the decision strategy developed in part (a)?
c. Using the expected value approach, what is the recommended decision without perfect information? What is its expected value?
d. What is the expected value of perfect information?

15. The Lake Placid Town Council decided to build a new community center to be used for conventions, concerts, and other public events, but considerable controversy surrounds the appropriate size. Many influential citizens want a large center that would be a showcase for the area. But the mayor feels that if demand does not support such a center, the community will lose a large amount of money. To provide structure for the decision process, the council narrowed the building alternatives to three sizes: small, medium, and large. Everybody agreed that the critical factor in choosing the best size is the number of people who will want to use the new facility. A regional planning consultant provided demand estimates under three scenarios: worst case, base case, and best case. The worst-case scenario

corresponds to a situation in which tourism drops substantially; the base-case scenario corresponds to a situation in which Lake Placid continues to attract visitors at current levels; and the best-case scenario corresponds to a substantial increase in tourism. The consultant has provided probability assessments of 0.10, 0.60, and 0.30 for the worst-case, base-case, and best-case scenarios, respectively.

The town council suggested using net cash flow over a 5-year planning horizon as the criterion for deciding on the best size. The following projections of net cash flow (in thousands of dollars) for a 5-year planning horizon have been developed. All costs, including the consultant's fee, have been included.

	Demand Scenario		
Center Size	**Worst Case**	**Base Case**	**Best Case**
Small	400	500	660
Medium	−250	650	800
Large	−400	580	990

a. What decision should Lake Placid make using the expected value approach?
b. Construct risk profiles for the medium and large alternatives. Given the mayor's concern over the possibility of losing money and the result of part (a), which alternative would you recommend?
c. Compute the expected value of perfect information. Do you think it would be worth trying to obtain additional information concerning which scenario is likely to occur?
d. Suppose the probability of the worst-case scenario increases to 0.2, the probability of the base-case scenario decreases to 0.5, and the probability of the best-case scenario remains at 0.3. What effect, if any, would these changes have on the decision recommendation?
e. The consultant has suggested that an expenditure of $150,000 on a promotional campaign over the planning horizon will effectively reduce the probability of the worst-case scenario to zero. If the campaign can be expected to also increase the probability of the best-case scenario to 0.4, is it a good investment?

SELF test

16. Consider a variation of the PDC decision tree shown in Figure 4.9. The company must first decide whether to undertake the market research study. If the market research study is conducted, the outcome will either be favorable (F) or unfavorable (U). Assume there are only two decision alternatives, d_1 and d_2, and two states of nature, s_1 and s_2. The payoff table showing profit is as follows:

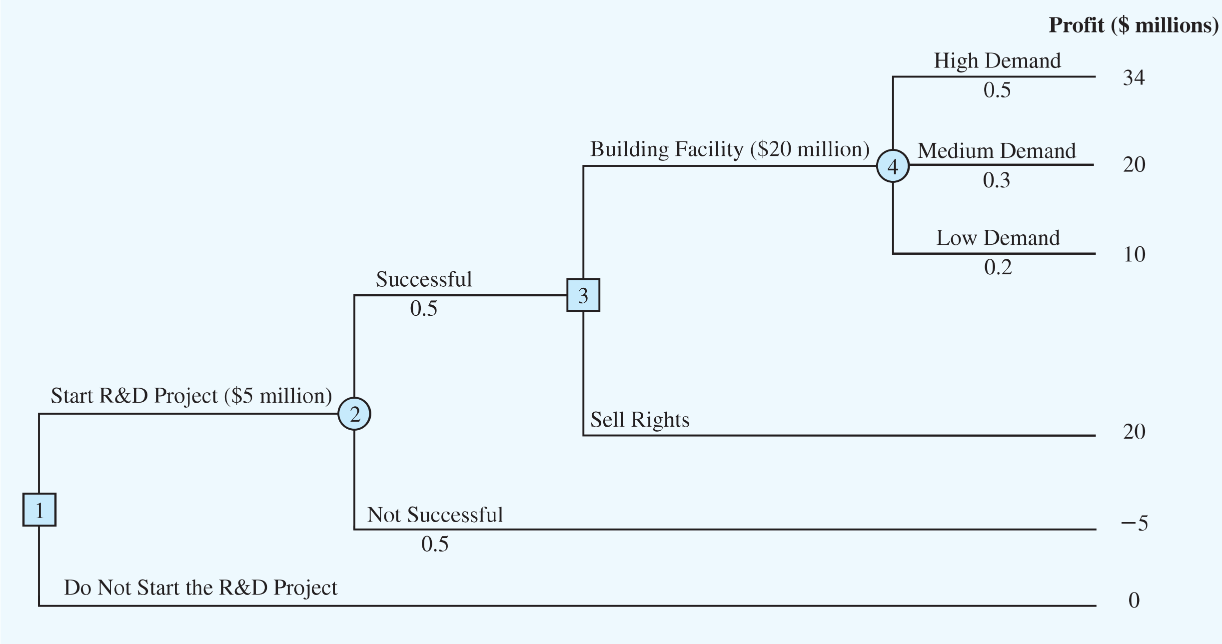
	State of Nature	
Decision Alternative	s_1	s_2
d_1	100	300
d_2	400	200

a. Show the decision tree.
b. Using the following probabilities, what is the optimal decision strategy?

$P(F) = 0.56$	$P(s_1 \mid F) = 0.57$	$P(s_1 \mid U) = 0.18$	$P(s_1) = 0.40$
$P(U) = 0.44$	$P(s_2 \mid F) = 0.43$	$P(s_2 \mid U) = 0.82$	$P(s_2) = 0.60$

17. Hemmingway, Inc., is considering a $5 million research and development (R&D) project. Profit projections appear promising, but Hemmingway's president is concerned because the probability that the R&D project will be successful is only 0.50. Furthermore, the president knows that even if the project is successful, it will require that the company build

FIGURE 4.16 DECISION TREE FOR HEMMINGWAY, INC.



a new production facility at a cost of $20 million in order to manufacture the product. If the facility is built, uncertainty remains about the demand and thus uncertainty about the profit that will be realized. Another option is that if the R&D project is successful, the company could sell the rights to the product for an estimated $25 million. Under this option, the company would not build the $20 million production facility.

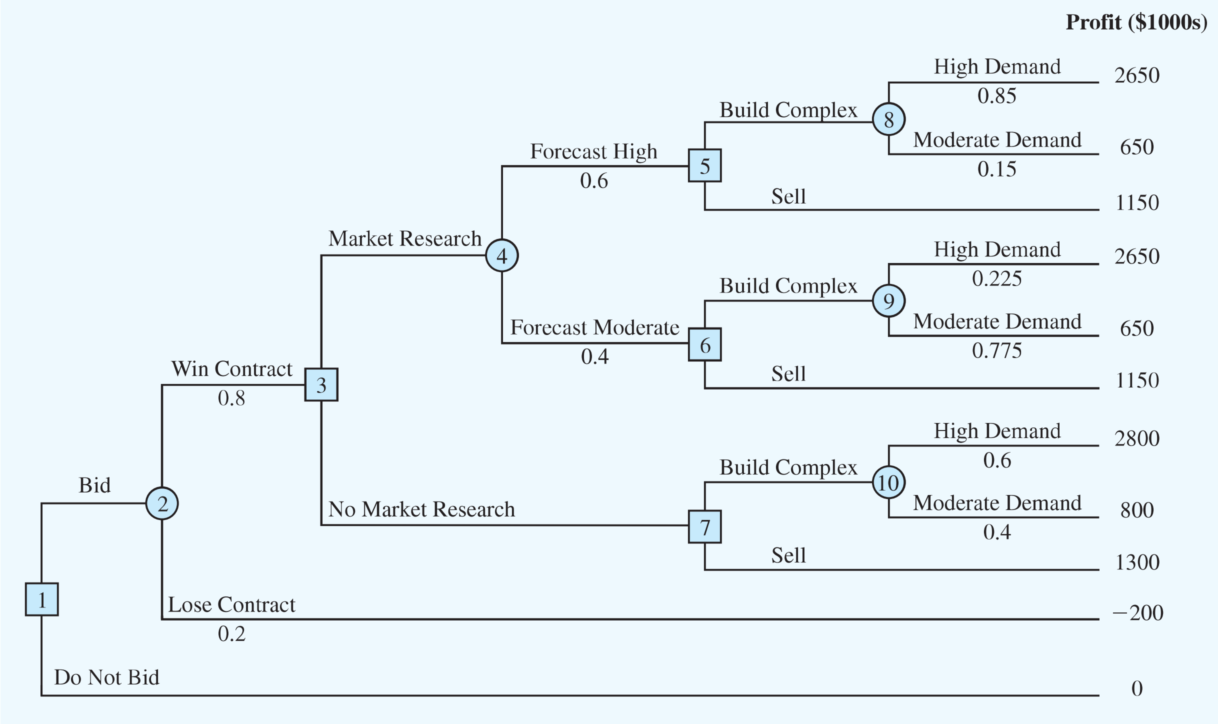
The decision tree is shown in Figure 4.16. The profit projection for each outcome is shown at the end of the branches. For example, the revenue projection for the high demand outcome is $59 million. However, the cost of the R&D project ($5 million) and the cost of the production facility ($20 million) show the profit of this outcome to be $59 − $5 − $20 = $34 million. Branch probabilities are also shown for the chance events.

a. Analyze the decision tree to determine whether the company should undertake the R&D project. If it does, and if the R&D project is successful, what should the company do? What is the expected value of your strategy?
b. What must the selling price be for the company to consider selling the rights to the product?
c. Develop a risk profile for the optimal strategy.

18. Dante Development Corporation is considering bidding on a contract for a new office building complex. Figure 4.17 shows the decision tree prepared by one of Dante's analysts. At node 1, the company must decide whether to bid on the contract. The cost of preparing the bid is $200,000. The upper branch from node 2 shows that the company has a 0.8 probability of winning the contract if it submits a bid. If the company wins the bid, it will have to pay $2,000,000 to become a partner in the project. Node 3 shows that the company will then consider doing a market research study to forecast demand for the office units prior to beginning construction. The cost of this study is $150,000. Node 4 is a chance node showing the possible outcomes of the market research study.

Nodes 5, 6, and 7 are similar in that they are the decision nodes for Dante to either build the office complex or sell the rights in the project to another developer. The decision to build the complex will result in an income of $5,000,000 if demand is high and $3,000,000 if

FIGURE 4.17 DECISION TREE FOR THE DANTE DEVELOPMENT CORPORATION



demand is moderate. If Dante chooses to sell its rights in the project to another developer, income from the sale is estimated to be $3,500,000. The probabilities shown at nodes 4, 8, and 9 are based on the projected outcomes of the market research study.

a. Verify Dante's profit projections shown at the ending branches of the decision tree by calculating the payoffs of $2,650,000 and $650,000 for first two outcomes.
b. What is the optimal decision strategy for Dante, and what is the expected profit for this project?
c. What would the cost of the market research study have to be before Dante would change its decision about the market research study?
d. Develop a risk profile for Dante.

19. Hale's TV Productions is considering producing a pilot for a comedy series in the hope of selling it to a major television network. The network may decide to reject the series, but it may also decide to purchase the rights to the series for either one or two years. At this point in time, Hale may either produce the pilot and wait for the network's decision or transfer the rights for the pilot and series to a competitor for $100,000. Hale's decision alternatives and profits (in thousands of dollars) are as follows:

	State of Nature		
Decision Alternative	**Reject, s_1**	**1 Year, s_2**	**2 Years, s_3**
Produce pilot, d_1	−100	50	150
Sell to competitor, d_2	100	100	100

The probabilities for the states of nature are $P(s_1) = 0.20$, $P(s_2) = 0.30$, and $P(s_3) = 0.50$. For a consulting fee of $5000, an agency will review the plans for the comedy series and indicate the overall chances of a favorable network reaction to the series. Assume that the agency review will result in a favorable (F) or an unfavorable (U) review and that the following probabilities are relevant:

$P(F) = 0.69$	$P(s_1 \mid F) = 0.09$	$P(s_1 \mid U) = 0.45$
$P(U) = 0.31$	$P(s_2 \mid F) = 0.26$	$P(s_2 \mid U) = 0.39$
	$P(s_3 \mid F) = 0.65$	$P(s_3 \mid U) = 0.16$

a. Construct a decision tree for this problem.
b. What is the recommended decision if the agency opinion is not used? What is the expected value?
c. What is the expected value of perfect information?
d. What is Hale's optimal decision strategy assuming the agency's information is used?
e. What is the expected value of the agency's information?
f. Is the agency's information worth the \$5000 fee? What is the maximum that Hale should be willing to pay for the information?
g. What is the recommended decision?

20. Embassy Publishing Company received a six-chapter manuscript for a new college textbook. The editor of the college division is familiar with the manuscript and estimated a 0.65 probability that the textbook will be successful. If successful, a profit of \$750,000 will be realized. If the company decides to publish the textbook and it is unsuccessful, a loss of \$250,000 will occur.

    Before making the decision to accept or reject the manuscript, the editor is considering sending the manuscript out for review. A review process provides either a favorable (F) or unfavorable (U) evaluation of the manuscript. Past experience with the review process suggests that probabilities $P(F) = 0.7$ and $P(U) = 0.3$ apply. Let s_1 = the textbook is successful, and s_2 = the textbook is unsuccessful. The editor's initial probabilities of s_1 and s_2 will be revised based on whether the review is favorable or unfavorable. The revised probabilities are as follows:

$P(s_1 \mid F) = 0.75$	$P(s_1 \mid U) = 0.417$
$P(s_2 \mid F) = 0.25$	$P(s_2 \mid U) = 0.583$

a. Construct a decision tree assuming that the company will first make the decision of whether to send the manuscript out for review and then make the decision to accept or reject the manuscript.
b. Analyze the decision tree to determine the optimal decision strategy for the publishing company.
c. If the manuscript review costs \$5000, what is your recommendation?
d. What is the expected value of perfect information? What does this EVPI suggest for the company?

21. A real estate investor has the opportunity to purchase land currently zoned residential. If the county board approves a request to rezone the property as commercial within the next year, the investor will be able to lease the land to a large discount firm that wants to open a new store on the property. However, if the zoning change is not approved, the investor will have to sell the property at a loss. Profits (in thousands of dollars) are shown in the following payoff table:

	State of Nature	
	Rezoning Approved	**Rezoning Not Approved**
Decision Alternative	s_1	s_2
Purchase, d_1	600	−200
Do not purchase, d_2	0	0

a. If the probability that the rezoning will be approved is 0.5, what decision is recommended? What is the expected profit?

b. The investor can purchase an option to buy the land. Under the option, the investor maintains the rights to purchase the land anytime during the next three months while learning more about possible resistance to the rezoning proposal from area residents. Probabilities are as follows:

Let H = High resistance to rezoning

L = Low resistance to rezoning

$$P(H) = 0.55 \qquad P(s_1 \mid H) = 0.18 \qquad P(s_2 \mid H) = 0.82$$
$$P(L) = 0.45 \qquad P(s_1 \mid L) = 0.89 \qquad P(s_2 \mid L) = 0.11$$

What is the optimal decision strategy if the investor uses the option period to learn more about the resistance from area residents before making the purchase decision?

c. If the option will cost the investor an additional $10,000, should the investor purchase the option? Why or why not? What is the maximum that the investor should be willing to pay for the option?

22. Lawson's Department Store faces a buying decision for a seasonal product for which demand can be high, medium, or low. The purchaser for Lawson's can order one, two, or three lots of the product before the season begins but cannot reorder later. Profit projections (in thousands of dollars) are shown.

	State of Nature		
Decision Alternative	**High Demand** s_1	**Medium Demand** s_2	**Low Demand** s_3
Order 1 lot, d_1	60	60	50
Order 2 lots, d_2	80	80	30
Order 3 lots, d_3	100	70	10

a. If the prior probabilities for the three states of nature are 0.3, 0.3, and 0.4, respectively, what is the recommended order quantity?

b. At each preseason sales meeting, the vice president of sales provides a personal opinion regarding potential demand for this product. Because of the vice president's enthusiasm and optimistic nature, the predictions of market conditions have always been either "excellent" (E) or "very good" (V). Probabilities are as follows:

$$P(E) = 0.70 \qquad P(s_1 \mid E) = 0.34 \qquad P(s_1 \mid V) = 0.20$$
$$P(V) = 0.30 \qquad P(s_2 \mid E) = 0.32 \qquad P(s_2 \mid V) = 0.26$$
$$P(s_3 \mid E) = 0.34 \qquad P(s_3 \mid V) = 0.54$$

What is the optimal decision strategy?

c. Use the efficiency of sample information and discuss whether the firm should consider a consulting expert who could provide independent forecasts of market conditions for the product.

SELF test

23. Suppose that you are given a decision situation with three possible states of nature: s_1, s_2, and s_3. The prior probabilities are $P(s_1) = 0.2$, $P(s_2) = 0.5$, and $P(s_3) = 0.3$. With sample information I, $P(I \mid s_1) = 0.1$, $P(I \mid s_2) = 0.05$, and $P(I \mid s_3) = 0.2$. Compute the revised or posterior probabilities: $P(s_1 \mid I)$, $P(s_2 \mid I)$, and $P(s_3 \mid I)$.

24. To save on expenses, Rona and Jerry agreed to form a carpool for traveling to and from work. Rona preferred to use the somewhat longer but more consistent Queen City Avenue. Although Jerry preferred the quicker expressway, he agreed with Rona that they should take Queen City Avenue if the expressway had a traffic jam. The following payoff table provides the one-way time estimate in minutes for traveling to or from work:

	State of Nature	
	Expressway Open	**Expressway Jammed**
Decision Alternative	s_1	s_2
Queen City Avenue, d_1	30	30
Expressway, d_2	25	45

Based on their experience with traffic problems, Rona and Jerry agreed on a 0.15 probability that the expressway would be jammed.

In addition, they agreed that weather seemed to affect the traffic conditions on the expressway. Let

$$C = \text{clear}$$
$$O = \text{overcast}$$
$$R = \text{rain}$$

The following conditional probabilities apply:

$$P(C \mid s_1) = 0.8 \qquad P(O \mid s_1) = 0.2 \qquad P(R \mid s_1) = 0.0$$
$$P(C \mid s_2) = 0.1 \qquad P(O \mid s_2) = 0.3 \qquad P(R \mid s_2) = 0.6$$

a. Use Bayes' theorem for probability revision to compute the probability of each weather condition and the conditional probability of the expressway open, s_1, or jammed, s_2, given each weather condition.
b. Show the decision tree for this problem.
c. What is the optimal decision strategy, and what is the expected travel time?

25. The Gorman Manufacturing Company must decide whether to manufacture a component part at its Milan, Michigan, plant or purchase the component part from a supplier. The resulting profit is dependent upon the demand for the product. The following payoff table shows the projected profit (in thousands of dollars):

		State of Nature	
	Low Demand	**Medium Demand**	**High Demand**
Decision Alternative	s_1	s_2	s_3
Manufacture, d_1	−20	40	100
Purchase, d_2	10	45	70

The state-of-nature probabilities are $P(s_1) = 0.35$, $P(s_2) = 0.35$, and $P(s_3) = 0.30$.

a. Use a decision tree to recommend a decision.
b. Use EVPI to determine whether Gorman should attempt to obtain a better estimate of demand.

c. A test market study of the potential demand for the product is expected to report either a favorable (F) or unfavorable (U) condition. The relevant conditional probabilities are as follows:

$$P(F \mid s_1) = 0.10 \qquad P(U \mid s_1) = 0.90$$
$$P(F \mid s_2) = 0.40 \qquad P(U \mid s_2) = 0.60$$
$$P(F \mid s_3) = 0.60 \qquad P(U \mid s_3) = 0.40$$

What is the probability that the market research report will be favorable?

d. What is Gorman's optimal decision strategy?

e. What is the expected value of the market research information?

f. What is the efficiency of the information?

Case Problem 1 Property Purchase Strategy

Glenn Foreman, president of Oceanview Development Corporation, is considering submitting a bid to purchase property that will be sold by sealed bid at a county tax foreclosure. Glenn's initial judgment is to submit a bid of \$5 million. Based on his experience, Glenn estimates that a bid of \$5 million will have a 0.2 probability of being the highest bid and securing the property for Oceanview. The current date is June 1. Sealed bids for the property must be submitted by August 15. The winning bid will be announced on September 1.

If Oceanview submits the highest bid and obtains the property, the firm plans to build and sell a complex of luxury condominiums. However, a complicating factor is that the property is currently zoned for single-family residences only. Glenn believes that a referendum could be placed on the voting ballot in time for the November election. Passage of the referendum would change the zoning of the property and permit construction of the condominiums.

The sealed-bid procedure requires the bid to be submitted with a certified check for 10% of the amount bid. If the bid is rejected, the deposit is refunded. If the bid is accepted, the deposit is the down payment for the property. However, if the bid is accepted and the bidder does not follow through with the purchase and meet the remainder of the financial obligation within six months, the deposit will be forfeited. In this case, the county will offer the property to the next highest bidder.

To determine whether Oceanview should submit the \$5 million bid, Glenn conducted some preliminary analysis. This preliminary work provided an assessment of 0.3 for the probability that the referendum for a zoning change will be approved and resulted in the following estimates of the costs and revenues that will be incurred if the condominiums are built:

Cost and Revenue Estimates

Revenue from condominium sales	\$15,000,000
Cost	
Property	\$5,000,000
Construction expenses	\$8,000,000

If Oceanview obtains the property and the zoning change is rejected in November, Glenn believes that the best option would be for the firm not to complete the purchase of the property. In this case, Oceanview would forfeit the 10% deposit that accompanied the bid.

Because the likelihood that the zoning referendum will be approved is such an important factor in the decision process, Glenn suggested that the firm hire a market research service to conduct a survey of voters. The survey would provide a better estimate of the likelihood that the referendum for a zoning change would be approved. The market research firm that Oceanview Development has worked with in the past has agreed to do the study for $15,000. The results of the study will be available August 1, so that Oceanview will have this information before the August 15 bid deadline. The results of the survey will be either a prediction that the zoning change will be approved or a prediction that the zoning change will be rejected. After considering the record of the market research service in previous studies conducted for Oceanview, Glenn developed the following probability estimates concerning the accuracy of the market research information:

$$P(A \mid s_1) = 0.9 \qquad P(N \mid s_1) = 0.1$$
$$P(A \mid s_2) = 0.2 \qquad P(N \mid s_2) = 0.8$$

where

A = prediction of zoning change approval
N = prediction that zoning change will not be approved
s_1 = the zoning change is approved by the voters
s_2 = the zoning change is rejected by the voters

Managerial Report

Perform an analysis of the problem facing the Oceanview Development Corporation, and prepare a report that summarizes your findings and recommendations. Include the following items in your report:

1. A decision tree that shows the logical sequence of the decision problem
2. A recommendation regarding what Oceanview should do if the market research information is not available
3. A decision strategy that Oceanview should follow if the market research is conducted
4. A recommendation as to whether Oceanview should employ the market research firm, along with the value of the information provided by the market research firm

Include the details of your analysis as an appendix to your report.

Case Problem 2 Lawsuit Defense Strategy

John Campbell, an employee of Manhattan Construction Company, claims to have injured his back as a result of a fall while repairing the roof at one of the Eastview apartment buildings. He filed a lawsuit against Doug Reynolds, the owner of Eastview Apartments, asking for damages of $1,500,000. John claims that the roof had rotten sections and that his fall could have been prevented if Mr. Reynolds had told Manhattan Construction about the problem. Mr. Reynolds notified his insurance company, Allied Insurance, of the lawsuit. Allied must defend Mr. Reynolds and decide what action to take regarding the lawsuit.

Some depositions and a series of discussions took place between both sides. As a result, John Campbell offered to accept a settlement of $750,000. Thus, one option is for Allied to pay John $750,000 to settle the claim. Allied is also considering making John a counteroffer of $400,000 in the hope that he will accept a lesser amount to avoid the time and cost of going to trial. Allied's preliminary investigation shows that John's case is strong; Allied is concerned that John may reject its counteroffer and request a jury trial. Allied's lawyers spent some time exploring John's likely reaction if they make a counteroffer of $400,000.

The lawyers concluded that it is adequate to consider three possible outcomes to represent John's possible reaction to a counteroffer of $400,000: (1) John will accept the counteroffer and the case will be closed; (2) John will reject the counteroffer and elect to have a jury decide the settlement amount; or (3) John will make a counteroffer to Allied of $600,000. If John does make a counteroffer, Allied decided that it will not make additional counteroffers. It will either accept John's counteroffer of $600,000 or go to trial.

If the case goes to a jury trial, Allied considers three outcomes possible: (1) the jury may reject John's claim and Allied will not be required to pay any damages; (2) the jury will find in favor of John and award him $750,000 in damages; or (3) the jury will conclude that John has a strong case and award him the full amount of $1,500,000.

Key considerations as Allied develops its strategy for disposing of the case are the probabilities associated with John's response to an Allied counteroffer of $400,000 and the probabilities associated with the three possible trial outcomes. Allied's lawyers believe that the probability that John will accept a counteroffer of $400,000 is 0.10, the probability that John will reject a counteroffer of $400,000 is 0.40, and the probability that John will, himself, make a counteroffer to Allied of $600,000 is 0.50. If the case goes to court, they believe that the probability that the jury will award John damages of $1,500,000 is 0.30, the probability that the jury will award John damages of $750,000 is 0.50, and the probability that the jury will award John nothing is 0.20.

Managerial Report

Perform an analysis of the problem facing Allied Insurance and prepare a report that summarizes your findings and recommendations. Be sure to include the following items:

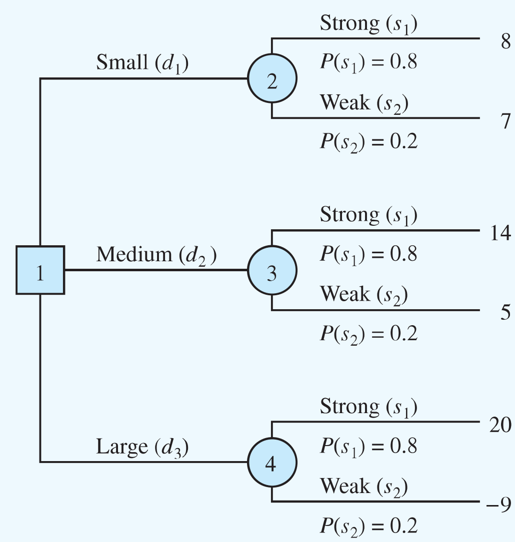
1. A decision tree
2. A recommendation regarding whether Allied should accept John's initial offer to settle the claim for $750,000
3. A decision strategy that Allied should follow if they decide to make John a counteroffer of $400,000
4. A risk profile for your recommended strategy

Appendix 4.1 Decision Analysis with TreePlan

TreePlan[2] is an Excel add-in that can be used to develop decision trees for decision analysis problems. The software package is provided at the website that accompanies this text. Instructions for installation and a manual containing additional information are also available at the website. In this appendix we show how to use TreePlan to build a decision tree and solve the PDC problem presented in Section 4.3. The decision tree for the PDC problem is shown in Figure 4.18.

[2]TreePlan was developed by Professor Michael R. Middleton at the University of San Francisco and modified for use by Professor James E. Smith at Duke University. The TreePlan website is *http://www.treeplan.com.*

FIGURE 4.18 PDC DECISION TREE



Getting Started: An Initial Decision Tree

We begin by assuming that TreePlan has been installed and an Excel workbook is open. To build a TreePlan version of the PDC decision tree, proceed as follows:

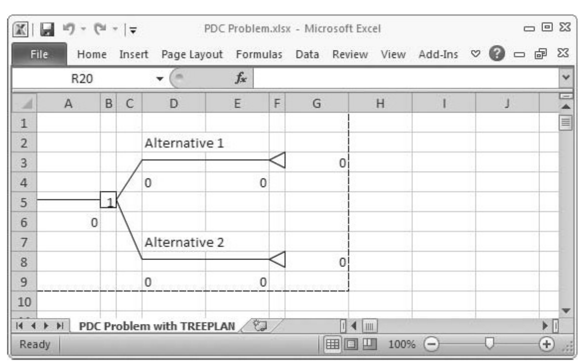
Step 1. Select cell A1

Step 2. Select the **Add-Ins tab** and choose **Decision Tree** from the **Menu Commands** group

Step 3. When the **TreePlan - New Tree** dialog box appears:
Click **New Tree**

A decision tree with one decision node and two branches (initially labeled as *Alternatives*) is provided in Figure 4.19.

FIGURE 4.19 A DECISION TREE WITH ONE DECISION NODE AND TWO BRANCHES DEVELOPED BY TREEPLAN



Adding a Branch

The PDC problem has three decision alternatives (small, medium, and large condominium complexes), so we must add another decision branch to the tree.

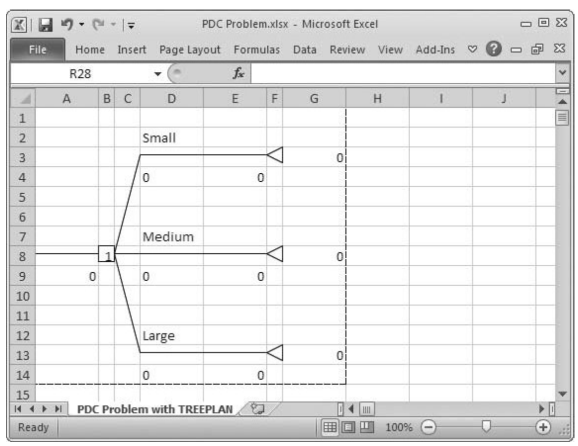
Step 1. Select cell B5
Step 2. Select the **Add-Ins tab** and choose **Decision Tree** from the **Menu Commands** group
Step 3. When the **TreePlan - Decision Node** dialog box appears:
Select **Add branch**
Click **OK**

A revised tree with three decision branches now appears in the Excel worksheet.

Naming the Decision Alternatives

The decision alternatives can be named by selecting the cells containing the labels Alternative 1, Alternative 2, and Alternative 3, and then entering the corresponding PDC names Small, Medium, and Large. After naming the alternatives, the PDC tree with three decision branches appears as shown in Figure 4.20.

FIGURE 4.20 TREEPLAN DECISION TREE WITH AN ADDITIONAL DECISION NODE AND LABELS ON THE BRANCHES



Adding Chance Nodes

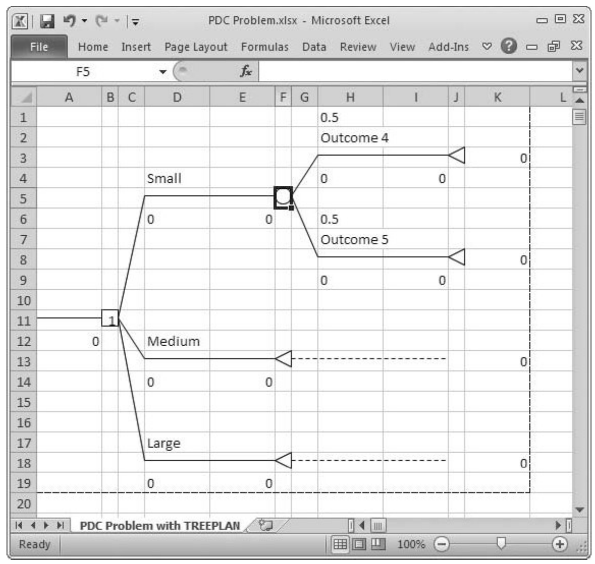
The chance event for the PDC problem is the demand for the condominiums, which may be either strong or weak. Thus, a chance node with two branches must be added at the end of each decision alternative branch. To add a chance node with two branches to the top decision alternative branch:

Step 1. Select cell F3
Step 2. Select the **Add-Ins tab** and choose **Decision Tree** from the **Menu Commands** group

Step 3. When the **TreePlan - Terminal Node** dialog box appears:
Select **Change to event node**
Select **Two** in the **Branches** section
Click **OK**

The tree now appears as shown in Figure 4.21.

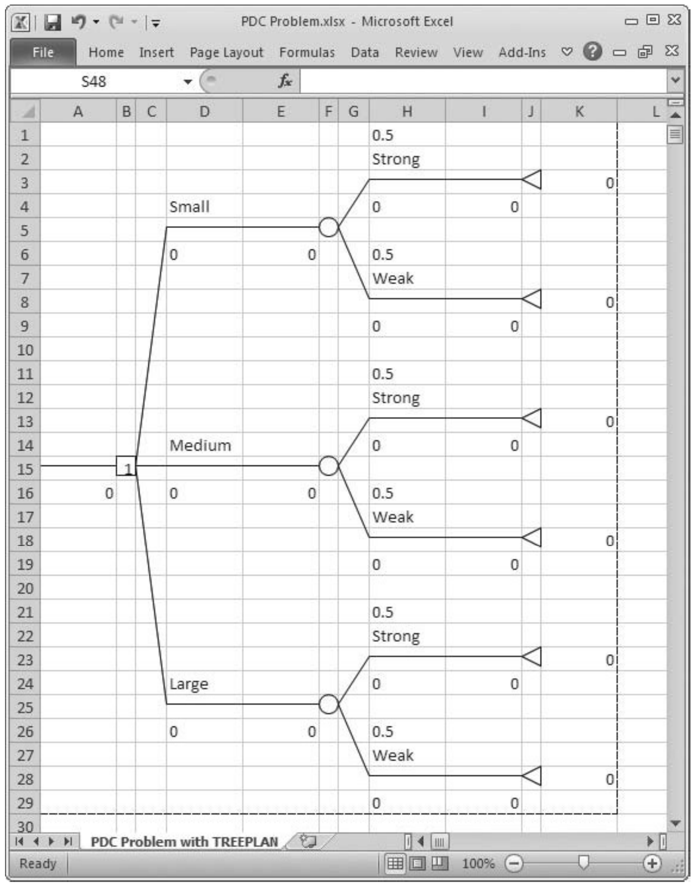
FIGURE 4.21 TREEPLAN DECISION TREE WITH A CHANCE NODE ADDED TO THE END OF THE FIRST DECISION BRANCH



We next select the cells containing Outcome 4 and Outcome 5 and rename them Strong and Weak to provide the proper names for the PDC states of nature. After doing so we can copy the subtree for the chance node in cell F5 to the other two decision branches to complete the structure of the PDC decision tree as follows:

Step 1. Select cell F5
Step 2. Select the **Add-Ins tab** and choose **Decision Tree** from the **Menu Commands** group
Step 3. When the **TreePlan Event** dialog box appears:
Select **Copy subtree**
Click **OK**
Step 4. Select cell F13
Step 5. Select the **Add-Ins tab** and choose **Decision Tree** from the **Menu Commands** group
Step 6. When the **TreePlan - Terminal Node** dialog box appears:
Select **Paste subtree**
Click **OK**

FIGURE 4.22 THE PDC DECISION TREE DEVELOPED BY TREEPLAN



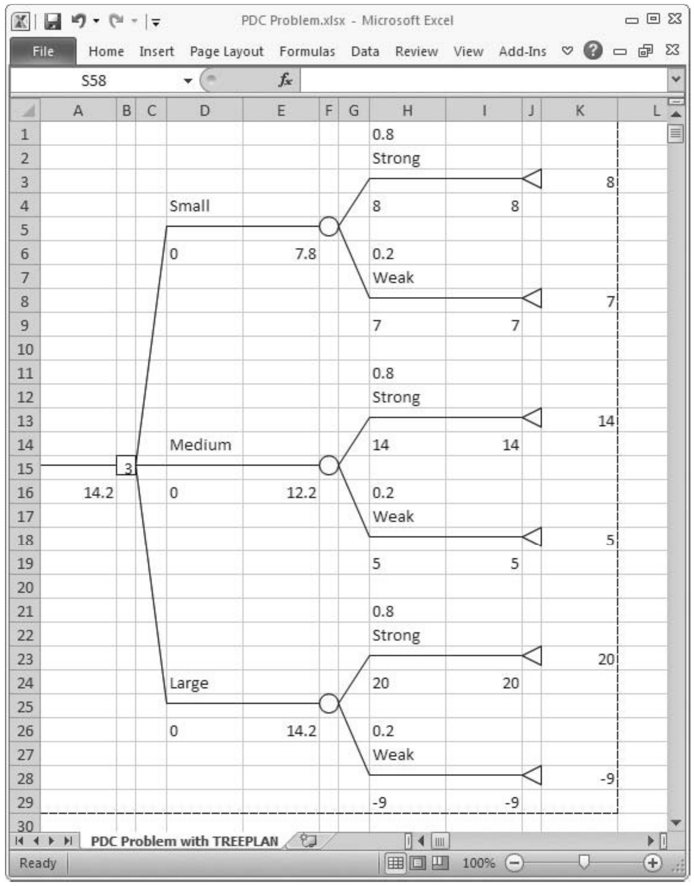
This copy/paste procedure places a chance node at the end of the Medium decision branch. Repeating the same copy/paste procedure for the Large decision branch completes the structure of the PDC decision tree as shown in Figure 4.22.

Inserting Probabilities and Payoffs

TreePlan provides the capability of inserting probabilities and payoffs into the decision tree. In Figure 4.19 we see that TreePlan automatically assigned an equal probability 0.5 to each of the chance outcomes. For PDC, the probability of strong demand is 0.8 and the probability of weak demand is 0.2. We can select cells H1, H6, H11, H16, H21, and H26 and insert the appropriate probabilities. The payoffs for the chance outcomes are inserted in cells H4, H9, H14, H19, H24, and H29. After inserting the PDC probabilities and payoffs, the PDC decision tree appears as shown in Figure 4.23.

Note that the payoffs also appear in the right-hand margin of the decision tree (column K in this problem). The payoffs in the right margin are computed by a formula that adds the payoffs on all of the branches leading to the associated terminal node. For the PDC problem,

FIGURE 4.23 THE PDC DECISION TREE WITH BRANCH PROBABILITIES AND PAYOFFS



no payoffs are associated with the decision alternatives branches so we leave the default values of zero in cells D6, D16, and D26. The PDC decision tree is now complete.

Interpreting the Result

When probabilities and payoffs are inserted, TreePlan automatically makes the backward pass computations necessary to determine the optimal solution. Optimal decisions are identified by the number in the corresponding decision node. In the PDC decision tree in Figure 4.20, cell B15 contains the decision node. Note that a 3 appears in this node, which tells us that decision alternative branch 3 provides the optimal decision. Thus, decision analysis recommends PDC construct the Large condominium complex. The expected value of this decision appears at the beginning of the tree in cell A16. Thus, we see the optimal expected value is \$14.2 million. The expected values of the other decision alternatives are displayed at the end of the corresponding decision branch. Thus, referring to cells E6 and E16, we see that the expected value of the Small complex is \$7.8 million and the expected value of the Medium complex is \$12.2 million.

Other Options

TreePlan defaults to a maximization objective. If you would like a minimization objective, follow these steps:

Step 1. Select **Decision Tree** from the **Menu Commands** group
Step 2. Select **Options**
Step 3. Choose **Minimize (costs)**
Click **OK**

In using a TreePlan decision tree, we can modify probabilities and payoffs and quickly observe the impact of the changes on the optimal solution. Using this "what if" type of sensitivity analysis, we can identify changes in probabilities and payoffs that would change the optimal decision. Also, because TreePlan is an Excel add-in, most of Excel's capabilities are available. For instance, we could use boldface to highlight the name of the optimal decision alternative on the final decision tree solution. A variety of other options TreePlan provides is contained in the TreePlan manual on the website that accompanies this text. Computer software packages such as TreePlan make it easier to do a thorough analysis of a decision problem.